비어파인 비선형 계통에 대한 적응 퍼지 슬라이딩 모드 제어기

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Adaptive Fuzzy Sliding-Mode Controller for Nonaffine Nonlinear Systems

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Abstract— **An adaptive fuzzy sliding-mode controller (SMC) for uncertain or ill-defined single-intput single-output (SISO) nonaffine nonlinear systems is proposed. By using the universal approximation property of the fuzzy logic system (FLS), it is tuned on-line to cancel the unknown system nonlinearity. We adopt a self-structuring FLS to guarantee global stability of the closed-loop system rather than semi-global boundedness. The control and adaptive laws are derived so that the estimated fuzzy parameters are bounded and the sliding condition is satisfied.**

Keywords—**fuzzy system, nonaffine nonlinear system, sliding-mode control**

I. Introduction

Sliding-mode controller (SMCs) are conventional but powerful in controlling nonlinear systems with imprecise model or bounded unknown disturbance [1], [2]. A key step in the design of SMCs is to introduce a proper transformation of tracking errors to generalized errors so that an nth-order tracking problem can be transformed into an equivalent 1st-order problem. A control law for the simple 1st-order systems is easily developed to achieve the so-called reaching condition.

Fuzzy logic systems (FLSs) have been successfully applied to many control problems. It is well-known that FLSs, as well as neural networks, can approximate certain classes of functions to a given accuracy. Wang [3] proved that the FLSs with a crisp fuzzifier, Gaussian membership function, max-product inference engine and center-average defuzzifier are universal approximators. Furthermore, the output of the system can be represented by a linear combination of the so-called fuzzy basis functions (FBFs). Later, Castro [4] has relaxed the restrictions on the components of FLSs and has proved that general FLSs also have an approximation abilities. Based on this property, intensive research on stable adaptive control algorithms using FLSs have been performed [5], [6], [7], [8], [9], [10], [11].

In recent years, hybrid schemes of adaptive FLS with SMC are proposed to combine their advantages [12], [13], [14], [15], [16]. The basic idea of the proposed control schemes is that unknown dynamics of the controlled system are estimated on-line using the generalized error signal and an additional SMC compensates for a lumped uncertainty including the estimation error and external disturbance. However, their works are all consider affine nonlinear systems whose dynamics are linear in the control. Moreover, employed FLSs have fixed structure, which results in inefficiently small or large dynamic order of the controller and semi-global stability of the closed-loop system.

The purpose of this paper is to develop an adaptive SMC for single-intput single-output (SISO) nonaffine nonlinear dynamical systems using FLS. We adopt a self-structuring FLS [17] to guarantee global-stability of the closed-loop system rather than semi-global boundedness. The control law and adaptive algorithms for FLSs are derived so that the estimated fuzzy parameters are bounded and the sliding condition is satisfied.

A. *Notations and Preliminaries*

The following notations and definitions will extensively be used throughout the paper. Let R be the real number, R^n and $R^{n\times m}$ represent the real n-vectors and the real $n\times m$ matrices, respectively. $|\mathbf{y}|$ denotes the usual Euclidean norm of a vector $\mathbf{y}$. In case where y is a scalar, $|y|$ denotes its absolute value. When the upper-bound of $|\mathbf{y}|$ or $|y|$ is denoted, b_y or ϵ_y is used for the readability. The ϵ_y means that the upper-bound can be made arbitrarily small by appropriately choosing design constants while b_y cannot.

II. Problem Formulation

We consider nonaffine nonlinear SISO systems

$$y^{(n)} = F(y, y^{(1)}, \cdots, y^{(n-1)}, u) + d(t) \tag{1}$$

where

$y \in R$	measured output
$u \in R$	control input
$y^{(i)}, i = 1, \cdots, n$	ith time derivatives of y
$F(\cdot) : R^{(n+1)} \to R$	unknown smooth nonlinear function
$d(t)$	bounded disturbance : $\lvert d(t)\rvert \le b_d, \forall t$.

It should be noted that the nonlinearity $F(\cdot)$ is an implicit function with respect to u.

Let $\mathbf{x} = [x_1\ x_2 \cdots x_n]^T = [y\ y^{(1)} \cdots y^{(n-1)}]^T \in R^n$ be the state vector. Then, (1) can be rewritten as a state space model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_n &= F(\mathbf{x}, u) + d(t) \\ y &= x_1. \end{aligned} \tag{2}$$

The following assumption is made for the controllability of the system.

Assumption 1: The following inequality holds

$$\frac{\partial F(\mathbf{x}, u)}{\partial u} > 0 \tag{3}$$

for all $(\mathbf{x}, u) \in R^n \times R$.

The reference signal $y_d(t)$ and its time derivatives $y_d^{(1)}(t), y_d^{(2)}(t), \cdots, y_d^{(n)}(t)$ are assumed to be bounded and we set $\mathbf{x}_d = [y_d(t)\ y_d^{(1)}(t), \cdots, y_d^{(n-1)}(t)]^T \in R^n$. We also define the tracking error as $e = y_d - y$ and corresponding error vector as $\mathbf{e} = [e\ e^{(1)}, \cdots, e^{(n-1)}]^T \in R^n$. In general, a sliding surface is defined by

$$s(\mathbf{e}) = \boldsymbol{\lambda}^T \mathbf{e} = 0 \tag{4}$$

where $\boldsymbol{\lambda} = [\lambda_1\ \lambda_2, \cdots, \lambda_{n-1}\ 1]^T$ with λ_i's being real and all roots of the polynomial $h(x) = x^{n-1} + \lambda_{n-1}x^{n-2} + \cdots + \lambda_2 x + \lambda_1$ being in the open left-half plane. Then, the tracking problem can be considered as the $\mathbf{e}$ remaining on the sliding surface $s(\mathbf{e}) = 0$. A sufficient condition for this behavior is to choose the control input so that [2]

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{5}$$

where η is a positive constant.

The aim of this paper is to design an adaptive fuzzy SMC for nonaffine nonlinear SISO systems (1) under plant uncertainties which guarantees boundedness of all the estimated variables of the closed-loop system and asymptotic stability of the sliding surface (4) such that the condition (5) is globally satisfied.

III. Controller Design

A. Feedback Linearizing Controller Design

The following feedback input-output linearizing method is a simplified one of the proposed schemes in [18], [19]. Feedback linearization is performed by rewriting (1) as

$$\begin{aligned} y^{(n)} &= F(\mathbf{x}, u) + d(t) \\ &= cu + \{F(\mathbf{x}, u) - cu\} + d(t) \end{aligned} \tag{6}$$

where c is a design constant. Let $\Delta(\mathbf{x}, u) \triangleq F(\mathbf{x}, u) - cu$ and the control input be determined as

$$u = \frac{1}{c}\left(u_{dc} - \hat{u}_{ad} + u_{sl}\right) \tag{7}$$

where u_{dc} is a control input to stabilize linearized dynamics and $\hat{u}_{ad}$ is an adaptive control signal designed to cancel $\Delta(\mathbf{x}, u)$ using FLS and u_{sl} is the SMC input proposed in the sequel. Substituting (7) into (6) yields

$$y^{(n)} = u_{dc} + \{\Delta(\mathbf{x}, u) - \hat{u}_{ad}\} + u_{sl} + d(t). \tag{8}$$

We determine u_{dc} as

$$u_{dc} = y_d^{(n)} + \bar{\boldsymbol{\lambda}}^T \mathbf{e} \tag{9}$$

where $\bar{\boldsymbol{\lambda}} = [0\ \lambda_1 \cdots \lambda_{n-1}]^T$.

B. Approximation of $\Delta(\mathbf{x}, u)$ Using FLS

A FLS is employed to approximate $\Delta(\mathbf{x}, u)$ whose inputs to the FLS are $\mathbf{x}$ and $u(= (u_{dc} - \hat{u}_{ad} + u_{sl})/c)$ since $\Delta(\cdot)$ is the function of them. However, in this formulation, the FLS is to be a recurrent FLS (RFLS) because the output of the FLS, $\hat{u}_{ad}$, is directly fed back into the FLS to produce the control input u. However, if we use RFLS, a fixed-

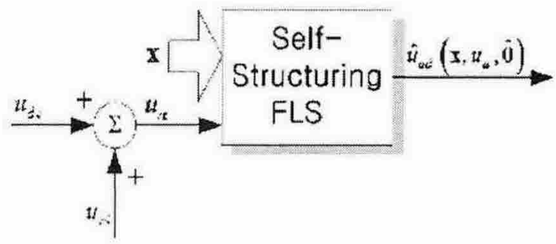

Fig. 1. Employed static FLS

point problem must be solved at every time instant and this imposes computational burden. To avoid this problem, we apply implicit function theorem to guarantee that an u_{ad}^* which satisfies

$$h(\mathbf{x}, u_\alpha, u_{ad}^*) \triangleq \Delta(\mathbf{x}, (u_\alpha - u_{ad}^*)/c) - u_{ad}^* = 0 \tag{10}$$

is a function of $\mathbf{x}$ and u_α where $u_\alpha = u_{dc} + u_{sl}$. By using the following Lemma 1, a static FLS can be employed to approximate $\Delta(\mathbf{x}, u)$ [11].

Lemma 1: Let the constant c satisfy the condition:

$$c > \frac{1}{2}\left(\frac{\partial F}{\partial u}\right). \tag{11}$$

Then, there exist a unique u_{ad}^* which is a function of $\mathbf{x}$ and u_α such that $u_{ad}^*(\mathbf{x}, u_\alpha)$ satisfies (10) for all $(\mathbf{x}, u_\alpha) \in R^n \times R$

Proof: Refer to [11], [20]. ■

Lemma 1 makes it possible to employ static FLS rather than RFLS to approximate u_{ad}^*, which enables the controller to avoid solving a fixed-point problem at every time instant. In this paper, $u_{ad}^*(\mathbf{x}, u_\alpha)$ is identified by a FLS which is known as a universal approximator [3], [4]. In what follows, we denote the input vector to the FLS as $\mathbf{X} = [\mathbf{x}^T\ u_\alpha]^T$ and its dimension as $m(= n + 1)$.

B.1 FLS and Fuzzy Basis Function

The FLS performs a mapping from $U \subset R^m$ to $V \subset R$. It comprises four principle components : fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. Many different choices are available within each block, and in addition, many combinations of these choices can result in a useful subclass of FLSs. The FLSs viewed as nonlinear systems are potential candidates for modeling and control of nonlinear systems. As proved in [21], [4], the FLS has universal function approximation property. We choose singleton fuzzyfier, product inference engine, center-average defuzzifier and the triangular membership function described as

$$A_{i_j}^j(x_j, p_{i_j}, q_{i_j}) = \begin{cases} \frac{x_j - p_{i_j} + q_{i_j}}{q_{i_j}} & p_{i_j} - q_{i_j} \le x_j < p_{i_j} \\ \frac{-x_j + p_{i_j} + q_{i_j}}{q_{i_j}} & p_{i_j} \le x_j < p_{i_j} + q_{i_j} \\ 0 & otherwise \end{cases} \quad (12)$$

where $j \in \{1, 2, \cdots, m\}$ and $i_j \in \{1, 2, \cdots, k_j\}$ with k_j being the number of MFs for X_j. The p_{i_j} and q_{i_j} are the center and half width of the triangular MF, respectively. The output of the FLS can be written as the linear combination of the FBFs :

$$\hat{u}_{ad}(\mathbf{X}) = \hat{\boldsymbol{\theta}}^T \boldsymbol{\xi}(\mathbf{X}) \quad (13)$$

where $\hat{\boldsymbol{\theta}}$ is a collection of the fuzzy consequence parameters and $\boldsymbol{\xi}(\mathbf{X})$ is a collection of fuzzy basis functions (FBFs). The FLS in the form of (13) with (12) is the most frequently used one in control application.
The function $u_{ad}^*(\mathbf{X})$ which satisfies (10) as

$$\begin{aligned} u_{ad}^*(\mathbf{X}) &= \hat{u}_{ad}(\mathbf{X}, \boldsymbol{\theta}^*) + \delta(\mathbf{X}) \\ &= \boldsymbol{\theta}^{*T} \boldsymbol{\xi}(\mathbf{X}) + \delta(\mathbf{X}) \end{aligned} \quad (14)$$

where $\delta(\mathbf{X})$ is the approximation error and $\boldsymbol{\theta}^*$ is optimal parameter required for analytical purpose satisfying

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \left[\sup_{\mathbf{X} \in R^m} |\hat{u}_{ad}(\mathbf{X}, \boldsymbol{\theta}) - u_{ad}^*(\mathbf{X})| \right] \quad (15)$$

According to the universal approximation theorem for the FLS [3], [4], there exists $\epsilon_\delta > 0$ such that the following inequality holds for all $\mathbf{X}$:

$$|\delta(\mathbf{X})| \le \epsilon_\delta \quad (16)$$

where $\epsilon_\delta > 0$ is an unknown finite constant. Note that, ϵ_δ can be arbitrarily small by making the structure (e.g., number of MFs and fuzzy rules) of the FLS sufficiently rich.

The self-structuring algorithm proposed in [17], [22] is adopted in this paper. Please refer to those papers for the detailed algorithm.

C. *Adaptive Laws and Stability Analysis*

In this section, we derive the sliding mode control input u_{sl} and adaptive laws for $\hat{\boldsymbol{\theta}}$ such that the sliding condition is satisfied. To show the original sliding condition (5), we adopt the technique in [23].

Lemma 2: Let the update law for the fuzzy parameter $\hat{\boldsymbol{\theta}}$ be determined as

$$\dot{\hat{\boldsymbol{\theta}}} = -\gamma \left(\boldsymbol{\xi}(\mathbf{X}) s + \sigma(t) |s| \hat{\boldsymbol{\theta}} \right) \quad (17)$$

where

$$\sigma(t) = \begin{cases} \frac{b_\xi}{\epsilon_\theta} & \text{if } |\hat{\boldsymbol{\theta}}| \ge \epsilon_\theta \\ 0 & \text{if } |\hat{\boldsymbol{\theta}}| < \epsilon_\theta \end{cases} \quad (18)$$

and γ, ϵ_θ are positive design constants and $|\boldsymbol{\xi}(\mathbf{x})| \le b_\xi$ with b_ξ being a computable constant. Then, $|\hat{\boldsymbol{\theta}}| \le \epsilon_\theta$.

Proof: omitted. ∎

Note that the switching function $\sigma(t)$ is adopted so that the FLS can keep the learned information. That is, all the previous update laws for FLS parameters [9], [10], [11], [12], [13], [14], [15] based on $\sigma-$ or $\epsilon-$modification of the conventional robust adaptive method [24], although they are adopted for the robustness and stability, lose their information since the updated parameters go to zero as time goes on. This is the serious demerit in the intelligent learning system. However, the adopted switching scheme prevents the information-losing situation if ϵ_θ is chosen large enough such that $\epsilon_\theta \ge |\boldsymbol{\theta}^*|$ while guarantees boundedness of the $|\hat{\boldsymbol{\theta}}|$. Note also that the ϵ_θ is freely designed constant, that is why we used the ϵ-denotation.

Lemma 3: There exist constants b_Δ and $b_{\tilde{\theta}}$ such that

$$|\Delta(\mathbf{x}, u) - \Delta(\mathbf{x}, u^*)| \le b_\Delta \left(b_{\tilde{\theta}} b_\xi + \epsilon_\delta \right) \quad (19)$$

for all $\mathbf{x} \in R^n$.

Proof: omitted. ∎

From (8) with (9), we obtain the following dynamics:

$$\begin{aligned} \dot{s} &= \boldsymbol{\lambda}^T \dot{\mathbf{e}} \\ &= \bar{\boldsymbol{\lambda}}^T \mathbf{e} + \left(y_d^{(n)} - y^{(n)} \right) \\ &= -\left\{ \Delta(\mathbf{x}, u) - \hat{u}_{ad}\left(\mathbf{X}, \hat{\boldsymbol{\theta}}\right) \right\} - d(t) - u_{sl} \\ &= -\left\{ \Delta(\mathbf{x}, u) - \Delta(\mathbf{x}, u^*) + u_{ad}^*(\mathbf{X}) - \hat{u}_{ad}\left(\mathbf{X}, \hat{\boldsymbol{\theta}}\right) \right\} \\ &\quad - d(t) - u_{sl} \\ &= -\left\{ \Delta(\mathbf{x}, u) - \Delta(\mathbf{x}, u^*) \right\} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\xi}(\mathbf{X}) - \delta(\mathbf{X}) \\ &\quad - d(t) - u_{sl}. \end{aligned} \quad (20)$$

We are now ready to present our main theorem.

Theorem 1: Consider the system (2) with assumption 1 being hold. The control input (7) with (9),(13) and

$$u_{sl} = k\,\text{sgn}(s) \quad (21)$$

where k is positive design constant. If the k is chosen sufficiently large such that

$$\eta := k - \left((b_\Delta + 1) \left(b_{\tilde{\theta}} b_\xi + \epsilon_\delta \right) + b_d \right) > 0 \quad (22)$$

the sliding condition (??) is satisfied.

Proof: omitted. ∎

Remark 1. In many applications, the sgn(·) in (21) is replaced by a saturation function of the form

$$\text{sat}(s) = \begin{cases} \text{sgn}(s) & \text{if } |s| > \epsilon_s \\ s/\epsilon_s & \text{if } |s| < \epsilon_s \end{cases} \quad (23)$$

or a smooth function $\tanh(\frac{s}{\epsilon_s})$ where $\epsilon_s > 0$ is a small design constant in order to remedy the control chattering.

The overall control scheme is illustrated in Fig. 2.

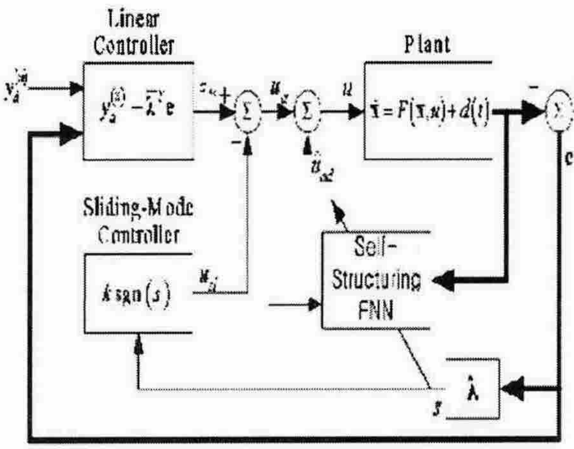

Fig. 2. Overall adaptive fuzzy SMC system

IV. Conclusion

The main contribution of this paper is to propose the adaptive fuzzy SMC for uncertain or ill-defined nonaffine nonlinear system. The lumped uncertainty including external disturbance is compensated by an additional SMC. The adaptive laws and control input are established to stabilize the closed-loop system in the Lyapunov sense and to guarantee that the sliding condition is globally satisfied.

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