Realization of Scattering Acoustic Holography using Plane-wave Decomposition

평면파 분리 방법을 이용한 산란 음향 홀로그래피의 구현 방법론

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Key Words : Scattered sound field(산란음장), Total sound field(전체음장), Directional function(방향함수), Spherical wave spectrum(구형파 스펙트럼), Plane-wave decomposition(평면파 분리)

ABSTRACT

When an object or objects, rigid or flexible, presents in incident sound field, the sound wave is scattered. This, we call, is scattered sound field. It, of course, depends on the amplitude and the direction of the incident sound field as well as the geometry and the surface impedance of the scatterer(object). This paper addresses the way to measure scattered sound field by using arbitrary incident sound wave. This means that the method can decompose the scattered field from measured sound field with respect to any magnitudes and directions of incident plane-waves.

1. Introduction

The scattered field depends on the characteristics of the incident field and the scatterer(Fig.1). The incident field can be characterized by its wavenumber and amplitude. It is noteworthy that the scattered field can not be measured directly, nor the incident plane-wave can be easily generated.

To obtain the scattered field, we have to extract the scattered field from the total field. Lee and Kim[1] proposed the method that extracts the scattered field using the difference between spatial transfer functions for the incident field and total field. To verify the method, they also proposed the helical wave spectrum transfer matrix that separates the scattered field affected by plane-wave from the measured scattered field.

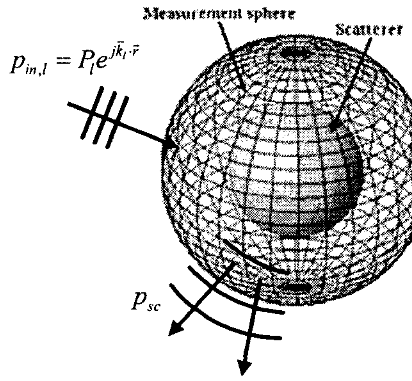

Figure 1. Total field composed by a plane-wave and a scattered one

When an arbitrary incident field is generated in a space, it is necessary to find out the amplitudes and directions of plane-waves of sound field. To identify the directions of plane-waves, Boaz Rafaely proposed the plane-wave decomposition method[2]. They[4] also proved it experimentally. However, we have to know not only directions, but also amplitudes of plane-waves.

In this paper, we present a theory to get the scattered field with respect to plane-waves of sound field. The restrictions of this method are also discussed.

2. Sound field description

Sound field generated in a space can be described in rectangular, cylindrical, or spherical coordinate. If an arbitrary sound source has finite extent, the spherical coordinate is the most suitable among them to express the sound field radiated from it.

Sound field distributed on the sphere($r = r_H$) can be expressed as the combination of spherical harmonics

$$p(r_H,\theta,\phi)=\sum_{m=0}^{\infty}\sum_{n=-m}^{m} P_{mn}(kr_H)Y_n^m(\theta,\phi), \quad (1)$$

$$Y_n^m(\theta,\phi)=\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}P_n^m(\cos\theta)e^{jm\phi}. \quad (2)$$

where $m=-n,\cdots,n$ and $n=0,1,2,\cdots$. $P_n^m(\cos\theta)$ are associated Legendre functions in the polar direction, and $e^{jm\phi}$ are harmonic functions in the azimuthal direction. $Y_n^m(\theta,\phi)$ are spherical harmonics, and hold the following relations;

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$$\int_0^{2\pi}\int_0^{\pi} Y_n^m(\theta,\phi) Y_{n'}^{m'}(\theta,\phi)^* \sin\theta d\theta d\phi = \delta_{nn'}\delta_{mm'} , \quad (3)$$

$$\sum_{m=0}^{\infty}\sum_{n=-m}^{m} Y_n^m(\theta',\phi')^* Y_n^m(\theta,\phi) = \delta(\cos\theta-\cos\theta')\delta(\phi-\phi') , \quad (4)$$

where $\delta_{mm'}$ and $\delta_{nn'}$ are the Kronecker delta functions, and $\delta(\cos\theta-\cos\theta')$ and $\delta(\phi-\phi')$ are the Dirac delta functions. $P_{mn}(kr_H)$ in eqn(1) are expansion coefficients on the sphere, and definded as the spherical wave spectrum[5]

$$P_{mn}(kr_H) = \int_0^{2\pi}\int_0^{\pi} p(r_H,\theta,\phi,k) Y_n^m(\theta,\phi)^* \sin\theta d\theta d\phi . \quad (5)$$

In the free-field, a unit incident plane-wave on the sphere is expressed as[5]

$$p_{in,l}(r_H,\theta,\phi) = e^{i\vec{k}_l\cdot\vec{r}_H} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} 4\pi i^n j_n(kr_H) Y_n^m(\theta_l,\phi_l)^* Y_n^m(\theta,\phi) \quad (6)$$

where $h_n^{(1)}(kr)$ are spherical Hankel function of the first kind, $j_n(kr)$ are spherical Bessel function of the first kind[6]. The amplitude of a unit incident wave is 1, and the incident direction is indicated by wave-number $\vec{k}_l$. The spherical wave spectrum of unit directional function[2] is defined as

$$W_{mn}(\theta_l,\phi_l) \equiv Y_n^m(\theta_l,\phi_l)^* , \quad (7)$$

However, when an incident field is generated by a specific sound source, it can be expressed as the combination of unit plane-waves

$$p_{in}(r_H,\theta,\phi) = \sum_{l=1}^{\infty} p_{in,l}(r_H,\theta,\phi) = \sum_{l=1}^{\infty} P_l \sum_{n=0}^{\infty}\sum_{m=-n}^{n} 4\pi i^n j_n(kr_H) W_{mn}(\theta_l,\phi_l) Y_n^m(\theta,\phi) . \quad (8)$$

P_l represent the amplitude of unit plane-waves.

If there is a unit incident plane-wave and a rigid sphere as a scatterer, the unit total field on the sphere($r=r_H$) is

$$p_{t,l}(r_H,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} b_n(kr_H,ka) W_{mn}(\theta_l,\phi_l) Y_n^m(\theta,\phi) , \quad (9)$$

$$b_n(kr_H,ka) = 4\pi i^n \left(j_n(kr_H) - \frac{j_n(ka)'}{h_n^{(1)}(ka)'} h_n^{(1)}(kr_H) \right) . \quad (10)$$

These coefficients can be obtained from the boundary condition of rigid surface; the radial velocity of the total field is zero on the surface($r=a$). $j_n(ka)'$ and $h_n^{(1)}(ka)'$ are derivatives of spherical Bessel functions and Hankel functions. The spherical wave spectrum of the unit total field is

$$P_{mn}^{t,l}(kr_H) = b_n(kr_H,ka) W_{mn}(\theta_l,\phi_l) . \quad (11)$$

Therefore, the spherical wave spectrum of the unit directional function[2] can be obtained by that of the unit total field,

$$W_{mn}(\theta_l,\phi_l) = \frac{P_{mn}^{t,l}(kr_H)}{b_n(kr_H,ka)} . \quad (12)$$

If an arbitrary incident field is generated in the free-field, the total field can be expressed as the combination of the unit total field,

$$p_t(r_H,\theta,\phi) = \sum_{l=1}^{\infty} p_{t,l}(r_H,\theta,\phi) = \sum_{l=1}^{\infty} P_l \sum_{n=0}^{\infty}\sum_{m=-n}^{n} b_n(kr_H,ka) W_{mn}(\theta_l,\phi_l) Y_n^m(\theta,\phi) . \quad (13)$$

The spherical wave spectrum of the total field is

$$P_{mn}^{t,l}(kr_H) = \sum_{l=1}^{\infty} P_l b_n(kr_H,ka) W_{mn}(\theta_l,\phi_l) . \quad (14)$$

Therefore, the spherical wave spectrum of a directional function can be obtained by of the unit total field,

$$\sum_{l=1}^{\infty} P_l W_{mn}(\theta_l,\phi_l) = \frac{P_{mn}^{t,l}(kr_H)}{b_n(kr_H,ka)} . \quad (15)$$

3. Plane-wave decomposition

Scattered sound field depends on not only directions, but also amplitudes of plane-waves of sound field. To find out the amplitudes of plane-waves, the directions of plane-waves have to be identified at first.

The direction of a unit incident plane-wave can be obtained by taking the inverse spatial Fourier transform of eqn(12),

$$\sum_{n=0}^{\infty}\sum_{m=-n}^{n} \frac{P_{mn}^{t,l}(kr_H)}{b_n(kr_H,ka)} Y_n^m(\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} W_{mn}(\theta_l,\phi_l) Y_n^m(\theta,\phi) = \delta(\cos\theta-\cos\theta_l)\delta(\phi-\phi_l). \quad (16)$$

It is marked at a point on the sphere due to Dirac delta functions[2] by eqn(4). The unit amplitude of this can be obtained by integrating eqn(16) over all region.

Directions of incident plane-waves can be obtained by taking the inverse spatial Fourier transform of eqn(15),

$$\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{P_{mn}^{l}(kr_H)}{b_n(kr_H,ka)}Y_n^m(\theta,\phi)$$
$$=\sum_{l=1}^{\infty}P_l\sum_{n=0}^{\infty}\sum_{m=-n}^{n}W_{mn}(\theta_l,\phi_l)Y_n^m(\theta,\phi) \qquad (17)$$
$$=\sum_{l=1}^{\infty}P_l\,\delta(\cos\theta-\cos\theta_l)\delta(\phi-\phi_l).$$

Moreover, the amplitudes of plane-waves can be obtained by integrating eqn(17) over each region ($\Omega_1-\varepsilon\le\Omega\le\Omega_1+\varepsilon$, $\cdots$, $\Omega_q-\varepsilon\le\Omega\le\Omega_q+\varepsilon$). However, the sound field can not be expressed as the infinite summation of spherical harmonics because the measurement points are finite on the sphere (θ_i,ϕ_i). Therefore, description for sound field is rewritten as $p(r_H,\Omega_i)$, $\Omega_i=(\theta_i,\phi_i)$.

Equation(16) describing the direction of a unit plane-wave is rewritten as below

$$\sum_{n=0}^{N_w}\sum_{m=-n}^{n}\frac{P_{mn}^{i,l}(kr_H)}{b_n(kr_H,ka)}Y_n^m(\Omega_i)$$
$$=\sum_{n=0}^{N_w}\sum_{m=-n}^{n}W_{mn}(\theta_l,\phi_l)Y_n^m(\Omega_i). \qquad (18)$$

The direction is expressed as polynomials[2], not Dirac delta function. The location of a peak value indicates the direction of a unit incident plane-wave(Fig.2).

Equation(17) describing the directions of plane-waves is rewritten as below

$$\sum_{n=0}^{N_w}\sum_{m=-n}^{n}\frac{P_{mn}^{l}(kr_H)}{b_n(kr_H,ka)}Y_n^m(d\Omega_i)$$
$$=\sum_{l}^{q}P_l\sum_{n=0}^{N_w}\sum_{m=-n}^{n}W_{mn}(\theta_l,\phi_l)Y_n^m(d\Omega_i). \qquad (19)$$

The locations of peak values indicate the directions of incident plane-waves(Fig.4). The amplitudes of plane-waves can be obtained by constructing q equations. They have q unknown variables(P_1 , $\cdots$, P_q) by substituting each direction (θ_l,ϕ_l) into eqn(19).

4. Scattered field by plane-wave decomposition

The scattered field affected by a unit plane-wave is

$$p_{sc,l}(r_H,\theta,\phi)$$
$$=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}S_n(kr_H,ka)W_{mn}(\theta_l,\phi_l)Y_n^m(\theta,\phi), \qquad (20)$$

$$S_n(kr_H,ka)=-4\pi i^n\frac{j_n(ka)'}{h_n^{(1)}(ka)'}h_n^{(1)}(kr_H). \qquad (21)$$

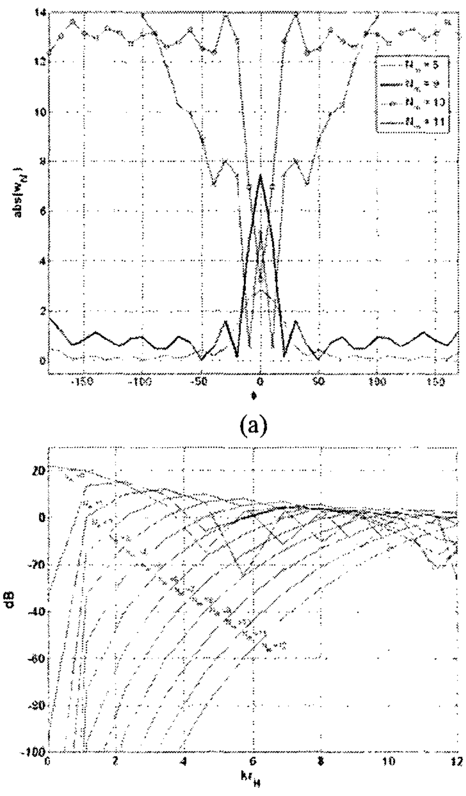
Especially, eqn(20) is denoted as the unit scattered field. Equation(21) result from the boundary condition of a rigid sphere.

An arbitrary scattered field can be expressed as the combination of the unit scattered field

$$p_{sc}(r_H,\theta,\phi)=\sum_{l=1}^{\infty}P_l\,p_{sc,l}(r_H,\theta,\phi)$$
$$=\sum_{l=1}^{\infty}P_l\sum_{n=0}^{\infty}\sum_{m=-n}^{n}S_n(kr_H,ka)W_{mn}(\theta_l,\phi_l)Y_n^m(\theta,\phi). \qquad (22)$$

Note from eqn(22) that the scattered field affected by an arbitrary incident field can be expressed as the combination of the spherical wave spectrum of the unit scattered field.

By using plane-wave decomposition, we can find out the directions and amplitudes of plane-waves. These components effect on an arbitrary scattered field in eqn(22). The measured scattered field can be computationally decomposed into scattered fields affected by each plane-wave. Therefore, the measured scattered field can be compared with the combination of analytical solutions.



(a)

(b)

Figure 2. (a) the directional function of a unit incident plane-wave incoming with the direction $\Omega_l=(90°,0°)$, (b) dB scale of $b_n(kr_H,ka)$

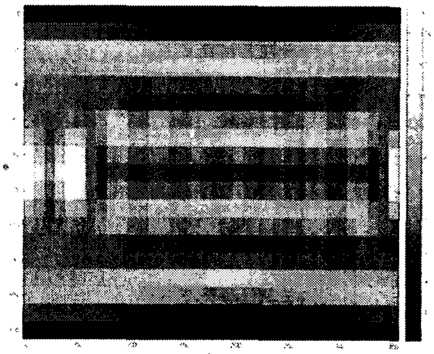


Figure 3. The directional function of two incident plane waves

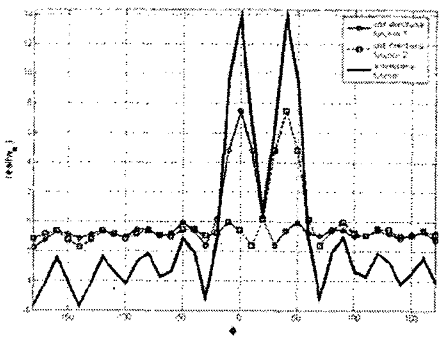


Figure 4. The directional function of two incident plane waves with each directional function of unit plane-waves

5. Simulation results

We generated an arbitrary incident field composed by two incident plane-waves for the simplest case. Their directions were $\Omega_1 = (90°, 0°)$, $\Omega_2 = (90°, 40°)$, and magnitudes were $(P_1 = 2, P_2 = 2)$. The sound field distributed on the sphere($r_H = 0.25m$) was measured on 614 points. Neighboring points were equiangle $10°$. The radius(a) of a rigid sphere was $0.1405m$. When interesting frequency was $1kHz$, $ka = 2.57$, and $kr_H = 4.57$.

In this case, maximum mode number N_{sys} was 17, when reconstructing the measured field on the sphere. Although the maximum mode number N_{sys} was determined by the measurement system, in the computation the maximum mode number N_w was determined by a unit directional function. In the figure 2, we could expand to $N_w = 9$ about a unit directional function in the given measurement system. If we expanded modes of $N_w > 9$, they caused error[2]. The directional function was amplified because the magnitude of eqn(10) had very low values at $N_w > 9$ (Fig.2.(b)). Therefore, we could find out the direction of two incident plane-waves denoted at the maximum peak value(Fig.2.(a)).

The directions of two incident plane-waves were identified by using eqn(19)(Fig.2); white points are the directions of two plane waves. Then, we generated two unit plane-waves by using eqn(18). The amplitudes was $P_1 = 2, P_2 = 2$. Therefore, we exactly found out the scattered field constructed by two analytical solutions.

6. Conclusion

In the simplest case, we can find out the directions and the amplitudes of two incident plane-waves. Therefore, we exactly decomposed the scattered fields with respect to each plane-wave from the scattered field affected by an arbitrary incident field. However, this method has the restriction that we have to exactly find out the directions of plane-waves of sound field. This restriction is caused by the different amplitudes of plane-waves, and by spatial resolution. To overcome this restriction, we have to more densely measure the sound field.

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