Direct Simulations of Aerodynamic Sounds by the Finite Difference and Finite Volume Lattice Boltzmann Methods

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Abstract: Direct simulations of aerodynamic sound, especially sound emitted by rapidly rotating elliptic cylinder by the finite difference lattice Boltzmann method (FDLBM). Effect of pile-fabrics for noise reduction is also studied by the finite volume LBM (FVLBM) using an unstructured grid. Second order time integration and third order upwind scheme are shown to be enough for these simulations. Sound sources are detected to be doublets for both cases. For the elliptic cylinder, the doublet is generated in the interaction between the vortex and the edge. For the circular cylinders, they are generated synchronizing with the Karman vortex street, and it is also shown that the pile-fabrics covering the surface of the cylinder reduces the strength of the source.

Introduction

The lattice Boltzmann method (LBM, Tsutahara 1999 and Succi 2002) is now a very powerful tool of computational fluid dynamics (CFD). This method is different from ordinary Navier-Stokes equations based CFD methods, and is based on the particle motions. The conventional lattice Boltzmann method is a finite difference form of the discrete BGK equation (partial differential equation), and the discrete BGK equation can be solved by FDM (Cao et.al 1997) or FVM (Ubertini 2003).

On the other hand, aero-acoustics is an important branch of the fluid dynamics. But direct simulations of the sound waves are still hard task by the following reasons: (1) the sound pressure is much smaller ($10^{-3}-10^{-4}$) than whole pressure variation and then the adequate accuracy is necessary. (2) The acoustic field spreads in very large region and the load to computers is still heavy. Now the direct simulation of sound waves on the base of the Navier-Stokes equations uses schemes of at least fifth-order accuracy in space and of fourth-order accuracy in time (Lele 1997, Inoue et.al 2002).

Discrete BGK equation

The governing equation of the FVLBM is the following discrete BGK equation (Tsutahara et.a.2002)

$$\frac{\partial f_i}{\partial t}+\mathbf{c}_i\cdot\nabla f_i^{*}=-\frac{1}{\phi}\left(f_i-f_i^{(0)}\right) \tag{1}$$

$$f_i^{*}=f_i-\frac{a}{\phi}\left(f_i-f_i^{(0)}\right) \tag{2}$$

where f_i is the particle distribution function at time t and position **r**, subscripts i denotes the direction of particle translation and the term on the RHS in Eq.(1) represents the collision of the particles, in which $f_i^{(0)}$ is the local equilibrium distribution function. ϕ is called the single relaxation time factor, and a in Eq.(2) is a positive constant and denote the negative viscosity.

The macroscopic variables, the density ρ, the flow velocity $\mathbf{u}$, and the internal energy e are given as

$$\rho=\sum_i f_i=\sum_i f_i^{(0)} \tag{3}$$

$$\rho\mathbf{u}=\sum_i f_i\mathbf{c}_i=\sum_i f_i^{(0)}\mathbf{c}_i \tag{4}$$

$$\frac{1}{2}\rho u^2+\rho e=\sum_i\frac{1}{2}f_i c_i^{\,2}=\sum_i\frac{1}{2}f_i^{(0)}c_i^2 \tag{5}$$

These variables are defined by the distribution function and, at the same time, by the local distribution function. This means that the mass, the momentum and the energy are conserved at the collision stage. For two-dimensional compressible flows, the D2Q21 velocity model is used and the velocity set is shown in Table 1.

Table.1 Velocity set in 2D21V model

i	Velocity vector	$\lvert c\rvert$
1	(0, 0)	0
2-5	(1, 0), (0, 1), (-1, 0), (0,-1)	1
6-9	(2, 0), (0, 2), (-2, 0), (0,-2)	2
10-13	(3, 0), (0, 3), (-3, 0), (0,-3)	3
14-17	(1, 1), (-1, 1), (-1,-1), (1,-1)	$\sqrt{2}$
18-21	(2, 2), (-2, 2), (-2,-2), (2,-2)	$2\sqrt{2}$

Direct simulation of sound emitted from a rapidly rotating elliptic cylinder

The discrete BGK equation is solved by the finite difference method and the ALE (Arbitrary Lagrangian Eulerian formulation) method is applied for simulation of sound emitted from a rapidly rotating elliptic cylinder.

The third order upwind scheme for space and the second order Runge-Kutta method for time integration are employed. The inner rotating grid is boundary-fitted coordinates and the outer still grid is cylindrical coordinates. The number of the grid is 421 x 301. The Reynolds number based on the chord length and the edge speed is 10,000. The Mach number of the edge speed is 0.2.

The sound pressure field and pressure near the cylinder are shown in Figs.1 and 2. The sound source is clearly shown to be a doublet created at the edge of the elliptic cylinder when the edge pass through the vortex emitted from itself.

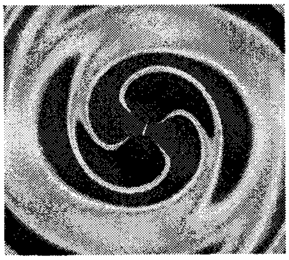

Fig.1 Sound pressure field at non-dimensional time t=6.24. Red part is high-pressure region and blue part is low pressure region

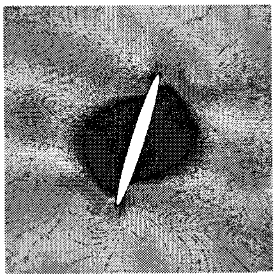

Fig.2 Pressure field near the cylinder at the same time as above. Doublets are clearly shown near the edges and they are the sound sources.

Simulation of effect of pile-fabrics for noise reduction

Pile-fabrics covering solid surface are well known to reduce aerodynamic sound (Nishimura et al. 1999). In this section, direct simulations of the Aeolian tone emitted from smooth circular cylinder and that covered by pile-fabrics are performed by the finite volume lattice Boltzmann method (FVLBM) using unstructured grids. The fluid dynamic effect of the pile-fabrics is realized by increasing the viscosity ten times of the fluid near the cylinder. The triangular cells are used and the number of cells is 23688.

Emitted sound is reduced remarkably as shown in Fig.3. The pressure fields in the vicinity of the smooth cylinder and that covered by the pile-fabrics are shown in Fig.4, and it is seen that the doublet (sound source) becomes weaker by the effect of the pile-fabrics.

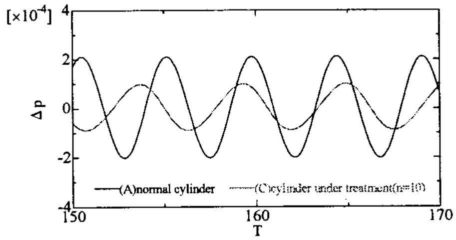

Fig.3 Comparison between the sound pressures for smooth cylinder and that covered by pile-fabrics.

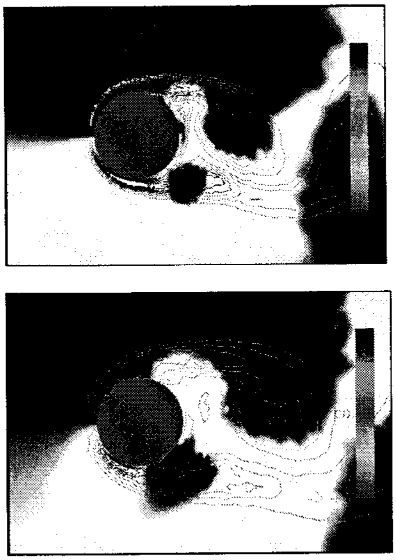

Fig.4 Comparison of the sound sources for smooth cylinder (above) and that for cylinder covered by pile-fabrics (below).

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