

# 자기부상 시스템에서 자속 궤환이 Open-Loop 특성에 미치는 영향에 대한 연구

## A Study on the Effect of the Magnetic Flux Feedback on Open Loop Characteristics in a Magnetic Suspension System

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**Key Words** : Magnetically Levitated System(자기력부상시스템), Control System(제어시스템), Flux Feedback(자속궤환), Air Gap Flux(공극자속)

### ABSTRACT

본 논문에서는 자속 궤환을 포함하는 자기부상 시스템의 open-loop 특성에 대해서 다룬다. 훌륭한 외란 억제 특성을 포함하고 파라메타 변동에 대해서 둔감한 제어를 설계하기 위해서 자속 궤환을 이용하는 것이 효과적일 수 있다. 적절한 센서에 의해서 측정 가능한 자속은 전류와 공극의 시간 변화에 대해서 선형적인 관계를 가지고 있으며, 자속의 궤환은 자기 부상시스템이 본질적으로 갖고 있는 전기적 액츄에이터와 기계적 플랜트 사이의 간섭에 의해서 발생하는 비선형성을 감소시키는 역할을 한다. 모의시험 결과를 통해서 자속의 궤환이 자기부상 시스템의 부하변동에 대해서 open-loop 성능 향상에 효과적임을 보인다.

### 1. Introduction

One of the ways that non-contact states between two objects are maintained is to employ a magnetic suspension technology [1]. This system is commonly known as Magnetically Levitated system (*Maglev*) which has been used in the vehicle suspension system and magnetic bearing system developed in the University of Virginia, U.S.A. in 1937 for the first time. There are various applications employing the magnetic suspension configuration as a key technology, such as the magnetically levitated train system, the high speed turbo compressors, the flywheel energy storage system, and the artificial heart pump [2][3][4][5][6].

The magnetically levitated train system can be divided into two parts based on the levitation method: one is a repulsive type using super

conductors. A disadvantage of this type of suspension system is needed for operation below the critical speed when the suspended object is stationary. The other type is using ferromagnetic or permanent magnet. This type of electromagnetic suspension system (EMS system) has one significant advantage in that it provides attraction force at zero speed, but such a system is inherently unstable. In order to overcome the inherent instability a active controller plays a very important role in the electromagnet suspension system to make the stable suspension and to maintain the suspended object within the nominal air gap. This active controller also should has a property of the robustness against the parameter variations and the external disturbance [2][5]. In many papers and literatures the design methodologies of the active magnetic suspension controllers have already been presented [4][6]. However the papers for the analysis of the open-loop characteristics of the magnetic suspension system have rarely presented. It is sometimes very valuable to make a focus on the inherent properties of the magnetic suspension system to achieve a design of the improved active controller against the system parameter variations. The purpose of this paper is to

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show the effectiveness of the feedback loop of the air gap in the open loop characteristic of the magnetic suspension system when a constant voltage is applied to the electromagnet coils.

In this paper we summarize the fundamental mathematical model of the EMS system, which does not include the flux feedback loop and we show a modified mathematical model including the flux feedback loop. In the flux feedback loop the estimated flux is linearly proportional to the magnetic excitation current and inversely proportional to the air gap. Simulation results verify that the magnetic suspension system including flux feedback loop has robustness against the parameter variations. Finally conclusions are summarized.

### 2. Fundamental Mathematical Model

Fig.1 shows a simple schematic diagram for EMS system which has the electromagnets as the suspension actuators, linear induction motor and reaction plate for the vehicle propulsion. As we see

in Fig. 1 the passenger vehicle and the bogie can be levitated by the electromagnets attraction force. Once the bogie is levitated the propulsion system (linear motor and reaction plate) is activated to move the passenger vehicle.

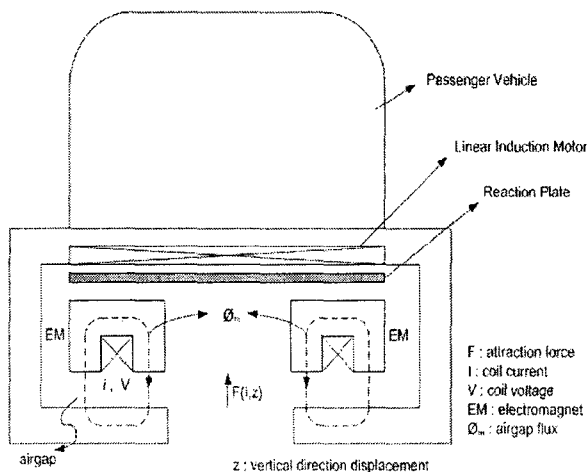


Fig. 1 Schematic diagram for EMS system

For the simplification of the mathematical model of the suspension system shown in Fig. 1 we modify

Fig. 1 to Fig. 2.

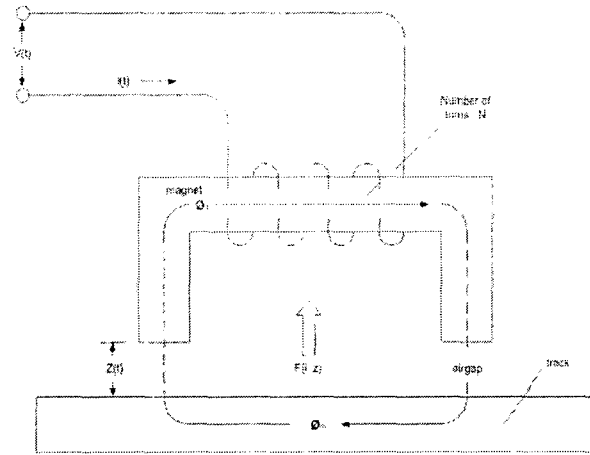


Fig. 2 Simplified schematic diagram

The mathematical model of this system is divided into two parts: One is the plant (mechanical) dynamics and the other is the actuator dynamics. The plant (mechanical) dynamics is

$$m\ddot{z} = F(i, z) - mg - f_d \quad (1)$$

where  $m$  is the total mass of the controlled object,  $g$  is the gravitational acceleration,  $f_d$  is the external disturbance force acting on the controlled object. In Eqn. (1)  $F(i, z)$  is the electromagnets attraction force which is proportional to the current deviation and inversely proportional to the air gap deviation, expressed such as:

$$F(i, z) = \frac{B^2 A}{\mu_0} = \frac{\mu_0 N^2 A}{4} \left( \frac{i(t)}{z(t)} \right)^2 \quad (2)$$

where  $B$  is the flux density of the magnetic core material,  $A$  is the cross sectional area of the pole face of the electromagnet,  $\mu_0$  is the permeability in the air space,  $N$  is the number of turns.

In order to drive Eqn. (2) it is necessary to check the relation between the inductance and the magnetic flux density.

$$L(i, z) = \frac{N}{i} \Phi_m = \frac{N}{i(t)} \frac{Ni(t)}{R_T} \quad (3)$$

where  $R_T = \frac{2z(t)}{\mu_0 A} = \frac{V_T}{\Phi_m}$  is the reluctance of the magnetic circuit.  $V_T$  is the electromotive force. Normally the reluctance in the magnetic core is

assumed to be negligible compared with the air gap, thus the coil inductance becomes

$$L(i, z) = \frac{\mu_0 N^2 A}{2z(t)} \quad (4)$$

$Li(t) = NBA$  and Eqn. (4) yield the magnetic flux density such as:

$$B = \frac{Li(t)}{NA} = \frac{N^2 \mu_0 A i(t)}{2z(t) NA} = \frac{\mu_0 N i(t)}{2z(t)} \quad (5)$$

Substituting Eqn. (5) into  $F(i, z) = \frac{B^2 A}{\mu_0}$  yields

Eqn. (2). Eqn. (2) has high nonlinearity and it is not easy to use Eqn. (2) without doing the linear approximation with respect to the nominal point  $(i_0, z_0)$ . For the linear approximation the Taylor Series Expansion is usually employed. From the Taylor Series Expansion the Eqn. (2) becomes

$$F(i, z) = k_i i(t) - k_z z(t) \quad (6)$$

where  $k_z = \frac{\mu_0 N^2 A i_0^2}{2z_0^3}$ ,  $k_i = \frac{\mu_0 N^2 A i_0}{2z_0^2}$  are the

coefficients for the linear approximation of Eqn. (2).

In Eqn. (3) the stiffness  $k_z$  has negative sign which means that once the attractive force of the electromagnets is activated the controlled object is attracted until the electromagnets stop attracting the controlled object. This is one of the reasons why the electromagnet suspension system should has the active controller to control the air gap deviation.

The actuator dynamics is

$$\begin{aligned} v(t) &= Ri(t) + \frac{d}{dt} [L(i, z)i(t)] \\ &= Ri(t) + \frac{\mu_0 N^2 A}{2z(t)} \frac{d}{dt} i(t) - \frac{\mu_0 N^2 A i_0}{2z(t)^2} \frac{d}{dt} z(t) \end{aligned} \quad (7)$$

where  $v$  is the coil voltage,  $R$  is the coil resistance,  $L(i, z)$  is the coil inductance which is the function of the air gap displacement. It should be noted there is a variation of the inductance with respect to the air gap displacement in the second term, and that the third term denotes a voltage which varies with changes in the air gap  $z(t)$  and

its rate of change similar to back EMF voltage.

By using equations (1), (6), (7) we can drive a state space equation such as:

$$\begin{bmatrix} \dot{z} \\ \dot{z} \\ \dot{z} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_z & 0 & k_i \\ m & 0 & m \\ 0 & k_z & -R/L \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1/m \\ 0 \end{bmatrix} f_d \quad (8)$$

Fig. 3 shows the simple block diagram for the open-loop EMS system which does not include the flux feedback signal.

The transfer function of the open loop system is induced by the Laplace transform of Eqn. (8) for each state variables, which is

$$z(s) = \left[ \frac{\left( \frac{k_i}{mR} \right)}{\left( 1 + \frac{L}{R}s \right) \left\{ s^2 + \frac{k_i^2}{mR \left( 1 + \frac{L}{R}s \right)} s - \frac{k_z}{m} \right\}} \right] v(s) \quad (9)$$

with  $f_d = 0$ . As we see in Eqn. (9) the negative stiffness  $k_z$  makes the system unstable, which means one of the poles of the characteristic equation exists in the right half plane of the s-plane.

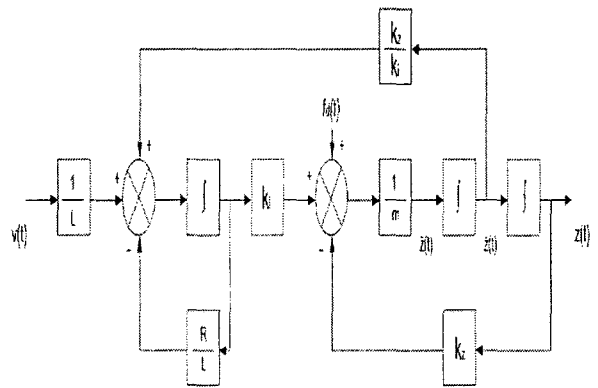


Fig. 3 Open-loop diagram without flux

### 3. Flux Feedback Loop

In this section we show an approach using flux feedback to estimate open loop characteristics. The flux which is produced by the core magnet is

proportional to the pole face area as:

$$\begin{aligned} \Phi_m &= BA & (10) \\ &= \frac{\mu_0 N A i}{2z} = k_\phi \frac{i}{z} \end{aligned}$$

The small deviation of the flux at some time is

$$\Delta\Phi_m = k_{f_i} i - k_{f_z} z \quad (11)$$

where  $k_{f_i} = \frac{\mu_0 A N}{2z_0}$ ,  $k_{f_z} = \frac{\mu_0 A N i_0}{2z_0^2}$ .

If the magnetic flux  $\Phi_m$  is fed back to the amplifier that activates the electromagnet the equation for the electromagnet actuator dynamics should be changed so that the force component which is included in the magnetic flux should be included in the actuator dynamic equation. This yields equation (12).

$$\begin{aligned} v(t) &= Ri(t) + \frac{d}{dt} [L(i, z)i(t)] + k_\phi \Delta\Phi_m \\ &= Ri(t) + \frac{\mu_0 N^2 A}{2z(t)} \frac{d}{dt} i(t) - \frac{\mu_0 N^2 A i(t)}{2z(t)^2} \frac{d}{dt} z(t) \\ &\quad + k_\phi (k_{f_i} i(t) - k_{f_z} z(t)) \\ &= Ri(t) + L \frac{d}{dt} i(t) - k_i \frac{d}{dt} z(t) \\ &\quad + k_\phi (k_{f_i} i(t) - k_{f_z} z(t)) \end{aligned} \quad (12)$$

Compare to the Eqn. (7) in Eqn. (12) the flux feedback term is included which is expressed as linear combination of the coil current and air gap deviation. A combination of the three equations (1), (6), (12) induce the state space model including the flux feedback such as:

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_z & 0 & k_i \\ \frac{k_\phi k_{f_z}}{L} & \frac{k_z}{k_i} & \frac{(-R - k_\phi k_{f_i})}{L} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} v + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} f_d \quad (13)$$

Fig. 4 presents the open-loop diagram with flux feedback loop. In this figure  $k_{\phi\phi}$  represents the flux feedback gain which includes the EMS force component.

The characteristic equation of the Eqn. (13) is

$$\begin{aligned} \det(sI - A) &= s^3 + \left( \frac{R}{L} + \frac{k_\phi k_{f_i}}{L} \right) s^2 \\ &\quad + \frac{k_\phi}{mL} (-k_i k_{f_z} + k_z k_{f_i}) + \frac{k_z R}{mL} \end{aligned} \quad (14)$$

In Eqn. (14) the gain of the flux feedback loop can be adjusted such that

$$k_\phi (-k_i k_{f_z} + k_z k_{f_i}) = k_z R \quad (15)$$

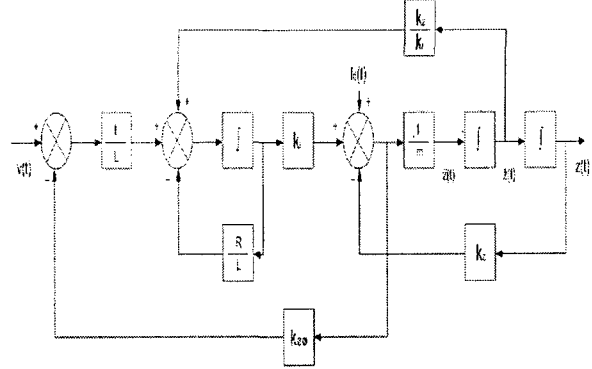


Fig. 4 Open-loop diagram with flux feedback

then the input-output transfer function is reduced to

$$z(s) = \frac{k_i/mL}{s^2 \left( s + \frac{R}{L} + k_\phi k_{f_i} \right)} v(s) \quad (16)$$

which indicates that flux feedback makes the system conditionally stable

Table 1. Parameters for EMS system

Variables	Value	Unit
Mass : $m$	100, 500	[Kg]
Coil Inductance : $L$	72	[mH]
Coil Resistance : $R$	0.7	[ $\Omega$ ]
Steady Current : $i_0$	1	[A]
Cross Sectional Area : $A$	$2.2 \times 10^{-5}$	[ $m^2$ ]
Number of turns : $N$	140	[turns]
Permeability : $\mu_0$	$4\pi \times 10^{-7}$	[H/m]

#### 4. Simulations

For the simulations to estimate the open-loop properties of the electromagnet suspension system we set the following parameters shown in Table 1. In the simulations to assess the effectiveness of the flux feedback loop we made the load variations in mass. Fig. 5 and Fig. 6 show the air gap deviation for the case of when the load acting on the controlled object is changed. When

the load is 100[Kg] (Fig. 5) without the flux feedback loop the controlled object contacts (dashed line) the electromagnets at time 50[sec]. In case of 500[Kg] (Fig. 6) it takes more time to contact the electromagnet than that of Fig. 5, however we see that the vibration amplitude becomes bigger and bigger with time. This is because of the inherent unstable characteristic of the magnetic suspension system.

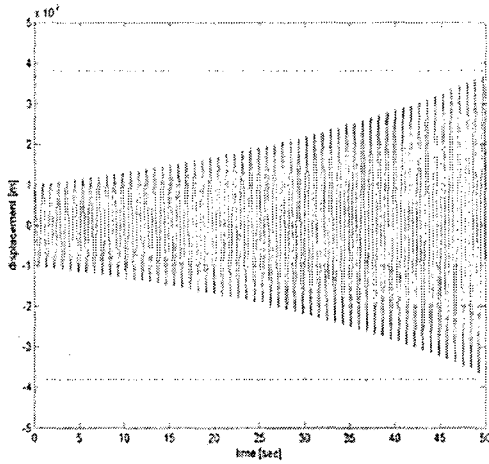


Fig. 5 Gap deviation without flux feedback ( $m=100$ [Kg])

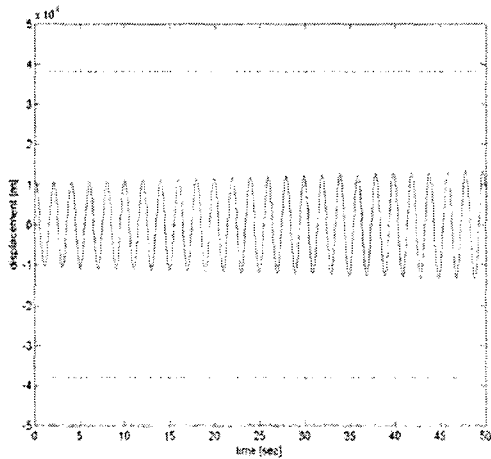


Fig. 6 Gap deviation without flux feedback ( $m=500$ [Kg])

On the contrary the simulation results with flux feedback loop shown in Fig. 7 and Fig. 8 present very good characteristics against the load

variations. Fig. 7 is for the case when the load is 100[Kg]. In this figure we see very good robustness when there is a parameter variations of the EMS system. The small vibration in Fig. 7 is because there is no feedback active controller. Thus any kind of feedback controller can eliminate this vibration. In Fig. 8 we see slower vibrations and bigger amplitude than that of Fig. 7. This is because of the much heavier load (500[Kg]).

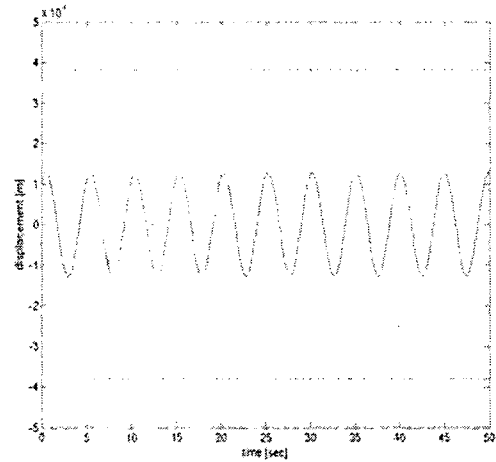


Fig. 7 Gap deviation with flux feedback ( $m=100$ [Kg])

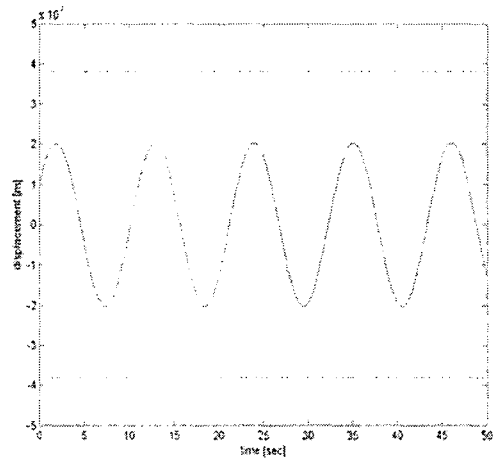


Fig. 8 Gap deviation with flux feedback ( $m=500$ [Kg])

### 5. Conclusions

In this paper we have dealt with the open loop

characteristics of the electromagnets suspension system which has the flux feedback loop. The flux feedback loop increases the system robustness against the parameter variations even if no active controller is employed. This property comes from the linear combination of the air gap displacement and the coil current which is different from the conventional open loop structure.

First, we showed the fundamental mathematical model which has no flux feedback loop, and then introduced the modified mathematical model including the flux feedback loop. Finally we achieved the robustness against the parameter variations of the modified open loop scheme by the simulations.

This modified open loop scheme can be applied to a system which has a frequent load variations such as magnetically levitated train system.

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