# 구조물의 능동제어를 위한 확률신경망 이론

# Probabilistic Neural Network for Vibration Control of Structures

김두기\* 장성규\*\* 김동현\*\*\* 이종재\*\*\*\* Doo Kie Kim Seong Kyu Chang Dong-Hyawn Kim Jong Jae Lee

\_\_\_\_\_\_

#### **ABSTRACT**

구조 재료와 시공기술의 발달로 구조물은 높고 길게 설계할 수 있게 되었으나, 그에 따른 진동 문제와 사용성에 관한 문제가 발생하였고 구조물의 과다한 변위는 구조물에 심각한 손상을 발생 시켰다. 이러한 구조물의 진동 문제를 해결하기 위하여 본 논문에서는 확률신경망이론을 사용한 구조물의 능동제어방법을 제안하였다. 구조물의 제어를 위하여 LQR 제어알고리즘을 이용하여 구 조물의 상태벡터와 제어력을 구한 후, 상태벡터를 입력으로 제어력을 출력으로 하는 확률신경망의 훈련패턴을 구성하였다. 제안된 방법을 사용하여 지진하중을 받는 3층 빌딩구조물을 제어하였고, 기존의 인공신경망의 제어 결과와 비교하였다.

\_\_\_\_\_\_

### 1. Introduction

Civil structures such as high-rise buildings, towers and long span bridges need to be designed safely and reliably under dynamic loads of earthquake, wind, and vehicle. Structures under construction and plan become longer and higher, and economic and beautiful design also comes to be possible with the development of new materials and construction techniques. However, such modern structures are susceptible to excessive structural vibrations, which may induce a problem of serviceability and a structural damage.

Pole assignment, optimal control, adaptive control, Fuzzy control, neural control, etc. have been developing as an active control technique to solve relevant structural vibration problems. Effective and reliable structural control theory started to be published by Yao<sup>1)</sup>. Structural active controls using an artificial neural network(ANN) were published by Chen<sup>2)</sup> and Ghaboussi<sup>3)</sup>. In the paper, they introduced learning ability of neural network in the design of a

<sup>\*</sup> 군산대학교 토목환경공학부 교수

<sup>\*\*</sup> 군산대학교 토목환경공학부 석사

<sup>\*\*\*</sup> 군산대학교 해양시스템공학부 교수

<sup>\*\*\*\*</sup> University of California, Irvine, 박사후연구원

controller to control structural vibration during earthquake. After that, Kim<sup>4)</sup> proposed ANN learning method using the cost function and performed the structural vibration control for three degrees of freedom(3DOF) structure.

In this paper we attempt to control structural vibration using a probabilistic neural network(PNN). The state vectors of the structure and control forces are used for training pattern of PNN. In which, control forces are made by linear quadratic regulator(LQR) control under El Centro(1940), Hachinohe(1968), California(1952) and KS artificial earthquakes. Northbridge earthquake(1994) are used to verify the proposed structural control. Control capability by PNN is compared with that by ANN.

## 2. Active Control Method using Probabilistic Neural Networks

### 2.1 State-space equation

The equation of motion of a structural system with n degrees of freedom subjected to an earthquake and the control force can be expressed as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [\overline{L}_c]\{f_c\} + [\overline{L}_e]\{f_e\}$$
 (1)

where [M], [C] and [K] are, respectively, the  $n \times n$  mass, damping and stiffness matrices;  $\{u(t)\}$  is the n-dimensional displacement vector;  $\{f_c(t)\}$  is the control force vector;  $\{f_c(t)\}$  is the excitation load vector proportional to the ground acceleration,  $\{\ddot{u}_g\}$ ;

and  $[\overline{L}_e]$  and  $[\overline{L}_e]$  are the location matrices of the control force and the excitation, respectively.

$$\{f_e\} = -[M]\{\ddot{u}_g\}$$
 (2)

$$[\overline{L}_c] = <1 \quad 0 >^T \tag{3}$$

$$\left[\overline{L}_{e}\right] = <1 \quad 1>^{T} \tag{4}$$

Premultipling  $[M]^{-1}$  in both sides of equation (1), the equation of motion is as follows

$$\begin{aligned} \{\ddot{u}\} + [M]^{-1}[C] \{\dot{u}\} + [[M]^{-1}[K] \{u\} \\ &= [M]^{-1}[\overline{L}_c] \{f_c\} + [M]^{-1}[\overline{L}_e] \{f_e\} \end{aligned} \tag{5}$$

The corresponding state-space equation is as follows

$$\{\dot{z}\} = [A]\{z\} + [L_e]\{f_e\} + [L_e]\{f_e\}$$
(6)

$$\{z(t)\} = \langle u(t) \quad \dot{u}(t) \rangle^T \tag{7}$$

$$[A] = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$
 (8)

$$[L_c] = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \overline{\mathbf{L}}_c \end{bmatrix}, \qquad [L_e] = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \overline{\mathbf{L}}_e \end{bmatrix}$$
(9)

Where  $\{z(t)\}$  is state vector; [A] is system matrix; and  $[L_c]$  and  $[L_e]$  are location matrices, respectively, corresponding to the locations of controllers and external excitations in the state space<sup>5)</sup>.

### 2.2 LQR Control Algorithm

In this paper, linear quadratic regulator(LQR) is used to compose training patterns of PNN. According to LQR control method, the control force  $\{f_c(t)\}$  is to be chosen in such a way that a performance index (J), defined as

$$J(f_c) = \int_0^t (\{z\}^T [Q]\{z\} + \{f_c\}[R]\{f_c\}) dt$$
 (10)

In equation (10), [Q] is a  $2n \times 2n$  positive semi-definite matrix; [R] is a  $m \times m$  positive semi-definite matrix; and control forces with m degrees of freedom are applied. [Q] and [R] are referred as weighting matrices. The optimal control force for LQR is denoted as follows

$${f_c} = -[G]{z} = -[R]^{-1}[B^T][S]{z}$$
 (11)

Where [G] is the control gain, and the solution of Riccati equation [S] is obtained from equation (12).

$$[A^{T}][S] + [S][A] - [S][B][R]^{-1}[B]^{T}[S] + [Q] = 0$$
(12)

Therefore, the control force is calculated by the product of the control gain([G]) and state vector of system( $\{z\}$ )<sup>6)</sup>.

# 2.3 Probabilistic neural networks

PNN is basically a pattern classifier that combines the well-known Bayes decision strategy with the Parzen non-parametric estimator of the probability density functions of different classes<sup>7)</sup>. PNN has gained interest because it offers a way to interpret the network's structure in the form of a probability density function and it is also easy to implement. An accepted norm for decision rules or strategies used to classify patterns is that they do so in a way that minimizes the "expected risk." Such strategies are called "Bayes strategies" and can be applied to problems containing any number of classes.

Consider the two-category situation in which the state of nature  $\theta$  is known to be either  $\theta_A$  or  $\theta_B$ . If it is desired to decide whether  $\theta = \theta_A$  or  $\theta = \theta_B$  based on a set of

measurements represented by the p-dimensional vector  $X' = [X_1 \dots X_j \dots X_p]$ , the Bayes decision rule becomes

$$d(\mathbf{X}) = \theta_A \quad \text{if} \quad h_A l_A f_A(\mathbf{X}) > h_R l_R f_R(\mathbf{X}) \tag{13a}$$

$$d(\mathbf{X}) = \theta_B \quad \text{if} \quad h_A l_A f_A(\mathbf{X}) < h_B l_B f_B(\mathbf{X})$$
 (13b)

where  $f_A(X)$  and  $f_B(X)$  are the probability density functions for categories A and B, respectively;  $l_A$  is the loss function associated with the decision  $d(X) = \theta_B$  when  $\theta = \theta_A$ ;  $l_B$  is the loss associated with the decision  $d(X) = \theta_A$  when  $\theta_A = \theta_B$  (the losses associated with correct decisions are taken to be equal to zero);  $h_A$  is the priori probability of occurrence of patterns from category A; and  $h_B = 1 - h_A$  is the priori probability that  $\theta = \theta_B$ . In the simplified case that assumes both loss function and a priori probability are equal to each other, the Bayes rule classifies an input pattern to the class that has its probabilistic density function(PDF) greater than the PDF of the other class. Therefore, the accuracy of the decision boundaries depends on the accuracy with which the underlying PDFs are estimated. Parzen<sup>8)</sup> showed how one may construct a family of estimates of  $f(\mathbf{X})$ , and Cacoullos<sup>9)</sup> has also extended Parzen's results to estimates in the special ease that the multivariate kernel is a product of univariate kernels. In the particular case of the Gaussian kernel, the multivariate estimates can be expressed as

$$f_{A}(\mathbf{X}) = \frac{1}{(2\pi)^{p/2} \sigma^{p}} \frac{1}{m} \sum_{i=1}^{m} \exp \left[ -\frac{(\mathbf{X} - \mathbf{X}_{Ai})^{T} (\mathbf{X} - \mathbf{X}_{Ai})}{2\sigma^{2}} \right]$$
(14)

where  $\mathbf{X}$  is the test vector to be classified;  $f_A(\mathbf{X})$  is the value of the PDF of category A at point  $\mathbf{X}$ ; m is the number of training vectors in category A; p is the dimensionality of the training vectors;  $\mathbf{X}_{Ai}$  is the  $i^{th}$  training vector for category A; and  $\sigma$  is the smoothing parameter. Note that although  $f_A(\mathbf{X})$  is simply the sum of small multivariate Gaussian distributions centered at each training sample, the sum is not limited to being Gaussian.

## 2.4 Active control using PNN

In order to apply the PNN theory in the vibration control of the structure, the rule base of the PNN needs to be composed. In this study, the rule base is made by a state vector and a control force as an input and an output, respectively. The state vector and control force are derived by LQR control algorithm for arbitrary earthquakes, and then the proposed PNN control algorithm is verified for new earthquakes. Fig.'s 1 and 2 show the basic control flowchart and controller using the PNN control algorithm.

### 3. Numerical Application

To illustrate the proposed method, a model of three story shear building with the active tendon control system is considered as shown in Fig. 3. The state vector of the structure and the control force that are derived by LQR control under El Centro, Hachinohe, California and KS artificial earthquakes are used for training pattern of the PNN. Then Northbridge earthquake is used to verify the proposed PNN control algorithm.

### 3.2 Comparison of the control capability of the PNN and the ANN

In this section, we compared the control capability of the PNN with that of the ANN. Controlled and uncontrolled responses under Northbridge earthquake are shown in Fig.'s 4 and 5. From the figures, the structural displacement and velocity responses have been suppressed effectively by the both control algorithm. In which, the decreasing rates of the maximum displacement in the first floor by the PNN and the ANN algorithm are 51.9% and 54.8%, respectively. Although the decreasing rates by the PNN and the ANN are not noticeable, the PNN control algorithm has many strong points over the ANN control algorithm.

The ANN needs a re-training process and much computational time in training the network. However, the PNN as a pattern classifier needs less time to determine the architecture of the network and to train the network. Moreover, the PNN provides the probabilistic viewpoints as well as deterministic classification results in order to consider the uncertainties in the control process.

#### 4. Conclusions

In this study, a control method using the PNN is proposed for a structural vibration subjected to earthquakes. The proposed algorithm is applied and verified for the vibration control of the three story shear building under earthquakes. The PNN control shows good results, and the results are compared with those of the ANN.

### Acknowledgment

This work is a part of a research project supported by Korea Ministry of Construction & Transportation (MOCT) through Korea Bridge Design & Engineering Research Center at Seoul National University. The authors wish to express their gratitude for the financial support.

### References

- 1. J. T. P. Yao., "Concept of structural control", J. Struct. Division, ASCE, ST 7, 1972, pp1567-1573.
- 2. Chen, H.M., Tsai, K.H., Qi, G.Z., Yang, J.C.S. and Amini, F., "Neural network for structural control", J. Comput. Civil Engrg., ASCE, Vol. 9, No. 2, pp.168-176.
- 3. Ghaboussi, J. and Joghataie, A., "Active control of structure using neural networks", J. Engrg. Mech., ASCE, Vol. 121, No. 4, pp.555-567.
- 김동현, 한상훈, 박우선, 서승남, 이인원, "구조물의 진동제어를 위한 신경망제어기의 학습법", 대한토목학회, 제 21 권, 제 5-1 호, 2001, pp.657-663.
- 5. 김두기, "구조동역학", 구미서관, 2005
- 6. Soong, T. T., "Active Structural Control", Longman Scientific and Technical, 1990
- 7. Specht, D. F., "Probabilistic Neural Networks", Neural Networks 3, 1990, pp.109-118.
- 8. Parzen, E., "On estimation of a probability density function and mode", Annals of Mathematical Statistics, 33, 1962, pp.1065-1076.
- 9. Cacoullos, T., "Estimation of a multivariate density", Annals of the Institute of Statistical Mathematics (Tokyo), 18(2), 1966, pp.179-189.

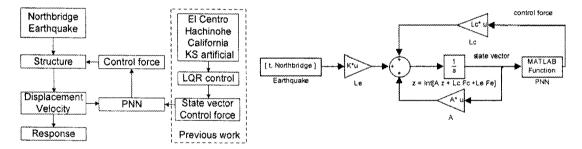


Fig. 1 Control flowchart

Fig. 2 Block diagram of PNN controller

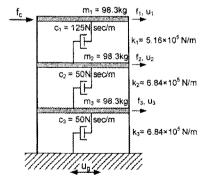


Fig. 3 Three story shear building

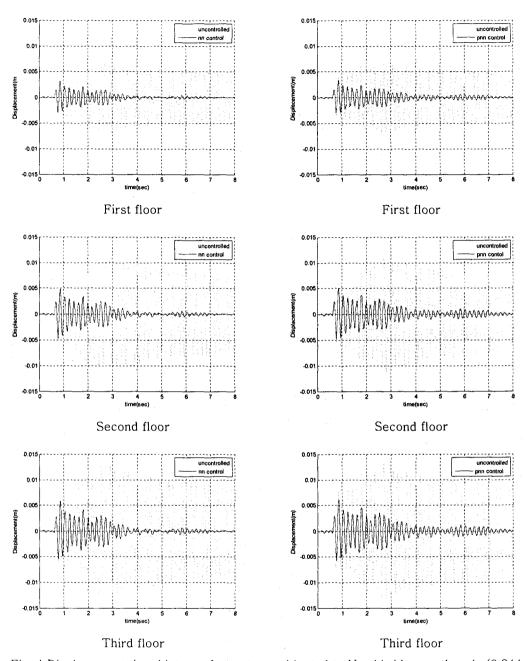


Fig. 4 Displacement time history of structure subjected to Northbridge earthquake(0.344g)

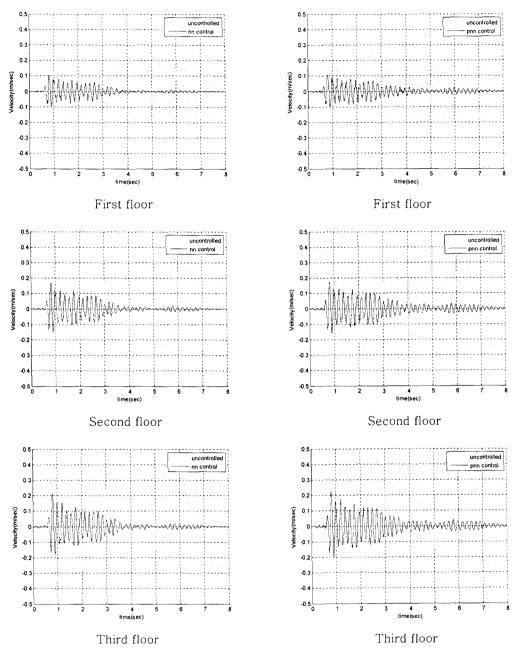


Fig. 5 Velocity time history of structure subjected to Northbridge earthquake(0.344g)