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상태-정합 성능 향상을 위한 지능형 디지털 재설계에 관한 새로운 충분조건들

New Sufficient Conditions to Intelligent Digital Redesign for the Improvement of State-Matching Performance

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Abstract

This paper presents new sufficient conditions to an intelligent digital redesign (IDR). The purpose of the IDR is to effectively convert an existing continuous-time fuzzy controller to an equivalent sampled-data fuzzy controller in the sense of the state-matching. The state-matching error between the closed-loop trajectories is carefully analyzed using the integral quadratic functional approach. The problem of designing the sampled-data fuzzy controller to minimize the state-matching error as well as to guarantee the stability is formulated and solved as the convex optimization problem with linear matrix inequality (LMI) constraints.

Key Words : Intelligent Digital Redesign (IDR), linear matrix inequality (LMI), sampled-data fuzzy control system, digital fuzzy control system.

1. Introduction

For the efficient DR of complex nonlinear systems, the so-called intelligent digital redesign (IDR) problem is addressed in [1]-[5]. Their common idea is to design the sampled-data fuzzy controller such that the optimal state-matching performance is achieved at any sampling points, in which a Takagi-Sugeno (T-S) fuzzy model is employed to describe the nonlinear systems. The concept of IDR was first introduced in [1]. The IDR problem in the presence of the parametric uncertainties was studied in [2]. The methods presented in [1], [2] allow to conveniently determine the sampled-data fuzzy gains without some complex optimization algorithms, but they are based on the local state-matching conditions, which are derived from the local dynamical behaviors of the continuous-time and the sampled-data fuzzy systems. To circumvent this weakness, the global state-matching conditions are presented in [3]-[5]. The result in [3] is based on the genetic algorithms that incur the computational burden. Very recently, in [4], [5], the convex optimization technique is employed to solve the IDR problem.

In this paper, the IDR problem of designing the sampled-data fuzzy controller that achieves the minimum (optimal), or near minimum, state-matching deviation as well as the stabilization is formulated and solved as the convex optimization problem with the linear matrix inequality (LMI) constraints. Specifically, the integral quadratic function is employed to analyze the state-matching deviation between the closed-loop trajectories carefully, which represents the energy of error responses between the continuous-time and the sampled-data fuzzy systems for all time. The estimation of state-matching deviation is given by using the linear matrix inequality (LMI) techniques.

2. Preliminaries

Consider the T-S fuzzy model described by

R_i: *If* $z_1(t)$ *is about* Γ_{i1} *and* $\cdots$ *and* $z_p(t)$ *is about* Γ_{ip}
then $\dot{x}(t) = A_i x(t) + B_i u(t)$

where $x(t) \in R^n$, $u(t) \in R^m$, $i \in \mathbf{I}_r = \{1,2,\cdots,r\}$, R_i denotes the i th fuzzy rule, $z_h(t)$ is the h th premise variable, and Γ_{ih}, $(i,h) \in \mathbf{I}_r \times \mathbf{I}_p$, is the fuzzy set of $z_h(t)$ in R_i. Using the center-average defuzzification, product inference, and singleton fuzzifier, its global dynamics is inferred as

$$\dot{x}(t) = A(\theta_t)x(t) + B(\theta_t)u(t) \qquad (1)$$

where $A(\theta_t) = \sum_{i=1}^{r}\theta_i(z(t))A_i$, $B(\theta_t) = \sum_{i=1}^{r}\theta_i(z(t))B_i$, $\theta_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$, $w_i(z(t)) = \prod_{h=1}^{p}\mu_{\Gamma_{ih}}(z_h(t))$, and $\mu_{\Gamma_{ih}}(z_h(t))$: $U_{z_h(t)} \subset R \to R_{[0,1]}$ is the membership function of $z_h(t)$ on the compact set $U_{z_h(t)}$. The following assumption is allowed in this paper:

Assumption 1 ([1-6]) The following two properties of $\theta_i(z(t))$: $U_z \to R_{[0,1]}$ hold:

A1) $\theta_i(z(t)) = \theta_i(z(kT))$ for $t \in [kT, kT+T)$, $k \in Z^+ = \{0,1,2,\cdots\}$, where T is sampling time.

A2) $\sum_{i=1}^{r}\theta_i(z(t)) = 1$.

Suppose that a continuous-time fuzzy controller for (1)

R_i : *If* $z_1(t)$ *is about* Γ_{i1} *and* $\cdots$ *and* $z_p(t)$ *is about* Γ_{ip}
then $u_c(t) = K_{ci} x_c(t)$

whose defuzzified output is given by

$$u_c(t) = K_c(\theta_t)x_c(t) \qquad (2)$$

has been predesigned such that the closed-loop system

$$\dot{x}_c(t) = (A(\theta_t) + B(\theta_t)K_c(\theta_t))x_c(t) \qquad (3)$$

is stable, where $K_c(\theta_t) = \sum_{i=1}^{r}\theta_i(z(t))K_{ci}$. The subscript '$c$' means the continuous-time control, while the subscript 's' will denote the sampled-data control in the sequel

In this paper, the desired sampled-data fuzzy controller takes the following form:

R_i : *If* $z_1(k)$ *is about* Γ_{i1} *and* $\cdots$ *and* $z_p(k)$ *is about* Γ_{ip}
then $u_s(t) = K_{si} x_s(k)$

where $u_s(t) = u_s(kT)$ is a piecewise constant control for $t \in [kT, kT+T)$, $k \in Z^+$. The overall sampled-data fuzzy controller is inferred as follows:

$$u_s(t) = K_s(\theta_k)x_s(k) \qquad (4)$$

where $K_s(\theta_k) = \sum_{i=1}^{r}\theta_i(z(k))K_{si}$. By interfacing an ideal sampler and a zero-order holder between (1) and (4), the closed-loop system is represented by

$$\dot{x}_s(t) = A(\theta_t)x_s(t) + B(\theta_t)K_s(\theta_k)x_s(k) \qquad (5)$$

for $t \in [kT, kT+T]$, $k \in Z^+$.

In the IDR, we will use the following discrete-time models:

$$x_c(k+1) = \widehat{G}(\theta_k)x_c(k) \qquad (6)$$

$$x_s(k+1) = (G(\theta_k) + H(\theta_k)K_s(\theta_k))x_s(k) \qquad (7)$$

where $\widehat{G}(\theta_k) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(k))\theta_j(z(k))\widehat{G}_{ij}$, $G(\theta_k) = \sum_{i=1}^{r}\theta_i(z(k))G_i$, $H(\theta_k) = \sum_{i=1}^{r}\theta_i(z(k))H_i$, $\widehat{G}_{ij} = e^{(A_i + B_i K_{cj})T}$, $G_i = e^{A_i T}$, and

$$H_i = \int_{kT}^{kT+T} e^{A_i(kT+T-\tau)} d\tau B_i$$

3. New Sufficient Conditions to IDR

The IDR problem is to design the sampled-data fuzzy controller (4) such that $x_s(t)$ of (5) closely matches $x_c(t)$ of (3) for all $t \in [0,\infty]$ under $x_c(0) = x_s(0)$. However, the previous works [1-5] attempted to minimize not $\| x_c(t) - x_s(t) \|$ for all $t \in [0,\infty]$ but $\| x_c(k) - x_s(k) \|$ at any $k \in Z^+$. As a result, their redesigned digital fuzzy controllers may produce the degraded state-matching for $t \in (kT, kT+T)$, $k \in Z^+$, even if $\| x_c(k) - x_s(k) \| = 0$ at any $k \in Z^+$. Therefore, we pursue the sampled-data fuzzy controller (4) such that the integral quadratic function $\int_0^\infty \| x_c(t) - x_s(t) \|^2 dt$ is minimized, or, equivalently, such that

$$Minimize_{K_{si}, i \in \mathbf{I}_r} \int_0^\infty \| x_c(t) - x_s(t) \|^2 dt \quad (8)$$

which describes the energy of error trajectory between (3) and (5). It is difficult to employ the condition (8) in IDR because computing (8) for infinite time space may be impossible.

Let $e(t) = x_c(t) - x_s(t)$, and rewrite (8) as

$$Minimize_{K_{si}, i \in I_r} \sum_{k=0}^{\infty} \int_{kT}^{kT+T} \| e(t) \|^2 dt = 0$$

It follows from (6) and (7) that

$$\begin{aligned}
\| e(k) \| &\le \| \widehat{G}(\theta_{k-1}) \| \| e(k-1) \| + \| \widehat{G}(\theta_{k-1}) - G(\theta_{k-1}) \\
&\quad - H(\theta_{k-1}) K_s(\theta_{k-1}) \| \| x_s(k-1) \| \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \| \| e(k-1) \| \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} - G_i - H_i K_{sj} \| \| x_s(k-1) \| \\
&\le (\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \|)^2 \| e(k-2) \| + (\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \\
&\quad - G_i - H_i K_{sj} \|)(\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \| + \sum_{i=1}^{r}\sum_{j=1}^{r} \| G_i \\
&\quad + H_i K_{sj} \|) \| x_s(k-2) \|
\end{aligned}$$

From the foregoing inequality, we have

$$\begin{aligned}
\| e(k) \| &\le (\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \|)^k \| e(0) \| + (\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} - G_i \\
&\quad - H_i K_{sj} \|)(\sum_{l=0}^{k-1}(\sum_{i=1}^{r}\sum_{j=1}^{r} \| \widehat{G}_{ij} \|)^{k-l-1} \\
&\quad \times (\sum_{i=1}^{r}\sum_{j=1}^{r} \| G_i + H_i K_{sj} \|)^l) \| x_s(0) \|
\end{aligned}$$

for any $k \in Z^+$. One can know that $\| e(k) \| = 0$ at any $k \in Z^+$ under $e(0) = 0$ if

$$\widehat{G}_{ij} - G_i - H_i K_{sj} = 0 \quad (9)$$

When the matrix equality (9) holds, (8) for $t \in [0,\infty)$ can be reduced to

$$\begin{aligned}
Minimize_{K_{si}, i \in \mathbf{I}_r} \int_0^T &\| (\widehat{G}(\theta_0) - G(\theta_0) - H(\theta_0) K_s(\theta_0)) \\
&\times x_s(0) \|^2 dt
\end{aligned} \quad (10)$$

for $t \in [0, T)$. From the fact that $\| (\widehat{G}(\theta_0) - G(\theta_0) - H(\theta_0)K_s(\theta_0)) x_s(0) \|^2 \le \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(0))\theta_j(z(0)) \times \| e^{(A_i + B_i K_{cj})t} - e^{A_i t} - \int_0^t e^{A_i \tau} d\tau B_i K_{sj}) x_s(0) \|^2$, we see that (10) can be converted to

$$minimize_{K_{si}, i \in I_r} \; L_{ij} + M_{ij} K_{sj} + K_{sj}^T M_{ij}^T + K_{sj}^T N_i K_{sj} \quad (11)$$

for $(i,j) \in I_r \times I_r$, where $L_{ij} = \int_0^T (e^{(A_i + B_i K_{cj})t} - e^{A_i t})^T \times (e^{(A_i + B_i K_{cj})t} - e^{A_i t}) dt$, $M_{ij} = \int_0^T (e^{(A_i + B_i K_{cj})t} - e^{A_i t})^T (\int_0^t e^{A_i \tau} d\tau) dt B_i$, and $N_i = \int_0^T (\int_0^t e^{A_i \tau} d\tau B_i)^T \times (\int_0^t e^{A_i \tau} d\tau B_i) dt$.

Problem 1 Find the sampled-data gains K_{si}, $i \in I_r$ such that the followings are satisfied:

■Minimize $\widehat{\gamma}$ subject to $\| \widehat{G}_{ij} - G_i - H_i K_{sj} \| \le \widehat{\gamma}$ and $L_{ij} + M_{ij} K_{sj} + K_{sj}^T M_{ij}^T + K_{sj}^T N_i K_{sj} \le \widehat{\gamma}$, $\forall (i,j)_r \times I_r$, where K_{si} is variable.

■The sampled-data closed-loop system (5) is globally exponentially stable in the sense of Lyapunov.

Theorem 1 *The main result of this section is summarized as follows: Suppose that the matrices* $Q = Q^T \succ 0$ *and* F_i *are optimal solutions to*

$$\text{minimize}_{F_i,Q} \quad \gamma \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma Q & (\bullet)^T & (\bullet)^T & (\bullet)^T \\ \widehat{G}_{\delta ij}Q - G_{\delta i}Q - H_{\delta i}F_j & -\gamma I & (\bullet)^T & (\bullet)^T \\ \omega^{\frac{1}{2}}W_{ij}^{(11)}Q + \omega^{\frac{1}{2}}W_{ij}^{(12)}F_j & 0 & -\gamma I & (\bullet)^T \\ \omega^{\frac{1}{2}}W_{ij}^{(21)}Q + \omega^{\frac{1}{2}}W_{ij}^{(22)}F_j & 0 & 0 & -\gamma I \end{bmatrix} \prec 0$$

$$\begin{bmatrix} \left(\begin{array}{l} \frac{G_{\delta i}Q + H_{\delta i}F_j + G_{\delta j}Q + H_{\delta j}F_i}{2} \\ +(\frac{G_{\delta i}Q + H_{\delta i}F_j + G_{\delta j}Q + H_{\delta j}F_i}{2})^T \end{array} \right) + X_{ij} & (\bullet)^T \\ \frac{T^{\frac{1}{2}}G_{\delta i}Q + T^{\frac{1}{2}}H_{\delta i}F_j + T^{\frac{1}{2}}G_{\delta j}Q + T^{\frac{1}{2}}H_{\delta j}F_i}{2} & -Q \end{bmatrix} \prec 0$$

$$1 \le i \le j \le r$$

$$\left[X_{ij}\right]_{r\times r} \succ 0 \qquad 1 \le i \le j \le r$$

for all $(i,j) \in I_r \times I_r$. *Then the state* $x_s(t)$ *of the sampled-data closed-loop fuzzy system (5) closely matches the state* $x_c(t)$ *of the continuous-time closed-loop fuzzy system (3) for all* $t \in [0,\infty)$, *and (7) is globally exponentially stable. When the above minimization problem is feasible, the sampled-data fuzzy gains are given by* $K_{si} = F_i Q^{-1}$, $i \in I_r$.

Proof: The proof is omitted because of the lack of space.

4. Closing Remarks

In this paper, an effective sampled-data controller have developed for the fuzzy systems. The design is based on some advanced IDR techniques via an integral quadratic functional approach. One advantage of this new IDR method is that a good state-matching performance at inter-sampling points as well as at sampling points is achieved.

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