Stress and Strain for Perated Tensile Specimen
- Experiemental Measurements and FEA Simulations

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ABSTRACT

The strain distribution in the vicinity of a hole in a tensile strip was measured using an image correlation method. The objective of this study is to evaluate the capability of predicting the strain component response using a constitutive model that was developed for use with paper materials. The need for a special constitutive model for paper derives from the characteristics of pronounced anisotropy and the fact that the material behaves differently under compressive loading than it does under tensile loading. The results of the simulation showed that predictions of strain distribution around the hole were in agreement with the experimental result trends, however, the agreement deteriorated as the edge of the hole was reached. It was observed that there is extensive inelastic strain that takes place around the hole prior to failure of the tensile strip. The simulation results showed that any difference between tensile and compressive behavior that may exist for paper material does not have any significant effect for the problem of this study because the level of compressive stress is quite low in comparison with compressive failure values.

INTRODUCTION

The presence of the hole in a tensile strip causes the stress and strain components to be much higher than would be the case without the hole [1]. Consequently, failure of the plate resulting from high loads will be initiated at the edge of the hole. The principal issue is the problem of how to best model the state of stress and strain in a situation where the solution varies through the plane of the structure. In the case of a brittle material, the solution can be achieved through the theory of linear elasticity. For a material such as paper, the elasticity solution is not appropriate when the load approaches the failure value since paper behaves in a strongly nonlinear manner. As a result of the complex nonlinear material behavior and because the stress and strain distribution varies from point to point in the field, the solution of problems of this type are often sought using the finite element method.

The finite element approach is most easily performed through the use of a commercially available software package. The choice of the software package is also influenced by the constitutive models available for the modeling of complex material behavior. In this respect, the study of problems involving paper materials presents a challenge because of the fact that paper exhibits a very nonlinear behavior, because its characteristics are very strongly anisotropic and because the material nonlinearity depends on whether the loading is tensile or compressive.

It is well known that the compressive strength of paper is markedly lower than the tensile strength [2]. On the other hand, it is generally agreed that the elastic modulus is the same for compression loading as it is for tensile loading. Since the compressive strength is lower than the tensile strength it is reasonable to believe that the nonlinear uniaxial stress-strain curve should differ for tensile and compressive loading. Furthermore, compressive failure in paper does not result in fracture, but rather is manifested by a pronounced nonlinear inelastic deformation.

A 3-D constitutive model was proposed by Shih and Lee [3] for the response of metals. This model is a classical time-independent plasticity model that incorporates the effect of material anisotropy and the property of having different behavior under uniaxial tension and compression. The Shih-Lee model has been implemented for use in the ANSYS software. Our experience with the Shih-Lee model is that it is difficult to implement this model for paper materials due to the rather greater degree of anisotropy for paper than would be expected for metals. In some cases, it does not appear to be possible to find any set of model parameters that will successfully predict the stress-strain curves for the machine direction, MD, and cross machine direction, CD. Furthermore, for the case of a plane stress problem as in the present work, the 3-D model requires the specification of material properties in the paper Z-direction. Information regarding the Z-direction mechanical response is very difficult to obtain. The Shih-Lee model was developed for use with metals and makes use of the assumption that the material is incompressible during plastic straining. This assumption is not appropriate for the case of paper materials.

There have been a number of other constitutive models that have been proposed for predicting the nonlinear mechanical behavior of paper materials. A 2-D model for paperboard was presented by Johnson and Urbanik [4]. This model does not permit the modeling of differences in tension and compression behavior. A J_2-flow theory model was proposed by Makela and Ostlund [5] and a continuum damage model was proposed by Isaksson and Hagglund [6]. Neither of these models permit difference in behavior for uniaxial tension and compression. A 3-D time-independent plasticity model was proposed by Xia, Boyce et al [7] for paper materials. They demonstated that the uniaxial stress-strain behavior of paper can be successfully predicted in the MD, CD and along a 45-degree orientation. This model also accounts for differences in tensile and compression behavior. Although we do not know of any published work where this constitutive model was used to solve a field problem involving paper materials, this model appears to have the necessary attributes for the analysis of complex problems involving paper. This model would probably be the choice of paper physics analysts if it were available in commercial software such as ANSYS or ABAQUS.

A mesomechanics constitutive model for paper materials was proposed by Ramasubramanian and Perkins [8] to predict the nonlinear stress-strain behavior of a uniaxial tensile strip based on the mechanics solution of a typical fiber that is bonded to the paper network. This model was used by Sinha [9] as the starting point for a general consitituve model for paper. Sinha and Perkins [10] discussed the implementation and use of this approach to perform 2-D finite element analysis using the ABAQUS software .This model is referred to as the method of effective mesoelements Perkins [11]. It is the objective of the present paper to investigate the application of this model to the problem of the finite element analysis of a paper tensile strip with a central hole.

Modeling

General Aspects of the Problem and the Approach

The mechanics problem consisted of a tensile strip with a central hole as illustrated in Figure 1. The loading was applied at the right edge by displacement control provided by an Instron machine. The left edge was fixed. The hole diameter was 0.635 cm. The sample length was 26 cm. Three specimen widths were studied: 1.6 cm, 4.3 cm and 8.9 cm. Because of the symmetry of the specimen and loading, the finite element analysis consisted of a ¼ model. Experimental measurement of the strains using an

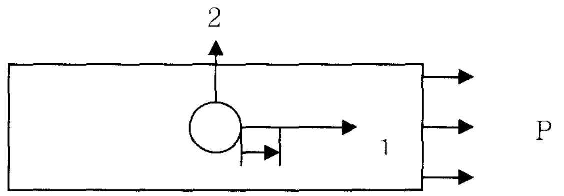

Figure 1. Tensile specimen with central hole.

image correlation method was focused on the regions emanating outward from the hole along the 1 and 2 coordinate directions. The specimen material consisted of a medium weight paper having a basis weight of 122 g/m^2 and a density of 676 Kg/m^3.

It is well known that for the problem of the tensile strip with a central hole there are regions where the stress components are tensile and other regions where the stress components are compressive. Referring to Figure 1, in the region where the 1-axis intersects the hole edge, the value of S_{22} has negative values. As mentioned above, and as discussed in detail by Bronkhorst and Bennett [2], the mechanical behavior of paper is dependent on whether the loading is tensile or compressive. Therefore, for finite element analysis of this problem it is appropriate to use a constitutive model that incorporates this characteristic.

The development of the method of effective mesoelements mentioned above was motivated by the desire to have a simple physically based constitutive model that could incorporate the difference in behavior under tensile and compressive loading. The effective mesoelement method is based on the mechanics of a typical mesoelement situated in the macroscopic deformation field. In the case of paper which is comprised of a network of pulp fibers, the mesoelement can be taken as a typical fiber that is bonded to its neighboring fibers. The fibers lie principally in the plane of the sheet and are preferentially oriented along the paper machine direction, the so-called machine direction, MD. Since there are more fibers oriented in the MD than in the cross-machine direction, CD, the stress-strain curve is higher for the MD than it is for the CD. Perpendicular to the sheet or Z-direction, the mechanical properties are on the order of 50 times lower than they are for the plane of the sheet. Therefore, paper materials exhibit a rather severe anisotropy. Most researchers model the system as having orthortropic symmetry. It is reasonable to assume that fibers loaded in compression can be expected to have a lower load carrying capacity than when they are loaded in tension. Whereas fibers in tension can be expected to fail by fracturing, fibers loaded in compression may fail due to an inelastic buckling. Up to the point of buckling, however, the compressively loaded fibers can be expected to behave in a similar fashion to that of the tensile loaded fibers.

The mechanics analysis of the mesoelement is essentially a one-dimensional analysis. The mesoelement deforms according to the mesoelement applied strain which is taken to be the macroscopic strain in the direction of the orientation of the mesoelement. The mesoelement is modeled as having a two-slope behavior consisting of an elastic part followed by a yield point and a second-slope for loading beyond the yield point. Mesoelements are assumed to behave differently depending on whether the mesoelement strain is tensile or compressive. The analysis consists of the calculation of incremental strain, therefore, the history of loading of the mesoelement must be taken into account. In addition, the mesoelement characteristics are assumed to be influenced by drying restraint that takes place during paper manufacture. Drying restraint of the sheet is believed to increase the initial modulus, yield point and tangent modulus when the mesoelement is in tension during drying. The model has the following parameters: Initial elastic modulus of mesoelement, yield point for tension and compression and second slope beyond the yield point for tension and compression. Each of these values also has an associated drying restraint parameter. In addition, the fiber orientation distribution and the sheet density are used to predict the macroscopic behavior taking into account the number of fibers crossing a line of unit length. The 2-D version of the constitutive model is, therefore, dependent on the specification of a total of 12 mesoelement parameters and 2 sheet properties of density and mesoelement orientation distribution. A trial and error method is used to select the parameters so that the model can accurately predict the nonlinear uniaxial stress-strain curve in the MD and CD material directions.

The complete set of analysis details for the 2-D system are provided by Sinha and Perkins [10].

It was found that in the present investigation it was only necessary to specify 9 of the total 14 parameters. The values of parameters that were observed to successfully model the sample material used in this study are shown in Table 1.

Table I. Effective mesoelement parameters

Parameter	Model 1
Sheet density	676 Kg/m^3
Elliptical fiber orientation distribution	5.0
Mesoelement initial modulus	7 GPa
Mesoelement tensile yield	29 MPa
Mesoelement compressive yield	29 Mpa
Mesoelement tensile tangent modulus	5e8 Pa
Mesoelement compressive tangent modulus	5e8 Pa
Drying restraint factor for the MD	0.04
Drying restraint tangent modulus hardening parameter	70.0

The physical implications of this set of parameters is that the ratio of the number of mesoelements in the MD to those oriented in the CD is 5. The effect of drying restraint is to harden tangent modulus of the mesoelements in the MD preferentially. The amount of the increase in tangent modulus is 1+.04*70.0=1.28. Therefore, the tangent modulus in the MD is 28 percent higher than the modulus in the CD. It is found that it is necessary to take drying restraint hardening into account in order to be able to simultaneously match the MD and CD stress-strain curves. As explained below, it was found that it was not necessary to assume that there is any difference between the tension and compression stress-strain behavior for the present problem.

The mesoelement model described above is based on the assumption that it is possible to define a single-valued function that provides the average mesoelement stress depending on the total mesoelement strain. As such, it can be used in problems that involve proportional loading and a single reversal of loading. The approach has not been investigated for use in cases where cyclic loading regimes are involved.

Finite Element Model

The tensile strip with central hole was modeled with a ¼ model due to the symmetry of the geometry and loading conditions. Loading was implemented by specifying the displacement of nodes on the boundary to correspond with the values used in the experiments. The model was a plane stress model with dimensions of length (26 cm), width (1.6cm, 4.3 cm or 8.9cm) and thickness (1.8 mm). The ABAQUS model used 6-node triangle elements (CPS6). The UMAT routine [12] was used to program the effective mesoelement constitutive model.

Experimental Characterization of the Paper Material

The paper material employed in the study is described in Part 1. Uniaxial tension stress-strain data were obtained using two methods: grip displacement and image correlation. The tensile specimen (length=23 cm, width=3.9 cm) was placed in the Instron test machine and loaded longitudinally using displacement control. The speed of the crosshead was 0.25 mm/min. Uniaxial tests were performed for samples having orientations MD, CD and 45 degrees. During the uniaxial loading and unloading period, images were taken at regular intervals of displacement for correlation using a Sony camcorder. The stress-strain curves obtained by image correlation and grip displacement yielded essentially identical results. The unloading curve was observed to exhibit a slope similar to that of the initial loading slope. The behavior is therefore strongly nonlinear, exhibits large plastic components and indicates that the material is strongly anisotropic.

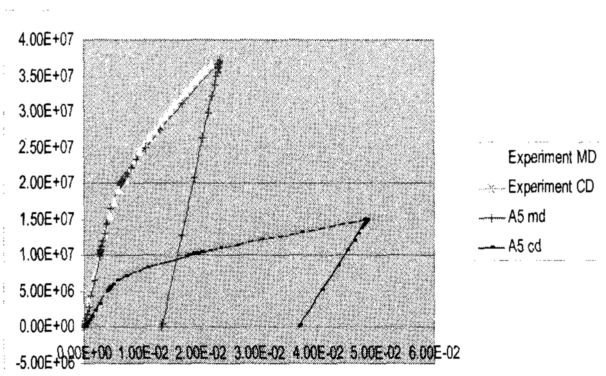

Figure 2. Experimental stress-strain curves for MD, CD and the results of the model simulation using constitutive models 1 and 2.

It is observed from Figure 2 that the model predicts both the CD and the MD curves very closely. The experimental lateral contraction ratios, LCR, and the model simulations are shown in Figure 3.

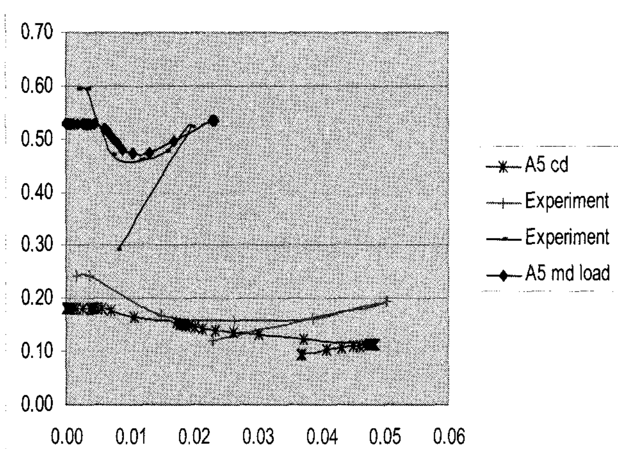

Figure 3. Lateral contraction ratios for MD and CD.

Note that the constitutive model provides results for LCR that are comparable to those obtained experimentally, especially given the variation that exists from point to point within a given sample as well as from sample to sample. The model shows a slight decrease in LCR beyond the elastic limit. Similarly, the experimental data shows a decrease in LCR, but then is followed by an increase in LCR at higher strains. Comparison of experimental and simulation behavior both for the stress-strain curves and for the LCR may be useful in the process of selecting the best set of model parameters.

Although it is widely accepted that the compressive loading stress-strain curve and failure point are very different than that for tensile loading, it turns out to be extremely difficult, if not impossible, to experimentally measure these characteristics [2]. An attempt was made to obtain the stress-strain curve with compressive loading using the cylinder test. The compressive stress-strain curves that were obtained were always lower than the corresponding curve obtained with tensile loading. However, close inspection of the results indicated that the initial slope, the elastic modulus, was different for compression than for tension. We believe that this result is not reasonable and we suspect that the cylinder results are simply inadequate to provide reliable data. It was not possible to compare LCR values for experiment and simulation in the case of compression loading. The true behavior of paper in uniaxial compression must await the development of a more satisfactory test procedure.

Tensile tests were conducted to determine the tensile strength of the paper material used in this research.

Comparison of Experimental Results and FEA simulation

Simulations and comparisons were carried out for a variety of directions of loading, load level and sample width. The results are quite similar in form, therefore, it suffices to illustrate the results with the example of the MD direction, load levels 3 and 4, and the 4.3 cm sample width. Figure 4 illustrates the comparison between experiment and simulation for strains E_{11} and E_{22} along the x_2 axis at load level 3. The applied strain for this load level is approximately 0.8 percent. The experimental and simulation results appear to converge to this value as the distance from the hole increases. It appears that the simulation and the experiment resemble each other in form, however, as the hole edge is approached the difference between experiment and simulation is significant.

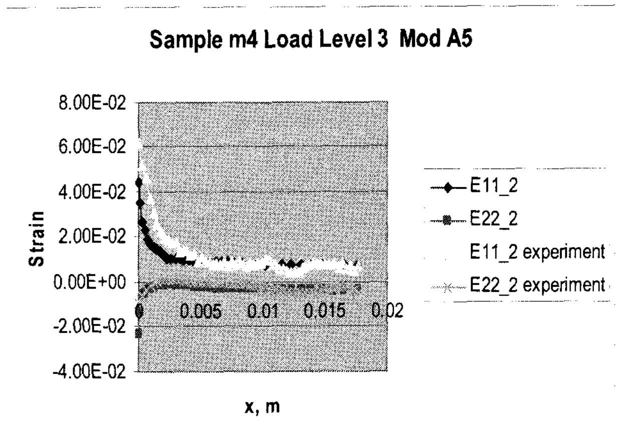

Figure 4. Comparison of experimental and simulation results for strains E_{11} and E_{22} along the x_2 axis.

The simulation model stresses S_{11} and S_{22} also along the x_2 axis are shown in Figure 5. It is noted that the applied stress at load level 3 is 23 MPa. This is quite close to the observed failure stress of 29 MPa.

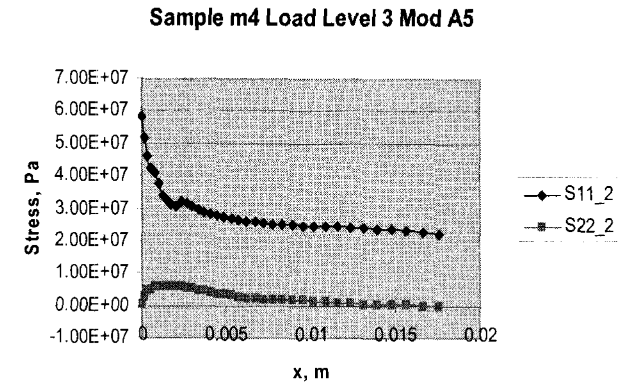

Figure 5. Simulation of stresses S_{11} and S_{22} along the x_2 axis for MD load level 3.

Inspection of Figures 4 and 5 shows that the conditions of failure are significantly above the failure stress and strain values. It is believed that the paper has failed progressively in a stable fashion in the vicinity of the hole at load level 3. This progressive failure is assumed to continue until eventual sample catastrophic failure occurs. The reason for the jump in the S_{11} curve is believed to be caused by the mesh structure combined with the extrapolation of results along the x_2 path. It is noted that the presumed progressive failure at the hole edge may have had a significant effect on the measurement of strains at this load level. It should also be noted that the simulation of stress S_{11} near the hole edge is meaningless if failure has actually taken place, as is suspected. No attempt was made to investigate the physical existence of the progressive failure after the tests were performed. The finite element simulation would be improved by taking into account element failure as the hole edge is approached. Work along these lines is planned for the future.

The comparison of simulation and experiment of stress components along the x_1 axis is shown in Figure 6. In this case, it is observed that the simulation and experiment are very different as the hole edge is approached. It is believed that the damage experienced near the hole edge and the presence of the hole in the image processing have rendered the experimental results unreliable.

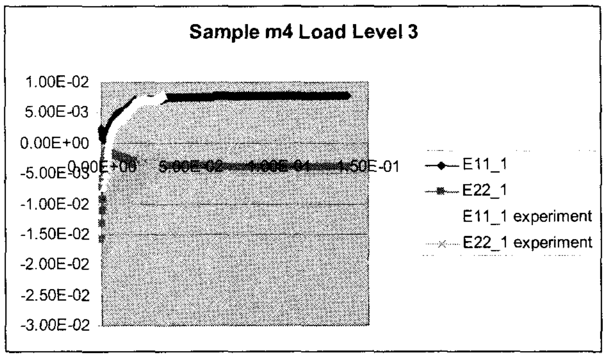

Figure 6. Simulation and experiment for strains E_{11} and E_{22} along the x_1 axis at load level 3.

The simulation stress components along the x_1 axis are shown in Figure 7.

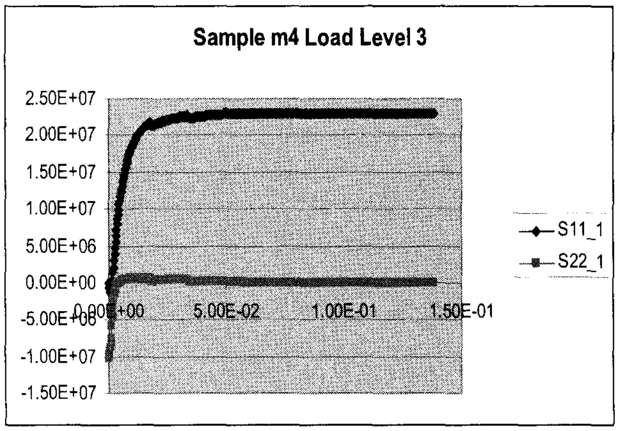

Figure 7. Stress components along the x_1 axis for MD load level 3.

Note that the stress S_{22} in Figure 7 is about -10 MPa as one approaches the hole edge. It may be suspected that this compressive stress may have been high enough to cause localized buckling of the paper in the vicinity of the hole. If localized buckling did actually take place, that could have had an effect on the validity of the experimental data shown in Figure 6 as well as the tendency for the experimental results to exhibit a significantly different behavior as a function of distance from the hole edge than that shown by the simulation.
As shown in Figure 7, it is apparent that the maximum compressive stress, S_{22} at the hole edge is predicted to be about 10 MPa. Perhaps the compressive stress does not reach a high enough value in this problem to make it necessary to take into account any difference in tension and compression behavior. Although, as discussed above, it was not possible to experimentally measure the compressive behavior of the paper, it is believed that paper compressive failure would be higher than the levels predicted by the simulation. As a result of this observation, it was judged unnecessary to introduce difference in compression parameters. Nevertheless, further work is

needed in this area.

RECOMMENDATIONS

As mentioned above, an attempt was made to measure the stress-strain curve under conditions of uniaxial compressive loading. This is referred to as edgewise compressive loading. As discussed by Bronkhorst and Bennett [2], all of the known methods for measuring the edgewise compressive strength have deficiencies. The distinction between actual material compression failure and nonlinear buckling is very difficult to separate. It is felt that a great need exists in this area in order to understand the true behavior of paper in nonlinear response up to and including failure under compressive loading.

Although it was found that the maximum compressive stress values that probably occurred along the x_1 axis at the hole edge may not have been high enough to have required the constitutive model to exhibit a difference in tension and compression behavior, this is an area for further research. The state of stress around the hole edge involves a combination of stress components. The question of how to model failure in such a complicated problem needs to be explored in greater detail. This area is of special importance in the case of the hole problem because it appears likely that extensive damage occurs at the hole edge prior to the eventual catastrophic failure of the sample.

Additional research for problems where the maximum compressive stress values reach higher levels should also be undertaken.

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