Ring-shaped Sound Focusing using Wavenumber Domain Matching
파수영역매칭을 통한 링 형상의 음향집적공간 형성

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Key Words : shaped-sound focusing (형상음향집적), wavenumber domain matching (파수영역매칭), acoustically bright shape(음향학적으로 밝은 형상)

ABSTRACT

Shaped Sound Focusing is defined as the generation of acoustically bright shape in space using multiple sources. The acoustically bright shape is a spatially focused region with relatively high acoustic potential energy level. In view of the energy transfer, acoustical focusing is essential because acoustic energy is very small to use other type of energy. Practically, focused sound shape control not a point is meaningful because there are so many needs to enlarge the focal region especially in clinical uses and others. If focused sound shape can be controlled, it offers various kinds of solutions for clinical uses and others because a regional focusing is essentially needed to reduce a treatment time and enhance the performance of transducers. For making the shaped-sound field, control variables, such as a number of sources, excitation frequency, source positioning, etc., should be taken according to geometrical sound shape. To verify these relations between them, wavenumber domain matching method is suggested because wavenumber spectrum can provide the information of control variables of sources. In this paper, the procedures of shaped sound focusing using wavenumber domain matching and relations between control variables and geometrical sound shape are covered in case of an acoustical ring.

1. Introduction

Study on sound focusing has been done by several people in various fields and there are also a lot of applications. In ultrasonic field, especially, several researches on sound focusing, time reversal(1) and inverse filtering(2), has been done actively for clinical applications such as lithotripsy. Sound wave has a goodness that it can penetrate into a body non-invasively. Recently, using the focused ultrasonic waves, ablation of tumor and hemostasis of internal bleeding are possible(3). Even though devices for this application are commercialized, the enlargement of focused area is very needed to treat patients safely and efficiently because these techniques are based on the only point focusing. The researches for sound shape control, utilizing the concept of acoustic brightness and contrast control was firstly proposed by Choi and Kim in 2002(4). These methods are very powerful and practical to find the optimal solution for focused region control. However, it is hard to interpret the relations between geometrical sound shape and control variables such as locating proper source positions, excitation frequency, and effective number of sources. To understand these relations more, it is needed to see them in wavenumber domain. Therefore, wavenumber domain matching method is proposed because the wavenumber spectrum can give clues to examine these relations.

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2. Theoretical Background

2.1 Wavenumber domain matching

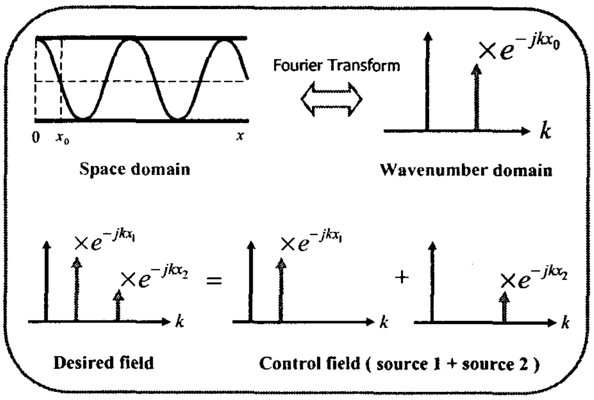


Fig. 1. Spatial Fourier transform and wavenumber domain matching. (x_i : position or initial phase component of a source)

One-to-one correspondence between space domain and wavenumber domain is the definite property of Fourier transform because of its existence of inverse Fourier transform (Fig. 1(top)). In other words, if the sound field affected by a source is considered, this sound field in space has a unique spectrum in wavenumber domain and the phase of wavenumber spectrum gives the information about where the source is positioned. Namely, this means that it is possible to express the wavenumber spectrum of the sound field wanted to make (desired field) as the summation of each wavenumber spectrum of control sources (control field) through the phase and magnitude control of each source used (Fig. 1. (bottom)). Wavenumber domain matching is started from this simple idea.

2.2 Shaped Sound Focusing using wavenumber domain matching

Shaped Sound Focusing is defined as the generation of acoustically bright shape in space using multiple sources as shown in Fig. 2. Wavenumber domain matching is based on wavenumber domain approach so that it can give the information which cannot obtain from the acoustic brightness and contrast control. In fact, it is hard to compare between two approaches because they have mutually complementary relations. Wavenumber domain matching can make it possible to focus the sound energy efficiently on the control region.

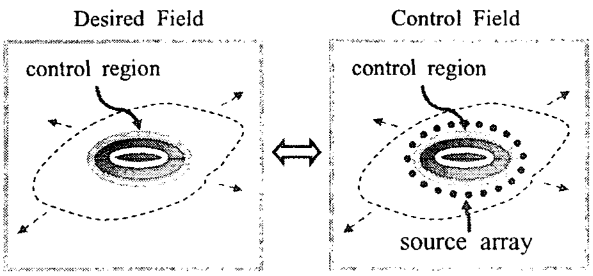


Fig. 2. Illustration of shaped sound focusing

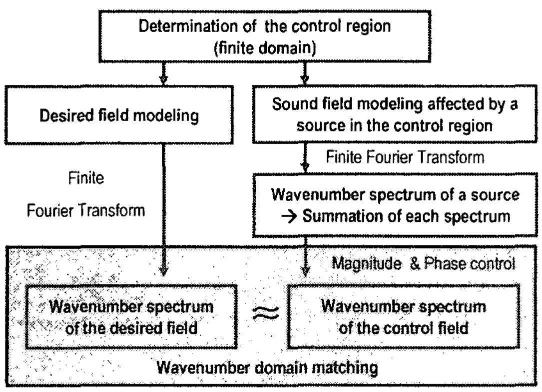


Fig. 3. Procedure diagram for shaped sound focusing using wavenumber domain matching

The procedure for Shaped Sound Focusing using wavenumber domain matching is shown in Fig. 3. The first step is the determination of the control region because it is only considered inside the control region. The next step is the mathematical modeling of desired field and control field in space and the transformation to wavenumber domain. The final step is matching process in wavenumber domain between the desired field and the control field through the phase and magnitude control of sources which are the core part of the wavenumber domain matching process.

3. Problem Definitions

3.1 Desired field : ring-shaped sound field

To start, a ring-shaped sound field is chosen. According to the procedure diagram for shaped sound focusing (Fig. 3), the desired field should be expressed mathematically. A ring can be expressed as Equation (1).

$$p_D(r,\theta,\phi;\omega)=\frac{\delta(r-r_0)}{2\pi r_0^{\,2}}\delta(\cos\theta)e^{ju\phi} \tag{1}$$

$e^{ju\phi}$ is the spatial phase term of the ring concerned about finite number of sources and r_0 is the radius of a ring (fig. 4).

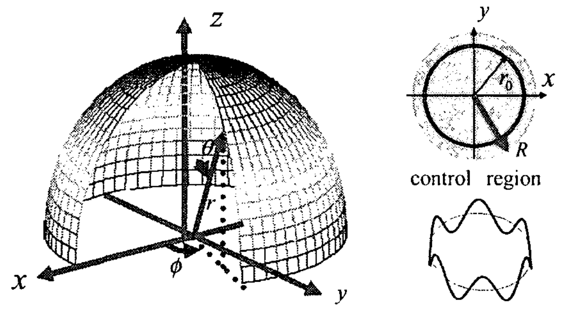


Fig. 4. Spherical coordinate system (left) and desired field described by magnitude (right top) and spatial phase for $u=6$ (right bottom)

3.2 Control field made by multiple monopole sources

Monopole sources is used, because of their simplicity, as control sources so that it is possible to express as Equation (2) with two initial phase terms in each direction (5).

$$p_M^{(i)}(\bar{r}\,|\,\bar{r}_s;\omega)=\frac{-jk\rho c S_{\omega,i}}{4\pi}\frac{e^{-jk|\bar{r}-\bar{r}_{s,i}|}}{|\bar{r}-\bar{r}_{s,i}|}e^{j\varphi_{\phi,i}}e^{j\varphi_{\theta}} \tag{2}$$

$$|\bar{r}-\bar{r}_{s,i}|=\sqrt{r^2+r_{s,i}^{\ 2}+2rr_{s,i}\cos\gamma_i}$$

$$\cos\gamma_i \equiv \sin\theta\sin\theta_{s,i}\cos(\phi-\phi_{s,i})+\cos\theta\cos\theta_{s,i}$$

$\bar{r}_{s,i}(r_s,\theta_s,\phi_{s,i})$: i-th source position vector

If several monopole sources are used for making a ring-shaped sound field, then the control filed can be described by a linear superposition (Equation (3))

$$p_C(\bar{r};\omega)=\sum_{i=1}^{N}p_M{}^{(i)}(\bar{r}\,|\,\bar{r}_{s,i};\omega) \qquad (3)$$

For the simplicity, a set of circular monopole source array with constant θ_s and angle difference $\Delta\phi_s(=2\pi/N)$ is considered and the control region is determined on $\theta=90^\circ$ plane.

4. Solution method

4.1 Transformation process : Fourier transform in spherical coordinate system

It is noteworthy that a finite domain has to be considered because of the finiteness of control region. Through finite domain transform, series expansions can be done as equation (4), (5):

4.2 Spectrum Matching processing in wavenumber domain (wavenumber domain matching)

(1) Wavenumber spectrum of the desired field

From this transformation process, series coefficients, which are wavenumber spectrum components, can be obtained for a ring-shaped sound field as:

$$C_{D,mnq}\Big|_{\theta=90^\circ}=\frac{2n+1}{2\pi R^3}\frac{j_n(k_{nq}r_0)}{[j_{n+1}(k_{nq}R)]^2}[P_n^{\ m}(0)]^2\delta_{m,u} \qquad (7)$$

($\delta_{m,u}$: Kronecker delta)

From Equation (7), it can be known that $C_{D,mnq}\big|_{\theta=90^\circ}$ has dominant value only for the case of $m=u$, $n+u=even$ and is getting much smaller as long as n increase (Fig. 5(a)).

(2) Wavenumber spectrum of the control field

After the same process of previous section, series coefficients of a monopole source at arbitrary position in space are calculated mathematically (Equation (8)). For a set of circular array, optimal $\varphi_{\phi,i}$ is given as $-(2\pi u/N)i+\pi$ to maximize the real part of $C_{A,mnq}^{(i)}\big|_{\theta=90^\circ}$ except $e^{j\varphi_\theta}$ term, which will be mentioned in the next section, and ε_m is Neumann's number (1 for $m=0$ and 2 for $m>2$).

$$C_{M,mnq}^{(i)}\Big|_{\theta=90^\circ}=\frac{-k^2\rho cS_\omega}{2\pi}\left(h_n^{(2)}(k_sr_s\sin\theta_s)\varepsilon_m(2n+1)\frac{(n-m)!}{(n+m)!}P_n^{\ m}(\cos\theta_s)P_n^{\ m}(0)\right)\frac{\int_0^R r^2 j_n(k_sr\sin\theta_s)j_n(k_{nq}r)dr}{R^3[j_{n+1}(k_{nq}R)]^2}[e^{jm\phi_{s,i}}e^{j\varphi_{\phi,i}}]e^{j\varphi_\theta} \qquad (8)$$

$$C_{A,mnq}\Big|_{\theta=90^\circ}=\sum_{i=1}^{N}C_{M,mnq}^{(i)}\Big|_{\theta=90^\circ}=e^{j\varphi_\theta}N\frac{k^2\rho cS_\omega}{2\pi}\left(h_n^{(2)}(k_sr_s\sin\theta_s)\varepsilon_m(2n+1)\frac{(n-m)!}{(n+m)!}P_n^{\ m}(\cos\theta_s)P_n^{\ m}(0)\right)\frac{\int_0^R r^2 j_n(k_sr\sin\theta_s)j_n(k_{nq}r)dr}{R^3[j_{n+1}(k_{nq}R)]^2}\delta_{u,m} \qquad (9)$$

$$p(\bar{r};\omega)=\sum_{q=1}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{n}C_{mnq,\omega}\psi_{mnq}(r,\theta,\phi;\omega) \qquad (4)$$

$$C_{mnq,\omega}=\frac{2n+1}{2\pi R^3[j_{n+1}(k_{nq}R)]^2}\frac{(n-m)!}{(n+m)!}\times\int_0^R\int_0^\pi\int_0^{2\pi}r^2p(\bar{r};\omega)\psi_{mnq}^{\ *}(r,\theta,\phi)\sin\theta d\phi d\theta dr \qquad (5)$$

(R : radius of control plane)

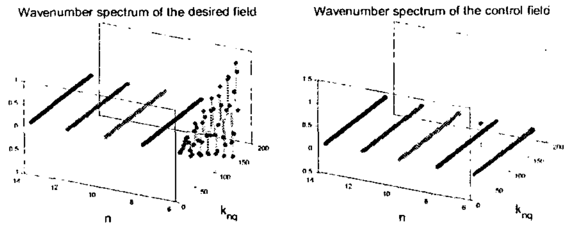
where $\psi_{mnq}\equiv j_n(k_{nq}r)P_n^{\ m}(\cos\theta)e^{jm\phi}$, $k_{nq}\equiv\alpha_{nq}/R$ and $j_n(\alpha_{nq})=0\,(\alpha_{nq}>0)$. As it is mentioned in the previous section, if it is concerned about $\theta=90^\circ$ plane, then Equation (4) can be written as:

$$p(r,\phi;\omega)=\sum_{q=1}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{n}[C_{mnq,\omega}P_n^{\ m}(0)]j_n(k_{nq}r)e^{jm\phi} \qquad (6)$$

where $C_{mnq}\big|_{\theta=90^\circ}\equiv C_{mnq}P_n^{\ m}(0)$.

In this case, the sound field affected by sets of circular array source is called the control field (Equation 9). To match the wavenumber spectrum, it is needed to see the wavenumber spectrum of the desired field which has the real value. This means there should be an effort to make the wavenumber spectrum of control field real. Equation (9) has similar characteristics to the desired field spectrum because of components in common between Equation (7) and (9), however, Equation (9) has a dominant value in case of $k_{uq}=k_s\cos\theta_s$ because of the orthogonality of Bessel functions (Fig 5(b)). This relation, $k_{uq}=k_s\cos\theta_s$, gives the clue to determine proper source position angle θ_s .

As shown in Fig. 6, additionally, it is obtained a guideline to determine proper source position angle θ_s. For small θ_s , series coefficient except spherical Bessel function component has very large value because of singularity of spherical Hankel function which is oppositely different from the characteristics of the desired field. Therefore it is known that the small θ_s is not recommended for sound shaping.



(a) (b)

Fig. 5. The normalized wavenumber spectrum of each sound field ($u=6, R=1m$) (a) the desired field spectrum for $r_0=0.3m$ (b) the control field for $k_s=27.5$, $\theta_s=90^\circ$ ($k_{66}=k_s\cos\theta_s$) [Note: If $\theta_s=62^\circ$, then $k_{65}=k_s\sin\theta_s$.]

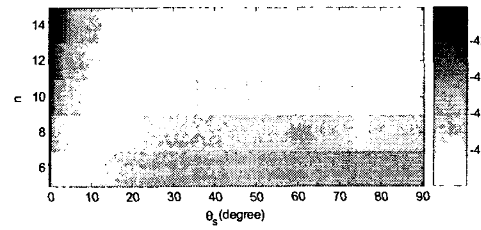


Fig. 6. Series coefficients distribution ($dB_{ref.max}$) with respect to n and θ_s except for spherical Bessel function components of Equation (9).

In summary, to make a ring-shaped sound field, several sets of circular array including azimuthally optimal phase term $\varphi_{\phi,i}$ are used. This circular array can be expressed as Equation (9). From Equation (9), if source position angle θ_s is controllable to satisfy the condition $k_{uq}=k_s\cos\theta_s$, then it can be determined effective source position angle θ_s and proper excitation frequency ($k_s=\omega_s/c$ where ω_s and c are an excitation frequency and speed of sound). Also we can learn a small θ_s is not proper for making a ring shaped sound field as shown in Fig. 6.

5. Simulation results and analysis

According to the previous section, computer simulation was taken for the acoustical ring with $r_0=0.3m$, $u=6$ using 5 sets of circular monopole source array. A set of circular monopole source array consists of 12 monopole sources, so that the total number of sources used is 60. Source position angle θ_s are determined as $38^\circ, 46^\circ, 54^\circ, 65^\circ. 90^\circ$ which satisfy the relation, $k_{uq}=k_s\cos\theta_s$ ($k_s=33.9$).

Fig. 7 shows the result of ring-shaped sound focusing using the proposed method. In this case, 10dB bandwidth is about 19cm. If the number of circular array is increase, it can be achieved much narrow band ring-shaped sound field. An initial phase term $e^{j\varphi_\theta}$ is actually not considered in Fig. 7 because it should be determined after the consideration of all order of Hankel function. If it can be determined the optimal initial phase $e^{j\varphi_\theta}$ from an optimization process, then we can achieve more contrast between focused region and unfocused region.

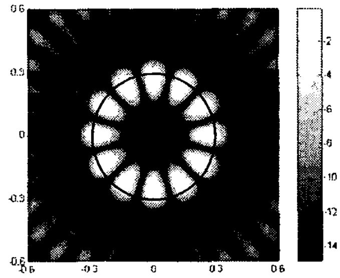


Fig. 7. Ring-shaped sound field using wavenumber domain matching. (pressure distribution normalized by maximum pressure level in dB scale)

6. Conclusions

For sound shape control using multiple sources efficiently, wavenumber domain matching method is proposed. Using this method, we can physically interpret the sound field affected by control sources (control field) and also make it similar to the desired sound field that defined. However, to achieve the better performance, which means maximum contrast between focused and unfocused region, much more studies on how to control the magnitude and phase of control sources effectively should be done in the future.

Acknowledgement

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