The Synchronization Method for Cooperative Control of Chaotic UAV

Young-Chul Bae

[a] *Division of Electronic Communication and Electrical Engineering, Yosu Nat'l University*
San 96-1, Dundeok-Dong, Yeosu-Si, Jennam, 550-749, Korea
Tel: +82-61-659-3315, Fax: +82-61-659-3310, E-mail: ycbae@yosu.ac.kr

여수대학교 공과대학 전자통신전기공학부
전남 여수시 둔덕동 산 96-1, 550-749
Tel: +82-61-659-3315, Fax: +82-61-659-3310, E-mail: ycbae@yosu.ac.kr

Abstract

In this paper, we propose a method to a synchronization of chaotic UAVs that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. The proposed methods are assumed that if one of two chaotic UAVs receives the synchronization command, the other UAV also follows the same trajectory during the chaotic UAVs search on the arbitrary surface.

Keywords: Chaos UAV; Synchronization; Unstable Limit cycle

Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a method to a synchronization of chaotic UAVs that have unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaotic UAVs meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Lorenz equation, Chua's equation, the obstacles reflect the chaotic UAVs.

Chaotic Mobile's equations

Chaotic UAV [24]

We assume that each UAV is equipped with standard autopilots for heading hold and mach hold. In order to focus on the essential issues, we will assume that altitude is held constant. Let $(x, y), \psi,$ and v denote the inertial position, heading angle, and velocity for the UAV respectively. Then the resulting kinematics equations of motion are

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \dot{\psi} &= \alpha_{\psi}(\xi^{c} - \psi) \\ \dot{v} &= \alpha_{v}(v^{c} - v) \end{aligned} \tag{1}$$

where ψ^c and v^c are the commanded heading angle and velocity to the autopilots, and α_ψ and α_v are positive constraints [22,23].

Assuming that α_v is large compared to α_ψ, Eq. (1) reduces to

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \dot{\psi} &= \alpha_{\psi}(\xi^{c} - \psi) \end{aligned} \tag{2}$$

Letting $\psi^{c} = \psi + (1/\alpha_{\psi})\omega$ and $v^c \approx v$, Eq. (2) becomes

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \psi^{\cdot} &= \omega \end{aligned} \tag{3}$$

Eq.(3) rewritten as follows,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{4}$$

Eq. (3) is similar to two wheel mobile robot equation (5).

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{5}$$

where (x,y) is the position of the robot and θ is the angle of the robot.

Chaos equations

In order to generate chaotic motions for the UAVs, we apply chaos equations such as a Chua's and Lorenz equation.

Chua's equation

We define the Chua's equation as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{6}$$

where

$$g(x) = m_{2n-1}x + \frac{1}{2}\sum_{k=1}^{2n-1}(m_{k-1} - m_k)(|x + c_k| - |x - c_k|)$$

Lorenz equation

We define the Lorenz equation as follows:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - bz \end{aligned} \tag{7}$$

where $\sigma = 10, r = 28, b = 8/3$. The Lorenz equation describes the famous chaotic phenomenon.

Chaotic Chua's UAVs

Combination of equation (4) and (6), we define and use the following state variables:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \alpha(x_2 - g(x_1)) \\ x_1 - x_2 + x_3 \\ -\beta x_2 \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \tag{8}$$

Using equation (8), we obtain the Chua's chaos UAV trajectories with Chua's equation. Fig. 2 shows the phase plane of Chua's equation.

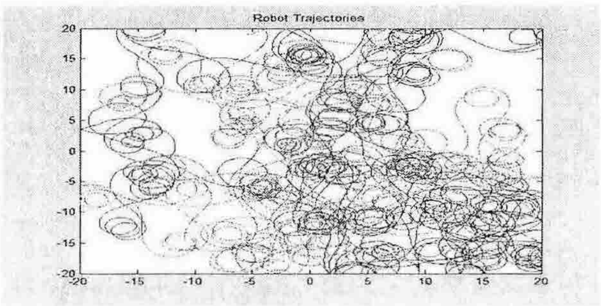


Fig. 2 Chua's UAVs trajectory.

2.4 Chaotic Lorenz UAVs

Combination of equation (4) and (7), we define and use the following state variables:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sigma(y - x) \\ \gamma x - y - xz \\ xy - bz \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \tag{9}$$

Using equation (9), we obtain the Lorenz chaotic UAVs trajectories with Lorenz equation. Fig.3 shows the phase plane of Lorenz equation.

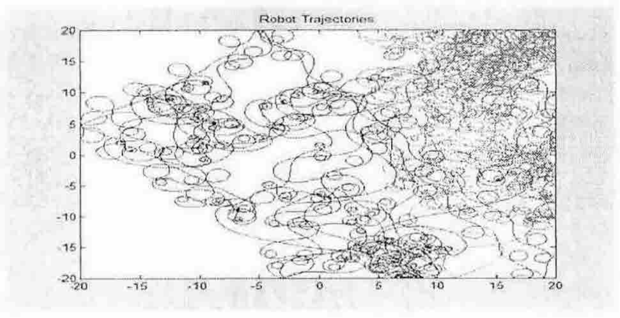


Fig. 3 Lorenz UAVs trajectory

Mirror mapping

Equations (8) - (9) assume that the UAVs moves in a smooth state space without boundaries. However, real UAVs move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the UAVs approach walls or obstacles using Equation. (10) and (11). Whenever the UAVs approach a wall or obstacle, we calculate the UAVs' new position by using Euation. (10) or (11).

$$A = \begin{pmatrix} \cos\ \theta & \sin\ \theta \\ \sin\ \theta & -\cos\ \theta \end{pmatrix} \tag{10}$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \tag{11}$$

We can use equation (10) when the slope is infinity, such as $\theta = 90$, and use equation (11) when the slope is not infinity.

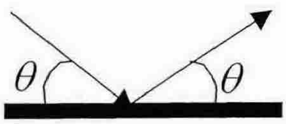


Fig. 4 Mirror mapping

The UAVs with Van der Pol equation obstacle

In this section, we will discuss the UAVs's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the UAVs can not move close to the obstacle and the obstacle is avoided.

VDP equation as a hidden obstacle

In order to represent an obstacle of the UAVs, we employ the VDP, which is written as follows:

$$\dot{x} = y$$
$$\dot{y} = (1 - y^2)y - x \tag{12}$$

From equation (12), we can get the following limit cycle as shown in Fig. 5.

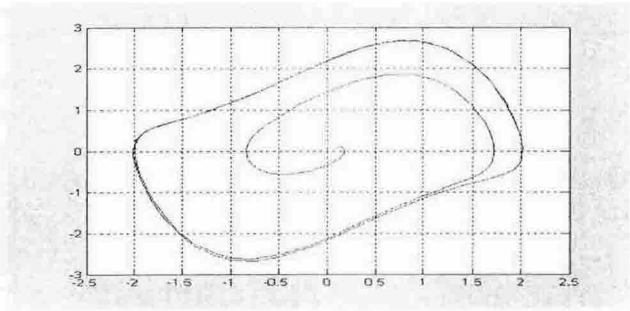


Fig. 5 Limit cycle of VDP

Magnitude of distracting force from the obstacle

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \tag{13}$$

where D_k is the distance between each effective obstacle and the UAVs.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \bar{x}_k \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1-(y_o - y)^2)(y_o - y) - (x_o - x) \end{bmatrix} \tag{14}$$

where (x_o, y_o) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual UAVs (l) and the enlarged coordinates (l/2L) of the magnitude of the virtual UAVs in VDP(x_k', y_k') as follows:

$$L = \sqrt[2]{(\vec{x}_{vdp}^{\ 2} + \vec{y}_{vdp}^{\ 2})}$$
$$I = \sqrt{(x_r^{\ 2} + y_r^{\ 2})}$$
$$x_k' = \frac{\vec{x}_k}{L}\frac{I}{2},\ y_k' = \frac{y_k}{L}\frac{I}{2} \tag{15}$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{bmatrix} \dfrac{\sum_{k}^{n}((1-\frac{D_k}{D_0})\vec{x} + \frac{D_k}{D_0}\vec{x}_k)}{n} \\ \dfrac{\sum_{k}^{n}((1-\frac{D_k}{D_0})\vec{y} + \frac{D_k}{D_0}\vec{y}_k)}{n} \end{bmatrix} \tag{16}$$

Using equations (13)-(16), we can calculate the avoidance method of the obstacle in the Chua's and Lorenz equation trajectories with one or more VDP obstacles.

Chaotic Synchronization

Chua's UAVs synchronization

In order to apply to coupled-synchronization theory in the Chua's UAVs, we compromised to state equation of Chua's UAVs is written as follows:

The state equation of main UAVs

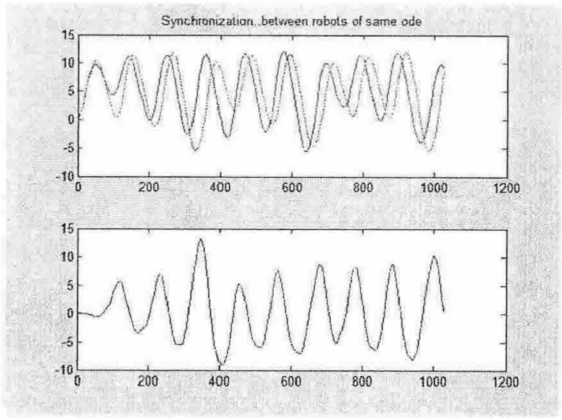
$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) + k \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= \upsilon \cos\ x_3 \\ \dot{y} &= v \sin\ x_3 \end{aligned} \tag{17}$$

The state equation of auxiliary UAVs

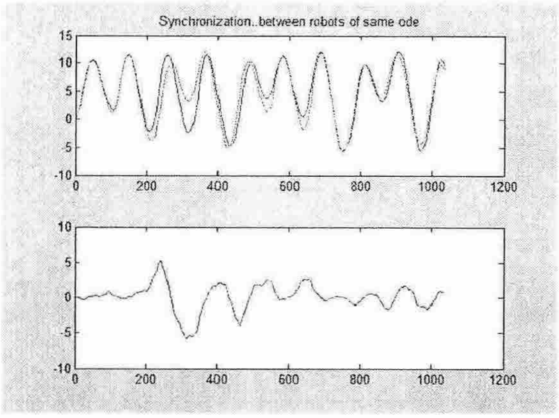
$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) + k' \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= \upsilon \cos\ x_3 \\ \dot{y} &= v \sin\ x_3 \end{aligned} \tag{18}$$

From equation (17) and (18), we apply coupled factors k or k' are 1.0, 2.0. 3.0 at the no obstacle and 3.0 hidden obstacle respectively. The results of Chua's UAVs synchronization are shown in Fig. 6.

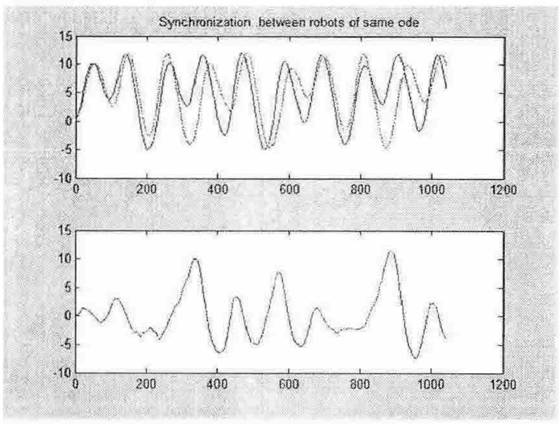
From Fig. 6 we can recognize synchronizations are generalized synchronization results and according to coupled factor difference, there are different results.



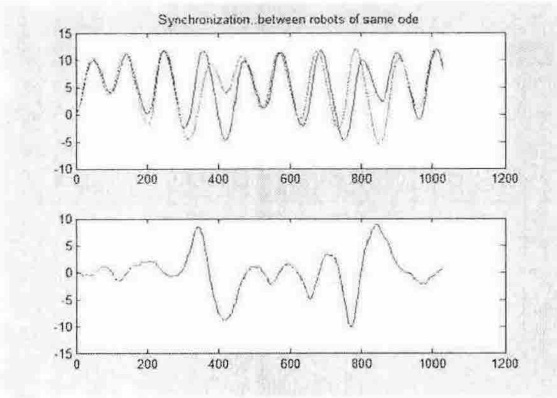
(a) k=1 , no obstacle



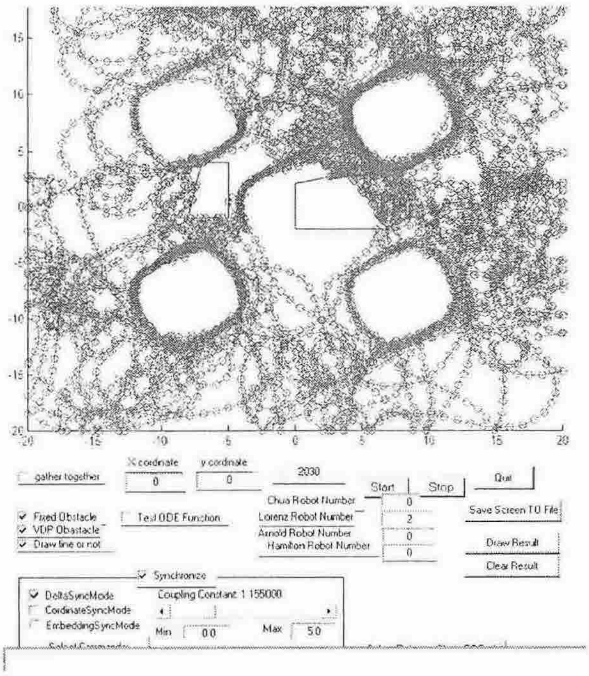
(b) k=2, no obstacle



(c) k=3, no obstacle



(d) k=3, hidden obstacle



(e) Chua's UAVs trajectory

Fig. 6 Chua's synchronization results

Lorenz UAVs synchronization

In order to apply to Driven-synchronization theory in the Lorenz's UAVs, we compromised to state equation of Lorenz's UAVs is written as follows:

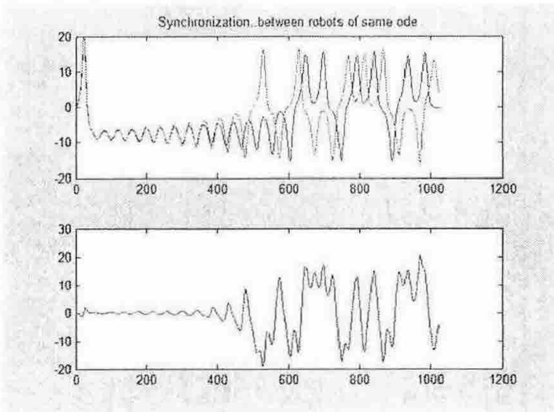
The state equation of main UAVs

$$\begin{aligned} \dot{x}_1 &= \sigma (y - x) \\ \dot{x}_2 &= \gamma x - y + xz \\ \dot{x}_3 &= xy - bz \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{19}$$

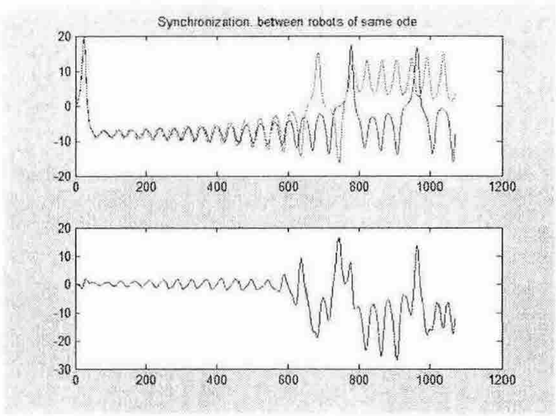
The state equation of auxiliary UAVs

$$\begin{aligned} \dot{x}_1 &= \sigma (y - x) \\ \dot{x}_2 &= \gamma x - y + xz \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{20}$$

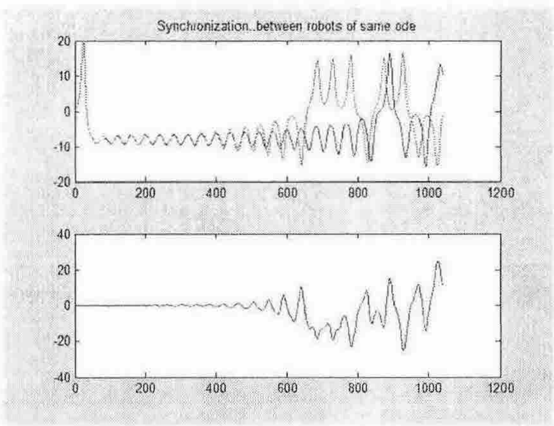
From equation (19) and (20), we can get the results of Lorenz UAVs synchronization are shown in Fig.7



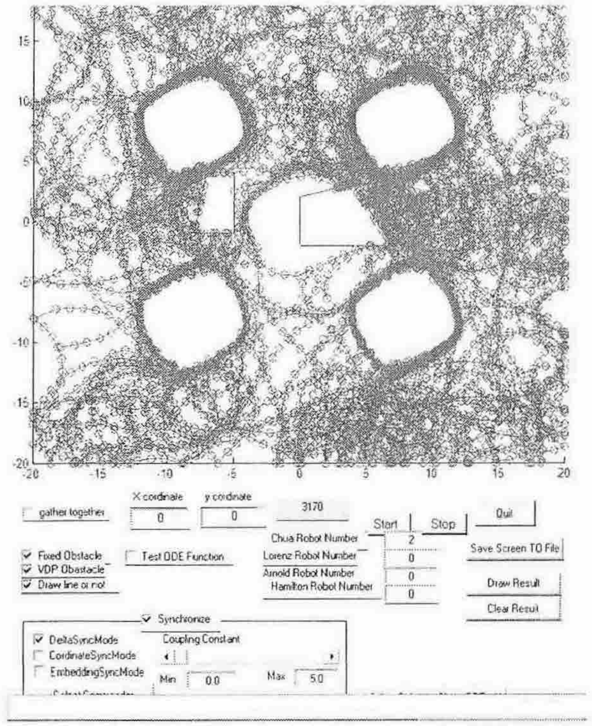
(a) No obstacle synchronization



(b) Fixed obstacle synchronization



(c) Hidden obstacle synchronization



(d) Lorenz UAVs trajectory

Fig. 7 Lorenz synchronization results

From Fig. 7 we can recognize synchronization result is generalized synchronization and also we can see that there are different synchronization results according to different obstacle such as fixed and hidden obstacles.

Concluding remark

In this paper, we proposed a chaotic UAVs, which employs a UAVs with Chua's equation and Lorenz equation trajectories, and also proposed a UAVs synchronization methods in which coupled-synchronization and driven synchronization.

We designed chaotic UAVs trajectories such that the total dynamics of the UAVs was characterized by a Chua's equation or Lorenz equation, and we also designed the chaotic UAVs trajectories to include an obstacle avoidance method. As a result, we realized that the result of synchronization is generalized synchronization.

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