

# Magnetic Field Inversion and Intra-Inversion Filtering using Edge-Adaptive, Gapped Gradient-Nulling Filters: Applications to Surveys for Unexploded Ordnance (UXO)

R. M. René <sup>1)</sup>, K.Y. Kim <sup>2)</sup>, C. H. Park <sup>3)</sup>

<sup>1)</sup> René Geophysics, Bloomington, Indiana, U.S.A.

<sup>2)</sup> Kangwon National University, Chuncheon, Korea

<sup>3)</sup> Korea Ocean Research & Development Institute, Ansan, Korea

**Abstract** : Estimations of depth, magnetic orientation, and strength of dipole moments aid discrimination between unexploded ordnance (UXO) and non-UXO using magnetic surveys. Such estimations may be hindered by geologic noise, magnetic clutter, and overlapping tails of nearby dipole fields. An improved method of inversion for anomalies of single or multiple dipoles with arbitrary polarization was developed to include intra-inversion filtering and estimation of background field gradients. Data interpolated to grids are flagged so that only nodes nearest to measurement stations are used. To apply intra-inversion filtering to such data requires a gapped filter. Moreover, for data with significant gaps in coverage, or along the edges or corners of survey areas, intra-inversion filters must be appropriately modified. To that end, edge-adaptive and gapped gradient-nulling filters have been designed and tested. Applications are shown for magnetic field data from Chongcho Lake, Sokcho, Korea and the U. S. Army's Aberdeen Proving Ground in Maryland.

**Keywords:** magnetic field inversion, intra-inversion filter, edge-adaptive filter, gapped filter, gradient-nulling filter, unexploded ordnance (UXO), Sokcho, Aberdeen Proving Ground

**요약** : 자기쌍극자 모멘트의 깊이, 방향, 크기 등에 관한 평가는 자기탐사자료로부터 폭발물과 기타 자성체를 구분하는데 유용한 정보를 제공한다. 이러한 평가는 지질학적 잡음, 자기분산체, 주변 쌍극자기장 중첩 등의 이유로 방해받을 수 있다. 임의의 극성을 갖는 단일 혹은 다중 쌍극자 이상체의 효과적 계산을 위하여, 역산내부 필터 및 배경 자기장의 구배 평가를 포함하는 개선된 역산법을 개발하였다. 관측값들은 보간하여 격자화하였으며, 관측점으로부터 가장 가까운 계산점만을 사용하도록 표시하였다. 이러한 자료에 역산내부 필터를 적용하기 위하여는 빈간격 필터가 필요하다. 게다가 상당히 빈 곳이 많은 자료나 조사지 가장자리 및 구석 부분 자료들을 처리하기 위해서는 역산내부 필터를 수정하여야 한다. 이러한 목적으로 가장자리에 적용가능한 빈간격 구배제거 필터를 고안하고 시험하였다. 한국 속초의 청초호와 미국 매릴랜드의 육군 애벌딘 무기시험장에서 기록한 자기탐사자료에 적용한 결과를 소개한다.

**주요어** : 자기장 역산, 역산내부 필터, 가장자리 적용 필터, 빈간격 필터, 구배제거 필터, 폭발물(UXO), 속초, 애벌딘 무기시험장

## 1. Introduction

Magnetic and electromagnetic methods are the principal methods used in surveys to locate, identify and discriminate unexploded ordnance. As only UXO require excavation and disposal, the costs of UXO cleanup are reduced by proper characterization of magnetic sources. Estimations of depth, magnetic orientation and strength of magnetic dipole moments aid discrimination between unexploded ordnance (UXO) and non-UXO using magnetic surveys. Because UXO are demagnetized by firing and impact they are likely to have less remanence than most non-UXO. Effective dipoles of demagnetized UXO will generally vary by less than 60 degrees from the direction of the earth's field.

Geologic noise, magnetic clutter, and overlapping tails of nearby dipole fields may degrade estimates of magnetic source parameters for target anomalies. Although intra-inversion estimates of background field gradients may improve some inversions, intra-inversion filtering may be more effective in many cases. In this method, a pre-inversion filter is applied to the observed field and then the same filter is applied to the fields of elements used to construct the inverse model. The component

elements of the inverse model may be unit dipoles, as in the present application, or small prisms, as in the case of growing-model inversion of gravity or magnetic fields. For magnetic field inversion, such filtering has been demonstrated on lake data using “profile-adaptive” filtering (Park et al., 2002) and on synthetic UXO data using 2-dimensional filters applied to gridded synthetic data (René et al., 2004).

This paper will present some improvements to the inversion method of René et al. (2004). These include the use of edge-adaptive, gapped gradient-nulling (EAGGN) intra-inversion filters and intra-inversion estimation of background field gradients. Applications of EAGGN filters are demonstrated for magnetic surveys of a lake in Korea and the U.S. Army’s Aberdeen Proving Ground, Maryland.

## 2. Inversion method

The input magnetic field,  $H_{ij}$ , is specified at grid locations  $(X_i, Y_j)$  with constant intervals of  $\Delta x$  and  $\Delta y$  on the measurement surface ( $Z = 0$ ). The simple least-squares method minimizes the objective function,  $\Phi$ , which depends on the inverse-modeled dipole’s field,  $D$ , observed field,  $H$ , dc-bias,  $K$ , and optionally, the background magnetic field gradients,  $G_x$  and  $G_y$ :

$$\Phi_{lmn} = \sum_{ij} [D_{ijlmn} + K_{lmn} + G_{xlmn}(X_i - X_o) + G_{ylmn}(Y_j - Y_o) - H_{ij}]^2; \quad (1)$$

where the magnetic anomaly of a test magnetic dipole at the location  $(X_l, Y_m, Z_n)$  is

$$D_{ijlmn} = \sum_k (U_{ijklmn} M_{klmn}), \quad (2)$$

$M_{klmn}$  are magnetic moments, and  $U_{ijklmn}$  are the magnetic field anomalies of unit-strength dipoles at the test magnetic dipole location in a user-specified window of possible dipole locations. Indices  $k$  indicate the unit dipole directions of magnetization. Rao et al. (1977) provide formulae for  $U_{ijklmn}$ . For each possible location, magnetic moments,  $M_{klmn}$ , dc bias,  $K_{lmn}$ , and, optionally, the gradients,  $G_{xlmn}$  and  $G_{ylmn}$  that minimize the objective function are obtained as solutions to a system of six or four linear equations, depending on whether the gradients are used. The reference point  $(X_o, Y_o)$  is at the data window’s center.

$M_{klmn}$ ,  $K_{lmn}$ , and optionally  $G_{xlmn}$  and  $G_{ylmn}$  are evaluated for all test locations. The best solution is then obtained for the dipole location,  $X_L$ ,  $Y_M$ , and  $Z_N$  that minimizes  $\Phi$ . An extension to simultaneous inversion of fields for  $N_D$  dipoles requires the solution of larger systems of equations. For two dipoles, a system of nine linear equations would be needed if  $G_x$  and  $G_y$  are estimated. René et al. (2004) demonstrated multidipole inversions with intra-inversion filtering of synthetic data where target fields overlapped significantly. If the grid spacing is sufficiently fine, then potential dipole locations can be tested at fewer than all the nodes of the dipole window on a first pass using increments of dipole positions greater than the grid intervals. Second or third passes using smaller windows of possible locations can then be used to refine the optimum solution for the dipole location. The initial dipole search window must be large enough to preclude the problem of local minima in  $\Phi$ .

**Intra-inversion Filtering.** In the case of overlapping fields of nearby UXO and geologic noise, two-dimensional highpass intra-inversion filtering that greatly distorts the target field may improve the inversion (René et al., 2004). In this method the observed magnetic field,  $H$ , is filtered and the fields of the unit dipoles,  $U$ , are filtered with the same filter (Park et al., 2002; René et al., 2004). The term “intra-inversion” filtering serves to distinguish this method from that of filtering only the input data with a filter that is considered mild enough so as not to significantly distort the target field. Pre-inversion filtering is often used in UXO inversions. The input data are filtered along the direction of profile with a demedian filter of sufficient length to remove geomagnetic variations without the necessity of using basestation measurements. This method also removes dc-bias and some low spatial frequency components of a regional field along the direction of profiling.

Where data are available at all grid nodes, intra-inversion filtering using equation (1) can be computationally efficient. For each depth  $Z_n$  it is only necessary to filter the unit-dipole field once at the corner location ( $l = m = 0$ ). The filtered unit dipole field is then shifted laterally in the  $x$ - and  $y$ -directions to obtain the field needed at each dipole test location. Where one cannot reasonably interpolate data to all nodes of a regular grid, intra-inversion filtering is less efficient yet potentially practical. Gapped filters will ignore data at grid nodes relatively distant from the measurement stations including nodes that extend beyond the edges of the survey area and that occur in data gaps where, for example, rough terrain or vegetation does not allow access by cart-mounted magnetometers.

### 3. Edge-adaptive and Gapped Gradient-Nulling Filters

For a symmetric filter with coefficients  $F_{mn}$ , the filtered magnetic field,  $H'$ , at the center of an  $M \times N$  filter is

$$H'_{ij} = \sum_{mn} F_{mn} H_{i+m,j+n}; \quad m = -M, M; \quad j = -N, N \quad (3)$$

Applying this filter to a constant gradient field,  $R_{ij} (= a X_i + b Y_j + c)$  yields a nulled filtered field,  $R' = 0$ , if the filter is zero-dc ( $\sum F_{m,n} = 0$ ) and symmetric ( $F_{-m,n} = F_{m,n}$ ;  $F_{m,-n} = F_{m,n}$ ). Such a filter is appropriately called gradient-nulling (GN). The GN filter may have rectangular, elliptic, or other symmetric geometry.

If data are not available at every point, one may derive a GN filter,  $F^*$ , that varies from point-to-point. Filter coefficients will only be defined for locations where input data are defined or flagged for use in inversion. The filtered magnetic field,  $H^*$ , is obtained only at nodes flagged for input:

$$H^*_{ij} = \sum_{m,n} F^*_{i,j,m,n} H_{i+m,j+n} E_{i,j,m,n}; \quad m = -M, M; \quad j = -N, N; \quad (4)$$

$E_{ijmn}$  is unity where  $H_{i+m,j+n}$  is defined for use, and zero elsewhere. Subscripts  $i$  and  $j$  for the filter coefficients,  $F^*_{i,j,m,n}$ , indicate that the filter varies from point-to-point.

These modified filter coefficients will be related to the original coefficients as follows:

$$F^*_{m,n,i,j} = F_{m,n} (1 + K_{xij} X_{i+m} + K_{yij} Y_{j+n}) C^*_{ijmn} E_{ijmn} \quad (5)$$

where  $C^*_{ijmn}$  is unity at the filter origin ( $C^*_{ij00} = 1$ ) and is otherwise equal to a constant,  $C_{ij}$  over the range of the filter coefficients. The requirement that  $F^*$  be gradient-nulling will determine the coefficients  $K_{xij}$ ,  $K_{yij}$ , and  $C_{ij}$ . That is, for any coefficients  $a$ ,  $b$ , and  $c$ , the filtered constant-gradient field,  $R'$  will be nulled:

$$R'_{ij} = a (\sum_{m,n} F^*_{i,j,m,n} X_{i+m}) + b (\sum_{m,n} F^*_{i,j,m,n} Y_{j+n}) + c (\sum_{m,n} F^*_{i,j,m,n}) = 0. \quad (6)$$

Each factor multiplying the coefficients  $a$ ,  $b$ , and  $c$  is therefore zero. By combining equations (5) and (6) and noting that the filter will only be applied with an origin at a point where the input data are defined ( $E_{i,j,0,0} = 1$ ), one obtains the following three equations:

$$K_{xij} (\sum_{m,n} F_{m,n} E_{ijmn} X_{i+m}^2) + K_{yij} (\sum_{m,n} F_{m,n} E_{ijmn} X_{i+m} Y_{j+n}) = - \sum_{m,n} F_{m,n} E_{ijmn} X_{i+m}; \quad (7)$$

$$K_{xij} (\sum_{m,n} F_{m,n} E_{ijmn} X_{i+m} Y_{j+n}) + K_{yij} (\sum_{m,n} F_{m,n} E_{ijmn} Y_{j+n}^2) = - \sum_{m,n} F_{m,n} E_{ijmn} Y_{j+n}; \quad (8)$$

$$C_{ij} [\sum'_{m,n} F_{m,n} E_{ijmn} (1 + K_{xij} X_{i+m} + K_{yij} Y_{j+n})] = - F_{0,0}; \quad (9)$$

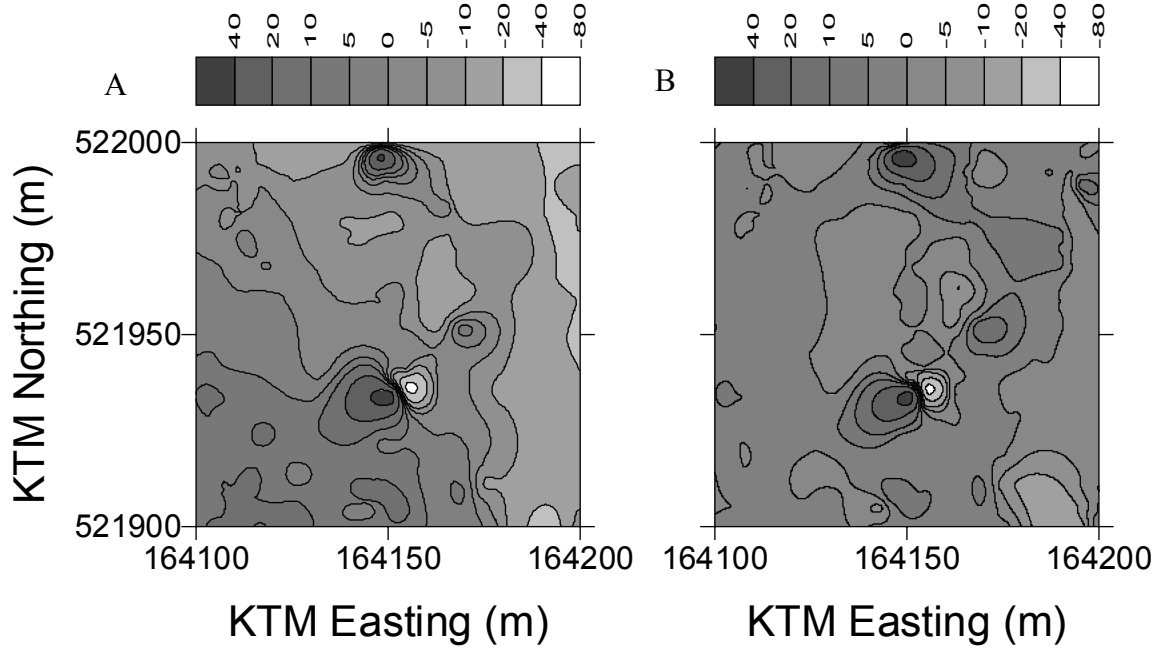
where the primed summation in equation (9) excludes the origin ( $m = n = 0$ ).

Equations (7-9) are generally well conditioned for more than a few data points input. They are solved for  $K_{xij}$ ,  $K_{yij}$  and  $C_{ij}$  at each location for the output filtered data. The filter output is at the center of the original filter window regardless of missing data points or extension of that filter beyond the bounds of the available data. If all input points are available,  $F^*$  will be identical to the original GN filter,  $F$ . If not all points are available because of sparse data interpolation or other data gaps, then  $F^*$  is a “gapped” gradient-nulling (GGN) filter. If some data points are not available because the filter is applied near an edge or corner of a survey area then  $F^*$  is an “edge-adaptive” gradient-nulling (EAGN) filter. If applied at edges or corners and with gaps,  $F^*$  is an edge-adaptive, gapped gradient-nulling (EAGGN) filter.

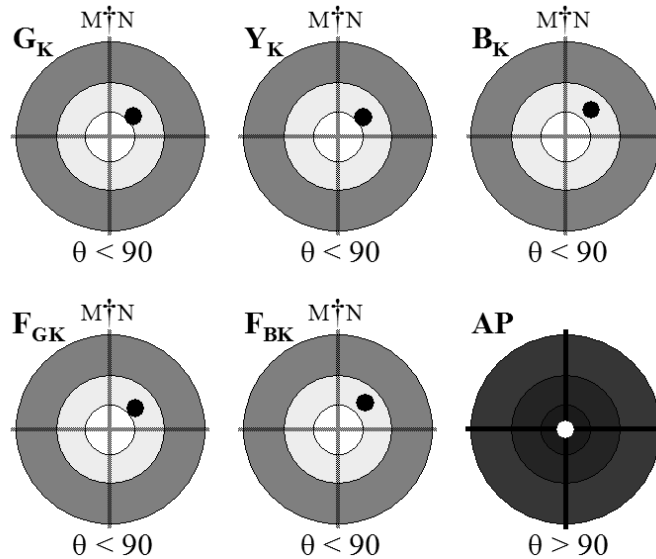
### 4. Applications to magnetic data in survey of Lake Chongcho at Sokcho, Korea

Figure 1 shows unfiltered and filtered magnetic field data from a magnetic survey of Lake Chongcho in Sokcho, Korea. The survey was conducted by KORDI in support of dredging at this East Sea port. The data were obtained in mostly north-south profiles towing a single magnetometer behind a wooden boat at nominal profile spacing of about 10 m (Park et al., 2004; René et al., 2006). These data include an anomaly (Figure 1) for which the magnetic orientations of the inverse-modeled dipoles are shown in Figure 2. For models  $G_K$ ,  $Y_K$ , and  $B_K$  unfiltered data were input. Both  $G_x$  and  $G_y$ , the north gradient  $G_y$  only, and dc-bias only were evaluated, respectively. Models  $F_{GK}$  and  $F_{BK}$  (Figure 2) input the EAGGN filtered input (Figure 1B) and were with and without gradient estimation

respectively. The 100-m x 100-m EAGGN filter was applied only to data at nodes nearest to measurement stations. Data were kriged using  $\Delta x = \Delta y = 0.5$  m so that the maximum distance for interpolation was only 0.25 m for nodes used in the filtering. The regional field is effectively removed by this filter (Figure 1B versus 1A). For purposes of inversion, a 20-m x 20-m data window with  $\Delta x = \Delta y = 0.1$  m contained only 19 measurement stations. The sparsity of data precluded use of 2-d intra-inversion filters. It is uncertain whether gradient estimation may have improved the estimation of dipole orientation. For additional details, the reader is referred to René et al. (2006).



**Figure 1.** (a) Unfiltered and (b) 10-m x 10-m GGN filtered magnetic field data from the survey of Chongcho Lake, Sokcho, Korea. The anomaly at (164150 m, 521940 m) was input to inversion.



**Figure 2.** Orientations of modeled dipoles for unfiltered data (top row), and filtered data (models  $F_{GK}$  and  $F_{BK}$ ). These bull's eye plots are stereographic projections onto a plane perpendicular to the earth's field. "Model AP" is a dipole antiparallel to the earth's field. Models  $G_K$  and  $F_{GK}$  estimated  $G_x$  and  $G_y$ . Model  $Y_K$  estimated  $G_y$  only. Models  $B_K$  and  $F_{BK}$  estimated the dc-bias only. Concentric circles indicate deviation angles in increments of 30 deg. [Modified from René et al. (2006).]

## 5. Applications to Aberdeen Proving Ground MTADS magnetic data

Figure 3 shows unfiltered and filtered magnetic field data from the Blind Test Area of Aberdeen Proving Ground (APG), Maryland. The data were acquired by the Naval Research Lab and AETC, Inc. using MTADS (Multi-sensor Towed Array Detection System) with eight Cs-vapor magnetometers towed in a cart by an all-terrain vehicle with low magnetic signature. The cross-line separation of magnetometers was 0.25 m. Measurements at 20-ms intervals yielded a nominal in-line station spacing of about 0.04 m. The height of the magnetometers above ground was 0.25 m and sensor positions were determined using DGPS. In this test area, inert UXO and magnetic clutter were emplaced at various depths and orientations at selected grid locations with at least 2-m grid intervals. The grid interval for the kriged magnetic data is 5 cm. The applied 2-m x 2-m EAGGN filter has only one-fourth as many points in the corners of the area as in the central areas (Figure 3B). The filtered output shows improved separation of target anomalies and it bears some resemblance to the analytic signal although the information about magnetic orientation of targets is preserved by the EAGGN filters.

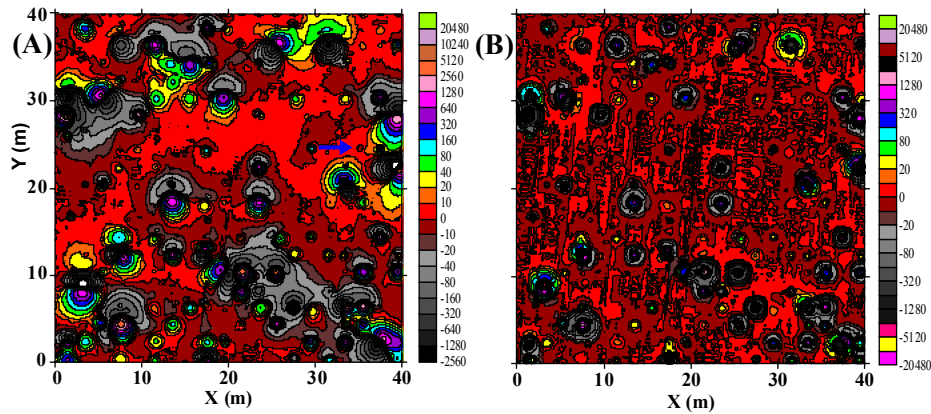
Figures 4A-4B show a very small negative anomaly of a 40-mm projectile buried at a depth-to-center of 0.40 m at APG grid location H03. This anomaly is severely disturbed by tails of larger anomalies. For 1.5- and 2-m data windows without intra-inversion filtering, inversion yielded poor locations and depths of 0.51 to 0.95 m, which are significantly greater than the depth of the projectile. Figure 4C shows the 0.5-m x 0.5-m EAGGN filtered anomaly in the 1.5-m data window used for inversion. The grid interval,  $\Delta x = \Delta y$ , is 1 cm. The edge-adaptive feature was applied in this case not along the edge of the survey area but rather along the edges of the data window used in inversion. This is generally more efficient and as effective as GGN filtering. The unfiltered and filtered inverse-modeled dipole's field are shown in Figures 4D and 4E, respectively. Figure 4F shows the inversion-derived background field, equal to the input data minus the inverse-modeled field. The modeled dipole's depth is 0.43 m, 0.3 m greater than the target. The dipole is also displaced horizontally by 0.06 m from the target's center. This minor error may result in part from positional errors of the magnetometers. The deviation angle, or difference between the inverse-modeled dipole and the direction of the earth's field, is 123 deg. The same magnetic orientation, within a few degrees, was also obtained for 1.5- and 2-m data windows using EAGGN intra-inversion filters with widths of 0.75 to 1.25-m. Computed depths ranged from 0.43 to 0.46 m. The target may have significant remanence or perhaps measurement errors affected the derived magnetic orientation. Demagnetized UXO will generally have deviation angles less than 60 deg (Nelson et al., 1996). For other results and additional details of inversions, the reader is referred to René and Kim (2006) and to the SERDP final report, to be published.

## 6. Discussion and conclusions

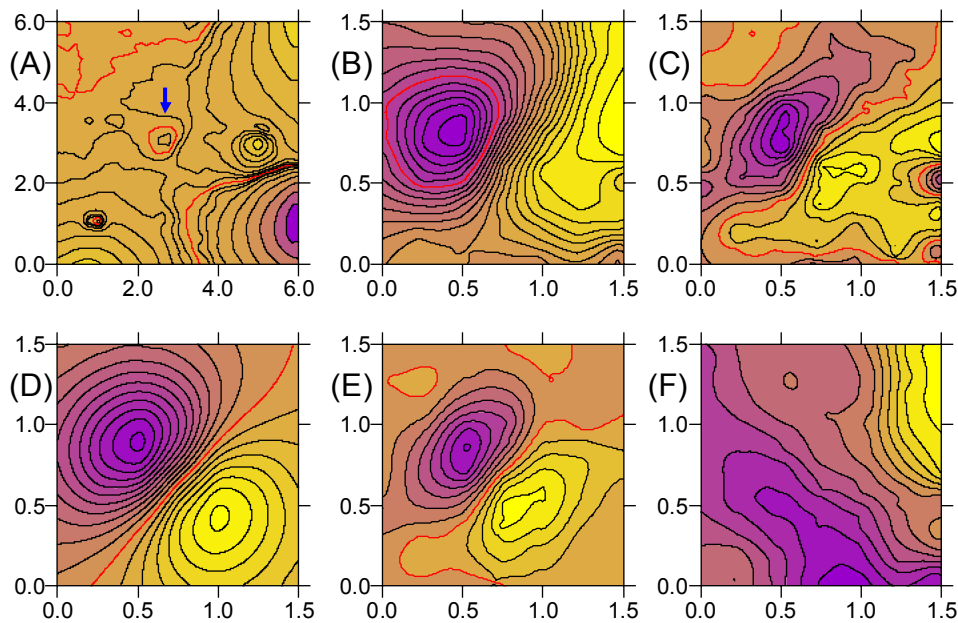
An edge-adaptive gapped gradient-nulling (EAGGN) filter has been developed and applications were demonstrated for magnetic data from a lake survey in Korea and the UXO test site of the Aberdeen Proving Ground. EAGGN intra-inversion filtering is particularly useful in the presence of geologic noise and overlapping fields of nearby UXO and magnetic clutter. The component unit-dipole fields can be separately filtered in the inversion process. One need not carefully design the filter to lessen distortion of the target field since this effect will be wholly compensated for. Rather one concentrates on selection of filters that will nearly null the noise. If consistent results for target dipole parameters are obtained with various appropriate filters then the results may be considered to be more reliable than if no testing with intra-inversion filtering had been applied. Where results are inconsistent using various appropriate intra-inversion filters, then those results may be less relied upon.

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**Figure 3.** (A) Unfiltered and (B) EAGGN-filtered APG magnetic fields. The blue arrow locates target H03. Contours are at 0 and  $\pm 2^n \cdot 5$  nT.



**Figure 4.** (A) H03 anomaly (blue arrow) with contours at 0 (red) and  $\pm 2^n \cdot 2$  nT; (B) same anomaly; (C) EAGGN filtered field; (D) dipole's field, (E) filtered dipole's field; and (F) inversion-derived background field. Peaks are yellow; lows are blue. Contour intervals: 1 nT (B, D, F) and 0.5 nT (C, E).

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