

# A Study on the Earth's Variation Prediction Using Geomagnetic Model

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## 지구자기 모델을 이용한 편차 추정에 관한 연구

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**ABSTRACT** : The objective of the project is to model and study the geomagnetic field structure and its secular variation in space and in time due to sources in the dynamic fluid outer core. the Earth's spherical harmonic model of the main field and of the secular variation gives the intensity and geomagnetic structure at any location around the Earth, assuming an undistorted, steady state field that no external sources or localized earth anomalies. To consider the practical use of a ship's digital compass in Earth's magnetic field, Earth's spherical harmonic model is searched for the related practical methods and procedures as a basic study in this work.

**KEY WORDS** : geomagnetic field, secular variation, geomagnetic variation, spherical harmonic model, digital compass

**요 약** : 이 연구의 목적은 지구 내부의 동적 코어에서 공간과 시간에 따라서 발생하는 지구 자기장 구조와 그에 따른 경년 자차변화를 모델링하고 연구하는 것이다. 지구의 주 자장과 경년 자차변화에 대한 구형 조화 모델은, 외부 간섭자장이나 내부의 불균일 등이 없다는 가정 하에 안정되고 왜곡되지 않은 지구상 어떠한 위치에서의 지구자장의 구조와 세기를 나타낼 수 있다. 이 연구에서는 선박용 디지털 컴퍼스를 이러한 지구자장에 적용하는 경우를 고려하여 지구의 원 조화 모델에 대한 실제 적용방법과 절차를 기술하였다.

**핵심용어** : 지구자장, 경년변화, 퍼스, 지자기 편차, 구 조화 모델, 디지털 컴퍼스

### 1. INTRODUCTION

The Earth's magnetic field is neither uniform, stationary, nor perfectly aligned with the planet, is approximately a magnetic dipole, with one pole near the north geographic pole and the other near the geographic South Pole. The Earth's magnetic field is generated in the fluid outer core by a self-exiting dynamo process. Electrical currents flowing in the slowly moving molten iron generated the magnetic field. In addition to sources in the earth's core the magnetic field observable at the Earth's surface has sources in the crust, in the ionosphere and magnetosphere. The intensity and structure of the Earth's magnetic field are always changing, slowly but erratically reflecting the influence of the flow of

thermal currents within the iron core. It varies on a range of scales and a description of these variations is now made, in the order low frequency to high frequency variations, in both the space and time domains. The field, as measured by magnetic sensor on or Earth's surface.

### 2. BASIC THEORY

#### 2.1 Source of magnetic field

Electric currents, which can be microscopic currents in wires, produce magnetic fields or microscopic currents associated with electrons in atomic orbits. The relationship between magnetic field strength  $H(A/m)$  and current density  $J$  can be shown as in eq.(1).

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$$\nabla \times H = j + \frac{\partial D}{\partial t} \quad (1)$$

where H is the magnetic field strength (in units of A/m), related to the magnetic flux B by a constant called the permeability,  $\mu(B = \mu H)$ , and J is the current density, defined by  $J = \int \rho_q v dV$ , where v is a vector field called the drift velocity that describes the velocities of that charges carries which have a density described by the scalar function  $\rho_q$ . In free space, the permeability  $\mu$  is the permeability of free space,  $\mu_0$ . Thus, in free space, the equation becomes

$$\nabla \times B = \mu_0 j + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad (2)$$

Maxwell's suggested to modify Ampere's Law as follows,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \quad (3)$$

Or,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (i + i_d) \quad (4)$$

The physical meaning of the displacement current is the ability of time-dependent electric field to create a magnetic field. The generation of magnetic field is linked to the rotation of the fluid metallic iron which makes up a large portion of the interior and the rotating conductor models leads to the term "dynamo effect" or "geo-dynamo".

So we surmise that circulating electric currents in the earth's molten metallic core are origin of the magnetic field.

### 2.2 The Earth's variation

The magnetic field of the earth is not a constant over time. Various changes occur in intensity and direction, including daily variations and ones due to the influence of the sun. The intensity and direction return to their initial states after a while and are known as temporal variations. The magnetic field also undergoes drift over long periods of time, or secular variations, which can eventually result in a reversal of the field. the variation of earth's depends on the position of the earth and in different latitudes the field intensity is different. The earth's shape is not smooth as like as Legendre polynomial of order 6, is known as  $P_6(\theta)$ .

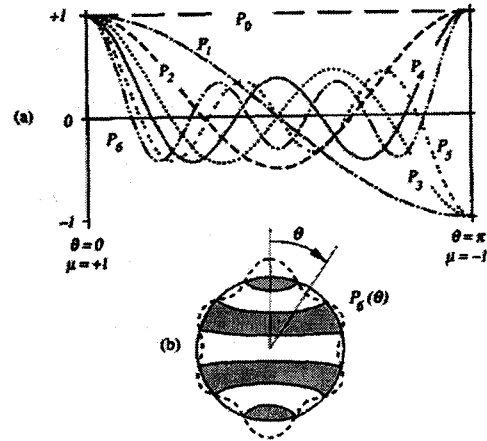


Fig. 1. A few low-degree Legendre functions. (a) Functions  $P_0(\mu)$  through  $P_6(\mu)$  are shown on the interval  $-1 \leq \mu \leq 1$ . (b) Functions  $P_6(\mu)$  is shown along the circumference of a circle: gray and white zone indicate areas where the function would be positive or negative, respectively, if wrapped around a sphere.

### 2.3 The effect of earth's on ship's compass

A ship has permanent magnetism when placed in a magnetic field such as the earth's field. The magnetism also depend on the how the ship is aligned in the magnetic field. The magnetic poles of the earth are not coincidental with the geographic poles. The angular difference between the true meridian and the magnetic meridian is called variation. This variation has different values at different locations on the earth. A ship is a combination of permanent, sub permanent and induced magnetism. Deviation is the difference between the compass heading and magnetic heading and variation is difference between the magnetic heading and true heading. The algebraic sum of the deviation and variation is the compass error. Uniform and no uniform magnetic field create deviations of the compass which have higher frequency characteristics.

### 2.4 Observation

The earth's magnetic field (B) is a vector quantity varying in space (r) and time (t). The field, as measured by a magnetic by a magnetic sensor on or above earth's surface, is actually a composite of several magnetic fields, generated by a variety of

sources. These fields are superimposed on each other and through inductive processes interact with each other. The most important of these geomagnetic sources are (a) the main field generated in earth's conducting fluid outer core ( $B_m$ ), (b) the crustal field from earth's crust/upper mantle ( $B_c$ ), (c) the combination disturbance field from electrical currents flowing in the upper atmosphere and magnetosphere, which also induce electrical currents in the sea and the ground ( $B_d$ ). Thus, the observed magnetic field is a sum of combination as in Eq. (5),

$$B(r, t) = B_m(r, t) + B_c(r) + B_d(r, t) \quad (5)$$

Where  $B_m$  is the dominating part of the field, accounting for over 95% of the field strength at the earth's surface. Secular variation is the slow change in time of  $B_m$ .  $B_c$ , the field arising from magnetized crustal rocks, varies spatially, but is considered here.  $B_c$  is usually much smaller in magnitude than  $B_m$ . The crustal field is constant over the time scales considered here. The field arising from currents flowing in the ionosphere and magnetosphere and their resultant induced currents in the earth's mantle and crust,  $B_d$  varies both with location and time.

To create an accurate main field model, it is necessary to have data with good global coverage and as low a noise level as possible. The Denis Oersted and the CHALLENGING Mini-satellite Payload (CHAMP) satellites data sets satisfy these requirements. Both satellites provide high quality vector and secular data at all latitudes and longitudes, but not during all latitudes needed for modeling.

The observatory data therefore provide valuable constraints on the time variations of the geomagnetic field. is used together, satellite and observatory data provide an exceptional quality data set for modeling the behavior of the main magnetic field in space and time.

## 2.5 Theoretical modeling

In equation (1)  $B_c$  has spatial variations on the

order of meters to thousands of kilometers and can't be fully modeled with low degree spherical harmonic models.  $B_c$  is usually smaller at sea than on land and decreases with increasing altitude (by the main magnetic field) or remnant or combination of both.

The field arising from currents flowing in the ionosphere and magnetosphere and their associated induced currents in the earth,  $B_d$  varies both with location and time. Fig. 2 shows the variations currents systems. The disturbance field can vary both regularly, with fundamental periods of one day and one year, as well as irregularly on time scales of seconds to days. The regular variations are both diurnal and annual and they are essentially generated by the daylight atmosphere.

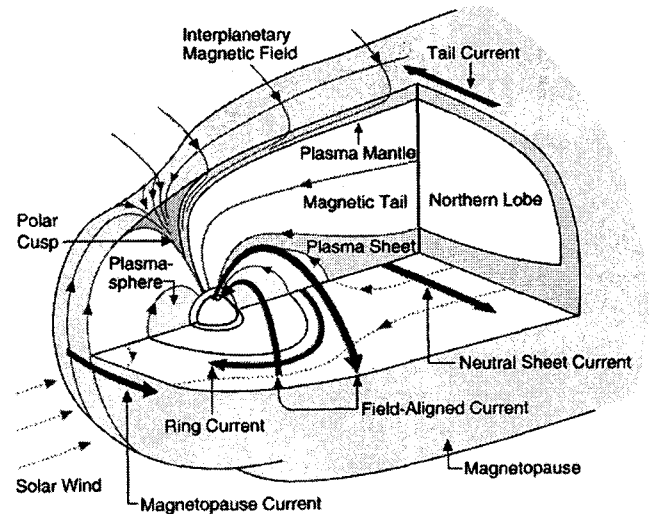


Fig. 2. Schematic of the magnetosphere showing the current flows and magnetic field lines (Nils Olsen, 2002).

## 3. MODEL PARAMETERIZATION

### 3.1 Model Parameterization

The geomagnetic field measured at the earth's surface or at satellite altitude is the sum of the field generated by sources internal or external to the solid earth. Away from its sources, the internal magnetic field  $B$  is a potential field and therefore can be written as the negative gradient of a scalar potential.

In M.K.S, magnetic field must satisfy the Maxwell's equation,  $\nabla \times B = \mu_0 J + \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$ , where  $\epsilon_0$  is the

permittivity of free space. If electric fields are not present or are slowly varying, displacement current can be ignored. Then  $\epsilon_0 \mu_0 \frac{\partial E}{\partial t} \ll \nabla \times B$ , If additionally there is almost no external current, then  $\mu_0 J = \nabla \times B$ . So, to a good approximation,  $\nabla \times B = 0$ . Therefore, in this approximation B can be represented as the gradient of a scalar function,  $B = -\nabla V$ . Because there are no magnetic monopoles, another of the Maxwell's equation gives,  $\nabla \cdot B = 0$ , everywhere, so V satisfies Laplace's equation. Therefore,

$$B(\phi', \lambda, r, t) = -\nabla V(\phi', \lambda, r, t) \quad (6)$$

This potential can be expanded in terms of spherical harmonics:

$$V(\phi, \lambda, r, t) = a \left( \sum_{n=1}^N \sum_{m=0}^n g_n^m(t) \cos(m\lambda) + h_n^m(t) \sin(m\lambda) \left(\frac{a}{r}\right)^{n+1} \bar{p}_n^m(\sin\phi) \right) \dots (7)$$

Where a (6371.2 km) is the standard earth's magnetic reference frame, and  $(g_n^m(t), h_n^m(t))$  are the time-dependent Gauss coefficients of degree n and m describing internal sources.  $\bar{p}_n^m(\sin\phi')$  Are the Schmidt semi-normalized Associated Legendre functions (Gradsteyn and Ryzhik, 1994), defined as

$$\begin{aligned} \bar{p}_n^m(\sin\phi') &= \sqrt{2 \frac{(n-m)!}{(n+m)!}} p_n^m(\sin\phi') \\ \bar{p}_n^m(\sin\phi') &= p_n^m(\sin\phi') \end{aligned} \quad (8)$$

### 3.2 Model Estimation

In matrix form, linear models are given by the formula

$$y = X\beta + \epsilon \quad (9)$$

Where, y is an n by 1 vector of responses,  $\beta$  is a m by 1 vector of coefficients, X is the n by m design matrix for the model,  $\epsilon$  is an n by 1 vector of errors.

For the first degree polynomial, the n equations in two unknowns are expressed in terms of y, X and  $\beta$  as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \times \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \quad (10)$$

The least squares solution to the problem is a vector b, which estimates the unknown vector of coefficients  $\beta$ . The normal equations are given by

$$(X^T X)b = X^T y \quad (11)$$

Where XT is the transpose of the design matrix X. solving for b,

$$b = (X^T X)^{-1} X^T y \quad (12)$$

To get the predicted response values,  $\hat{y}$

$$\hat{y} = Xb = Hy \quad (13)$$

$$H = X(X^T X)^{-1} X^T \quad (14)$$

Where H is the projection matrix.

The residuals are given by  $r = y - \hat{y} = (I - H)y$ . Weighted least squares regression minimizes the error

estimate  $S = \sum_{i=1}^n w_i (y_i - \hat{y}_i)^2$ , where  $w_i$  are the

weights. The weights modify the expression for the parameter estimates b in the following way,

$b = \hat{\beta} = (X^T W X)^{-1} X^T W y$ , W is diagonal elements of the weight matrix w.

### 3.3 Coordinate transformation

Satellite data are already located in a geocentric coordinate system but surface data are almost invariably located in a geodetic coordinate system, i.e. relative to the mean sea surface of the earth, which can be approximated by an ellipsoid.

The observations of the northerly, easterly and vertically down intensities X, Y and Z relative to an ellipsoid, are transformed into the northerly, easterly and vertically down intensities relative to a sphere  $X', Y'$  and  $Z'$ :

$$\begin{aligned} X' &= X \cos \psi + Z \sin \psi \\ Y' &= Y \\ Z' &= -X \sin \psi + Z \cos \psi \end{aligned} \quad (15)$$

Where  $\psi$  is the difference between geocentric and geodetic latitude in the sense  $\psi = \phi' - \phi$ .

### 3.4 Secular variation prediction

Prediction of future change in the magnetic field were derived from the long-term observatory annual mean data as well as polynomial extrapolation of the model based on satellite data and observatory hourly mean values. Annual mean data were utilized by determining and applying linear predictor filters to series of first differences to result in estimates of secular variation up to 2010.0 (Macmillan and Quinn, 2000). Linear prediction is successful at extrapolating signals which are smooth and oscillatory, though not necessarily periodic. Shown prediction bounds as follows:

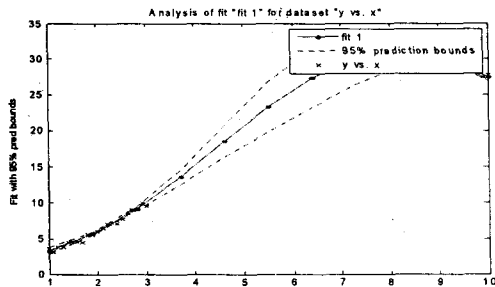


Fig. 3. Secular variation's prediction bounds.

## 4. RESULTING MODEL

### 4.1 Purpose and using world magnetic model data

World magnetic model software presented for all seven magnetic components for both the main field and secular variations (sv). The area covered is from  $80^\circ$  north to  $80^\circ$  south latitude at  $40^\circ$  intervals and for  $0^\circ$  to  $360^\circ$  east longitude at  $60^\circ$  intervals. Results for H, X, Y, Z, and F are shown in nT for the main field and secular variation in nT/year. Declination is considered positive east, inclination and vertical component are positive down.

The following figure 4 is shown the magnetic magnitude cover the world.

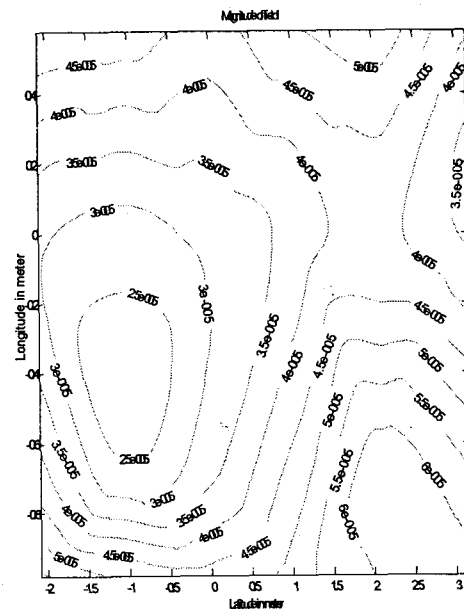


Fig. 4. The magnetic magnitude cover the world.

Our main purpose is to construct magnetic field in the local area like Mokpo city, of Korea peninsula. Later future work to establish earth's magnetic field in very small area.

Why need to evaluate the magnetic field intensity and direction of the field? Using local magnetic field is to adopt in the digital compass. Magnetic north is the points in the directions of the horizontal component of the magnetic where the located and not to geomagnetic nor magnetic north poles. The geomagnetic poles (dipole poles) are the pole positions based on the first three terms of the geomagnetic model. These poles are slowly drifting by the geomagnetic secular variation, which is caused by a slow variation of the electric currents flowing in the earth's core. To construct the total magnetic field, declination and inclination with secular variation is shown in the figure 5, figure 6 and figure 7.

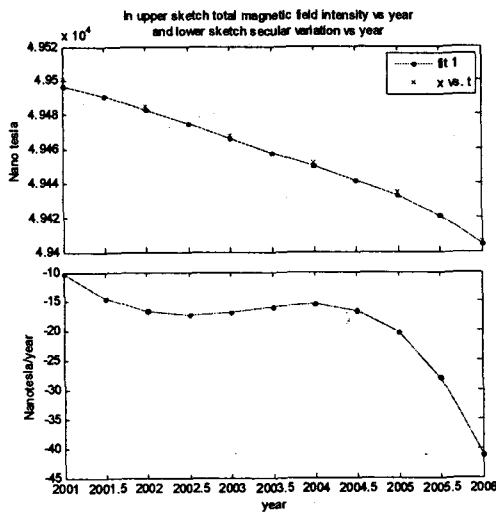


Fig. 5. total magnetic field and secular variation 2001–2006, Mokpo.

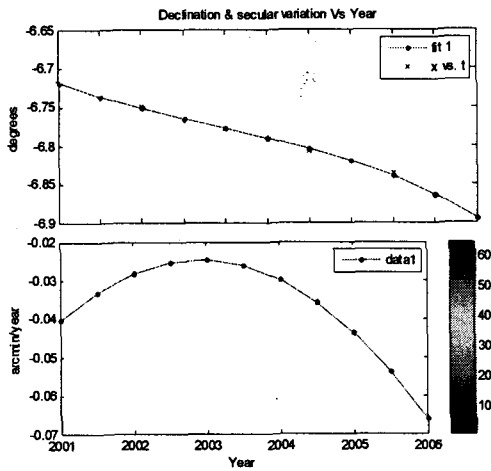


Fig. 6. Declination and secular variation.

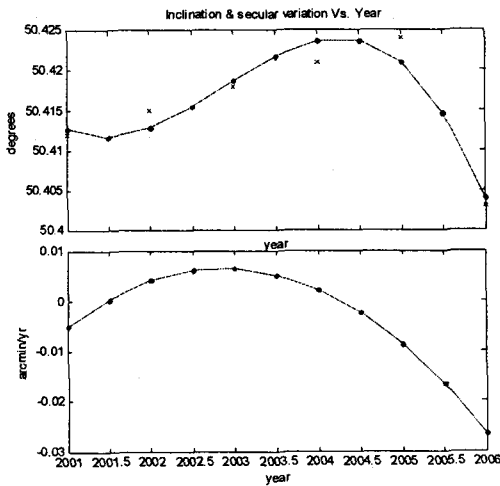


Fig. 7. Inclination and secular variation.

## 4.2 Accuracy and limitation

The accuracy at any given time for all of these depends on the geomagnetic latitude. The errors are least from the equator to mid-latitudes and greatest near the magnetic poles. The estimated RMSE for the various magnetic components of our plotted figure are: Declination: 0.004 deg. Inclination: 0.0033 deg. Total magnetic field: 1.751 nanoTesla

It is important that model describe the long wavelength spatial magnetic fluctuations due to earth's core. The geomagnetic model and charts characterize the portion of the earth's magnetic field that is generated by dynamo action in the earth's fluid outer core. a magnetic sensor or compass or magnetometer may observe spatial and temporal magnetic anomalies when referenced to the WMM. Declination anomalies of the order of 3 or 4 degrees are not uncommon but are of small spatial extent and are relatively isolated.

## 5. CONCLUSIONS

Geomagnetic model of the earth main magnetic field that portion of the field generated in the earth's core. The model is satellite based model and more accurate. The earth's field intensity is decreasing from year to year that is shown in the world magnetic data chart. Earth's variation deviate the position of magnetic pole of the earth this disturbance effects on ship's compass.

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