# Electrostatic Interference Model of EHD Spraying from an Array of Cone Jets in Electrospray Micro-Thruster

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## ABSTRACT

Onset voltage plays a crucial role in the design of a spray microthruster. This paper presents an analytical electrostatic model to predict the behavior of onset voltage in an array of emitters. The basic idea of this method is to superimpose the electric potentials obtained from each individual emitter in an array of emitters. The results show that if one emitter operates and the other neighboring emitters are dry, the potential required for cone-jet spraying generally increases as the emitter spacing decreases (due to electrical shielding). However at very close spacing the potential can decrease. If all emitters operate at the same time, the phenomenon that even at very close spacing the onset voltage required for cone-jet spraying increases merely as the emitter spacing decreases.

Key Words: Onset voltage, Microthruster, Array of emitters, Interference

## 1. Introduction

Colloid thruster was firstly proposed and studied in 1960s, with a working principle of electrohydrodynamic (EHD) spraying. A colloid thruster consists of a source emitter and an extractor, between which a high voltage is applied (Fig. 1). The minimum applied voltage to start the electrospraying is called the onset voltage of a colloid thruster. Due to high efficiency and high specific impulse ability, the colloid thruster appears to be an alternative to the conventional ion engine. However, the thrust produced by the colloid thruster was not high enough to meet the needs of propulsions for spacecrafts in the past. To overcome a requirement of enhancement the thrust, arrays of emitters should be used to fabricate microcolloid thruster.

The behavior of onset voltage of an array of emitters is different from that of single emitter due to electrical shielding. So the electrical inteference between emitters should be investigated. The effort of this paper is to develope a model to predict the electrical interaction between emitters. The model estimates the onset voltage of an array of cone jets in microcolloid thruster design with respect to emitter spacing

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## 2. Overview of prior works

Computation of the potential of a source emitter and ground extractor system was done by some researchers. A simplest and most commonly used is Jones and Thong's equation

[1]. The author mentioned the problem as an infinite earthed plate at a distance $z_0$ from a semi-infinite line of charge. A solution of this system was obtained by the method of images. The resulting potential is

$$V(r,z)=\frac{-\sigma}{4\pi\varepsilon_0}\ln\left[\frac{\left\{r^2+(z_0-z)^2\right\}^{1/2}+(z_0-z)}{\left\{r^2+(z_0+z)^2\right\}^{1/2}+(z_0+z)}\right] \quad (1)$$

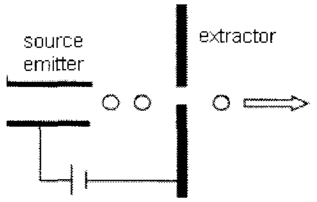


Fig. 1 Basic principle of colloid thruster

Recent work of Jijun Xiiong [2] shows the potential of a source emitter and grounded extractor system. The basic idea of this method is to simplify the source emitter and the extractor as two hyperboloids of two sheets of a set of equal potential surfaces. The contour of potential represents a set of hyperboloids of two sheets. Potential distribution of source emitter and grounded extractor system solved from Laplace's equation is presented as:

$$V=\frac{V_0}{\ln\left(2+2\frac{z_0}{r_c}-\sqrt{\left(2\frac{z_0}{r_c}+1\right)\left(3\frac{z_0}{r_c}+2\right)}\right)}\ln\frac{c-s}{c+s} \quad (2)$$

In summary, the preceding models give a clear view of electric potential distribution of single emitter of microcolloid thruster. These equations will be used as a part of electrostatic interference model presented next

## 3. Electrostatic Interference Model

### 3.1. Electrostatic Field Distribution

On the basic of the solution for the potential of a charged emitter in the presence of a grounded extractor obtained from Xiong's or Jones and Thong's models, we find, by inspection, the potential of an array of emitters in the presence of a grounded extractor using superposition theory. Here, electrostatic inteference model based on Xiong's solution is developed. The same procedure can be applied for electrostatic inteference model based on Jones and Thong's solution.

Let's consider a emitter order i (Fig. 2). Potential at a point A on the line from the apex of the meniscus to the electrode perpendicularly (z direction) will be contributed by potentials created by emitter i

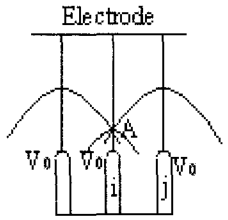


Fig. 2 Array of emitters of microthruster

and its neighboring emitters j ( j is from 1..N except i). Therefore the potential at a point on that line of nozzle i will be

$$V_i(z)=\sum_{j=1}^{N}A_jV_{ij}(z) \quad (3)$$

where $A_j$, a fraction of potential contribution of nozzle j on nozzle i, is unknown and $V_{ij}$, potential at point A contributed by nozzle j, is known variables for i=1 to i=N.

Equipotential surface equation of emitter order j in its coordinate is

$$\frac{x^2}{\lambda}+\frac{y^2}{\lambda}-\frac{z^2}{c^2-\lambda}=-1 \quad (4)$$

This surface passes through the point A. Therefore we have

$$\frac{d_{ij}^2}{\lambda}-\frac{z_A^2}{c^2-\lambda}=-1 \quad (5)$$

Where $d_{ij}$ is the distance between emitter order j to emitter order i. From Eq (5), we get

$$\lambda = \frac{c^2 - d_{ij}^2 - z_A^2 + \sqrt{\left(c^2 - d_{ij}^2 - z_A^2\right)^2 + 4d_{ij}^2 c^2}}{2} \quad (6)$$

Potential at point A contributed by nozzle j

$$V_{ij}(z) = C \ln \frac{c - \sqrt{c^2 - \lambda}}{c + \sqrt{c^2 - \lambda}} \quad (7)$$

Where

$$C = \frac{V_0}{\ln\left(2 + 2\frac{z_0}{r_c} - \sqrt{\left(2\frac{z_0}{r_c} + 1\right)\left(3\frac{z_0}{r_c} + 2\right)}\right)}$$

$$c = \sqrt{\frac{2r_c^2 z_0^2}{r_c^2 + 2r_c z_0} + z_0^2}$$

$$\lambda = \frac{c^2 - d_{ij}^2 - z_A^2 + \sqrt{\left(c^2 - d_{ij}^2 - z_A^2\right)^2 + 4d_{ij}^2 c^2}}{2}$$

Boundary conditions: $V_i = V_0$ (i=1..N) at $z_A = z_0$

Finally a set of linear equations is set up

$$V_0 = \sum_{j=1}^{N} A_j V_{ij}(z_0) \quad (8)$$

It is clearer if these equations are written in matrix form

$$\begin{bmatrix} V_0 \\ V_0 \\ \vdots \\ V_0 \\ V_0 \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1N-1} & V_{1N} \\ V_{21} & V_{22} & \cdots & V_{2N-1} & V_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ V_{N-11} & V_{N-12} & \cdots & V_{N-1N-1} & V_{N-1N-1} \\ V_{N1} & V_{N2} & \cdots & V_{NN-1} & V_{NN} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{N-1} \\ A_N \end{bmatrix} \quad (9)$$

By solving (9), the coefficients $A_j$, functions of $V_0$ and $V_{ij}$, are specified. As a result, the maximum electric field at the tip of the source emitter can be represented by

$$E_i(z_0) = -\frac{dV_i}{dz} = -\sum_{j=1}^{N} A_j \frac{dV_{ij}}{dz}(z_0) \quad (10)$$

Where

$$\frac{dV_{ij}}{dz}(z_0) = \frac{-4z_0 cC}{\sqrt{\left(d_{ij}^2 + z_0^2 - c^2\right)^2 + 4d_{ij}^2 c^2}\sqrt{2\left(c^2 + d_{ij}^2 + z_0^2 - \sqrt{\left(d_{ij}^2 + z_0^2 - c^2\right)^2 + 4d_{ij}^2 c^2}\right)}}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{N-1} \\ A_N \end{bmatrix} = V_0 \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1N-1} & V_{1N} \\ V_{21} & V_{22} & \cdots & V_{2N-1} & V_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ V_{N-11} & V_{N-12} & \cdots & V_{N-1N-1} & V_{N-1N-1} \\ V_{N1} & V_{N2} & \cdots & V_{NN-1} & V_{NN} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} = V_0 [B]_{Nx1}$$

### 3.2 Onset voltage

The onset voltage is estimated based on the balance between the surface tension and the force of the electrostatic field on the surface of droplet at the tip of source emitters of a microcolloid thruster. The starting electric field for jetting shown in [2] is

$$E = 2\sqrt{\frac{\delta}{r_c \varepsilon_0 \tan(a)}} \quad (11)$$

Substituting (11) into (10) yields the onset voltage of emitter i

$$V_0 = -2.\frac{\sqrt{\frac{\delta}{r_c \varepsilon_0 \tan(\alpha)}}}{\sum_{j=1}^{N} B_j \frac{dV_{ij}}{dz}(z_0)} \quad (12)$$

## 4. Results and discussion

The result of present model based on Jones and Thong's solution is compared to experimental data in J.D.Regele's work [3] shown in Fig.3. The EHD array used in the experiment, consist of four capillaries, each with a 0.8 mm ID and 1.27mm OD, arranged with three at the vertices of an equilateral triangle and the fourth located at the centroid. In that experiment, the only center capillary operates, its neighboring capillaries are dry. The experimental data show that the potential required for cone-jet spraying generally increases as the emitter spacing decreases (due to electrical shielding), but at very close spacing the potential can decrease. Physically, the roll-off of the atomization potential is due to the height difference between the liquid cone of the spraying capillary and the dry outer capillaries, which adds to the vertical component of the electric force acting on the fluid jet. In a real array, however, this effect will diminish since all capillaries spray, and the center cone-jet is electrically shielded by the outer cone-jet tips. The result of present model based on Jones and Thong's solution

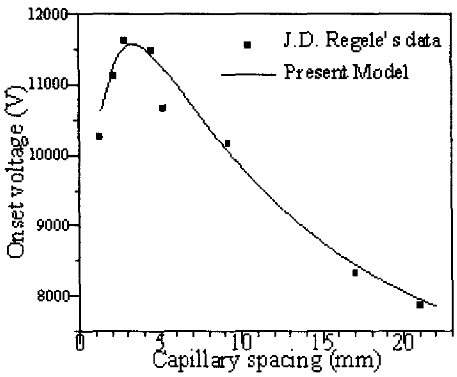


Fig. 3 Comparison of the onset voltage determined by present model and J.D. Regele's experimental data

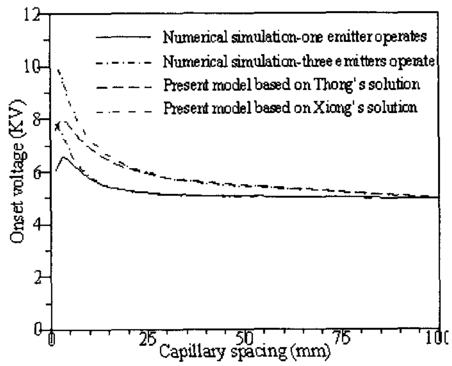


Fig. 4 Comparison of the onset voltage determined by numerical simulation and present model

shows an agreement with experiment data. In Jones and Thong's model, the effect of meniscus shape wasn't included. Therefore the same effect of the height difference between the liquid cone of the spraying capillary and the dry outer capillaries is observed when Jones and Thong's solution used in electrostatic interference model.

The results of present model are compared to numerical simulation based on finite element method. The EHD array used in numerical simulation and the present model consists of three emitters each with a 0.5 mm ID and 1.5mm OD arranged in a straight line (Fig. 4). The distance between the tip of emitters and grounded extractor is 7mm.

The results show that the present model based on Jones and Thong's solution can capture the trend of onset voltage if the emitter in question operates and the neighboring emitters are dry. Meanwhile the present model based on Xiong's solution can capture the trend of onset voltage which increases merely as the emitter spacing decreases if all three emitters operate. This is explained that Xiong solved the electrostatic problem of single emitter which included the effect of the shape of meniscus. So there is no difference between the liquid cones of the spraying emitters. But there exists a big difference in magnitude between numerical simulation and present model. Therefore, the present model should be evaluated by experiment which will be done in the future work.

## 5. Conclusion

We have presented an electrostatic inteference model for an array of emitters. The model incorporates most of the important feature of device geometry and thus provides a good description of device operation.

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