Feasibility Test of the Numerical Optimization for the Fast IMRT Planning

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In this study, we have tested the feasibility of the convex non-linear objective model and the line search optimization method for the fluence map optimization (FMO). We've created the convex nonlinear objective function with simple bound constraints and attained the optimal solution using well-known gradient algorithms with an Armijo line search that requires sufficient objective function decrease. The algorithms were applied to 10 head-and-neck cases. The numbers of beamlets were between 900 and 2,100 with a 3 mm isotropic dose grid. Nonlinear optimization methods could efficiently solve the IMRT FMO problem in well under a minute with high quality for clinical cases.

Key words: IMRT, FMO, Line search

INTRODUCTION

Intensity modulated radiation therapy (IMRT) creats very complex non-uniform dose distributions that allow the delivery of sufficiently high radiation doses to targets while limiting the radiation dose delivered to healthy tissues. These dose distributions are obtained by dynamically blocking different parts of the beam and the application of optimization techniques is essential. The problem of designing an optimal radiation density profile in the patient is often referred to as the fluence map optimization (FMO) problem.[1] The goal of this problem is to design a radiation therapy treatment plan that delivers a specific level of radiation dose, a so-called prescription dose, to the targets, while on the other hand sparing critical structures by ensuring that the level of radiation dose received by these structures does not exceed some structure-specific tolerance dose. Not only for the IMRT but for the image guided radiation therapy (IGRT), the fast inverse planning should be guaranteed. In this study, we have tested the feasibility of the convex non-linear objective model and the line search optimization method for the FMO.

MATERIALS AND METHODS

1. The convex nonlinear objective function

Although the linear programming (LP) model for the FMO was proven to be a feasible and robust method for the clinical use,[1] still it requires the complicated form of objective function and constraints. Even though the convex quadratic programming (QP) showed the better performance than LP, QP was not much different from the LP model. Due to these reasons, we've created the convex nonlinear objective function with simple bound constraints. It's an essentially unconstrained problem, and has only one feasible solution (global minimum). The solution can be attained easily using known numerical optimization methods. We employed the voxel-based penalty functions that were approximated in LP and QP already.[1]

In nonlinear programming (NLP), penalty functions were applied directly as followed form

$$f = \sum_{s=1}^{S} \sum_{i=1}^{N} \frac{\alpha_s}{v_s} (d_i - t_s)^{n_s},$$

where α_s, v_s, t_s, n_s denote the importances, scaling factors, thresholds and powers of specific organ structure *s*, respectively, while d_i is the dose value at *i* th voxel. The algorithms were implemented in an in-house planning system.

2. The numerical optimization with line search method

Well-known gradient algorithms were employed along with an Armijo line search that requires sufficient objective function decrease[2,3]. Each iteration of a line search method computes a search direction p_k and then decides how far to move along that direction. For the solution vector x, the iteration is given by

$$x_{k+1} = x_k + \alpha_k p_k$$

where the positive scalar α_k is called the step length. The success of a line search method depends on effective choices of both the direction p_k and the step length α_k. Most line search algorithm require p_k to be a descent direction because the property that $p_k^T \nabla f_k < 0$ guarantees that the objective function *f* can be reduced along this direction. The search direction has the form

$$p_k = -\nabla f_k.$$

This method is often called the steepest descent (SD) method. In this study, we had considered only the simple SD algorithm with Armijo's method. A simple projected-direction interior-point method was used to enforce fluence non-negativity.

3. The feasibility test

The algorithms were applied to 10 head-and-neck cases. The numbers of beamlets were between 900 and 2,100 with a 3 mm isotropic dose grid. The number of targets, OARs and total voxels were shown at the Table 1. All the tests were performed using Dothan 1.6 GHz CPU with 1GB RAM.

Table 1. The summary of clinical cases used in this study.

	# of targets	# of OARS	# of beamlets	# of voxels
Case 1	2	12	1,017	201,390
Case 2	3	9	1,195	203,014
Case 3	2	6	1,745	454,761
Case 4	2	10	1,804	459,158
Case 5	2	11	1,182	206,152
Case 6	3	11	1,015	137,048
Case 7	2	8	1,178	242,855
Case 8	2	8	1,090	201,534
Case 9	2	8	921	167,535
Case 10	4	10	2,073	351,292

RESULTS

The tested methods were found to be very fast and converged in small number of iterations. Every tested line search methods showed the good performance even though the trials were quite case dependent. However only one case, proposed

Table 2. The result of optimization applied to clinical cases varying the λ_0 and λ_{step} values.

	$\lambda_0=1, \lambda_{step}=0.1$		$\lambda_0=10^{-4}, \lambda_{step}=0.5$	
	Time (sec)	f* ($\times 10^8$)	Time (sec)	f* ($\times 10^8$)
1	18.7	4.06	15.6	4.06
2	39.2	3.50	32.3	3.50
3	40.9	2.55	28.3	3.23
4	48.6	3.23	33.2	3.44
5	22.5	3.45	15.3	3.23
6	16.0	3.23	11.8	2.73
7	25.3	2.72	17.3	3.01
8	31.6	3.01	21.8	3.04
9	18.6	3.04	12.6	3.04
10	53.1	4.40	35.9	4.41

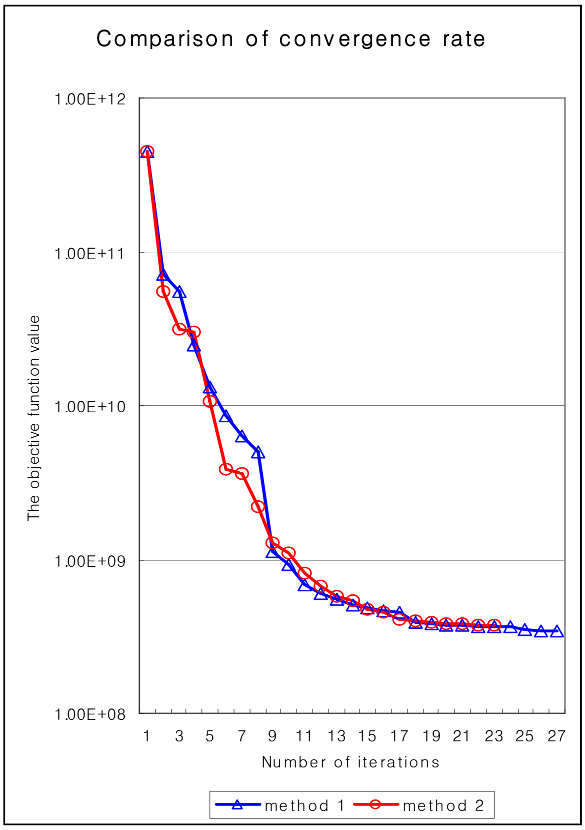

Fig. 1. The convergence trend of SD line search methods varying λ_0 and λ_{step} values for the patient case 5. Method 1 denotes the large initial value and large step size while method 2 is contrary.

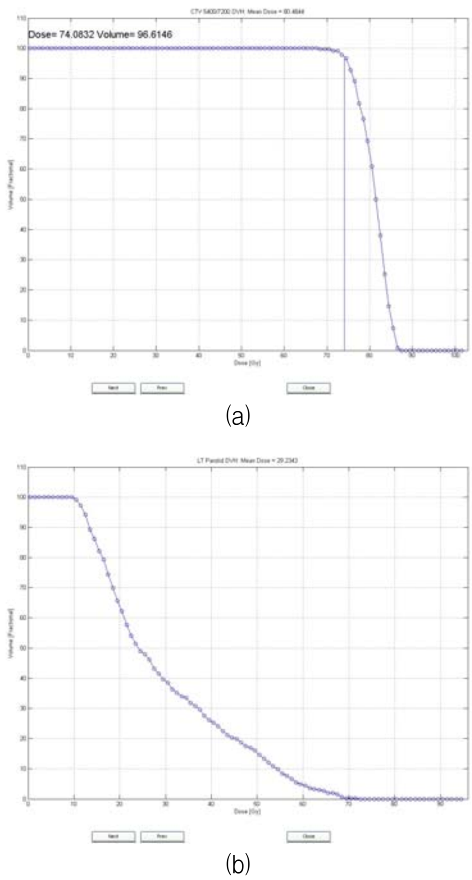

Fig. 2. The DVH of patient case 5. (a) GTV (b) LT parotid.

method failed to attain the optimal solution. During the feasibility test, we've varied the λ_0 and λ_{step} values. The smaller λ_0 showed the better performance than larger (generally unity) values (Table 2, Fig. 1). The plan results satisfied the criteria of RTOG-H0022[4)] and it can be seen from the DVH (Fig. 2).

CONCLUSION

Nonlinear optimization methods could efficiently solve the IMRT FMO problem well in a minute with high quality for clinical cases. Even though a simple SD algorithm was highly successful, it needs to be improved by employing other line search method e.g. quasi-Newton algorithm to perform a robust line search. Quasi-Newton methods have been successful in solving many large-scale nonlinear programming problems. We will test the algorithm's robustness with a larger population of cases in future.

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