Robust FIR filter for Linear Discrete-time System

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Abstract: In this paper, a robust receding horizon finite impulse response(FIR) filter is proposed for a class of linear discrete time systems with uncertainty satisfying an integral quadratic constraint. The robust state estimation problem involves constructing the set of all possible states at the current time consistent with given system input, output measurements and the integral quadratic constraint.

Keywords: Robust, finite impulse response, receding horizon, integral quadratic constraint, set

1. Introduction

An estimation problem deals with recovering some unknown parameters or variables from measured information in physical or mathematical models. For the last 40 years, the estimation problem has been widely used in science and engineering areas.

Among estimation problems, a state estimator which is called the filter has been widely investigated and often combined with control design. In state estimation problems, the state variables are assumed to be unmeasurable and thus unknown. Since the initial state information is itself a state, it is reasonable to assume that the initial state information is also unmeasurable and thus unknown.

In state space models with control inputs, filters for state estimation can have a finite impulse response (FIR) structure or an infinite impulse response (IIR). For the state x_k at time k, the linear FIR filter without a priori information on the initial state can be represented in the following batch form:

$$\hat{x}_{k|k}= \sum_{i=k-N}^{k-1} H_{k-i}y_i + \sum_{i=k-N}^{k-1} L_{k-i}u_i \tag{1}$$

for some gains H_{k-i} and L_{k-i} [1], [2], [3]. The IIR filter has a similar form as (1) with $k-N$ replaced by the initial time k_0. For IIR and FIR filters, initial states are denoted by x_{k_0} and x_{k-N} , respectively. It is noted that the linear FIR filter (1) does not include an initial state term and its gains H_{k-i} and L_{k-i} are independent of initial state information, while the standard Kalman filter does include an initial state term and its gains are dependent of initial state information. As shown in (1), FIR filters utilize only finite measurements and inputs on the most recent time interval $[k-N,\ k]$ called the receding horizon or the horizon, which can avoid long processing time due to the large data sets in case of IIR filters when time increases. The FIR structure has inherent properties such as a bounded input-bounded output (BIBO) stability and robustness against temporary modeling uncertainties and round-off errors.

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The above mentioned papers, however, did not consider the uncertain systems. But the filters require the robustness with respect to the model uncertainty. There are some attempts to investigate the robust Kalman filter [4], [5], [6], [7], [8], which needs the assumption of known initial state. In many cases, however, it may difficult to obtain the correct initial information because it may be costly and require many experience. Therefore, in the current paper, a new robust FIR filter will be investigated that is called the receding horizon robust FIR (RHRF) filter, which is independent on a *priori* information of the initial state. The RHRF filter represented in a batch form will be obtained by directly solving an optimization problem with the integral quadratic constraint.

The current paper is organized as follows. In Section 2, an RHRF filter with a batch form is proposed and finally the conclusions are stated in Section 3.

2. Receding Horizon Robust FIR Filter

Consider a linear discrete-time state space model:

$$x_{k+1} = (A + G\Delta_{1,k}E_1)x_k + (B + G\Delta_{1,k}E_2)u_k, \tag{2}$$
$$y_k = (C + \Delta_{2,k}E_1)x_k + \Delta_{2,k}E_2u_k \tag{3}$$

where $x_k \in \Re^n$ is the state, $u_k \in \Re^l$ and $y_k \in \Re^q$ are the input and measurement, respectively; A, B, C, G, E_1 and E_2 are given matrices with full row rank, $\Delta_{1,k}$ and $\Delta_{2,k}$ are uncertainty matrices satisfying

$$\Delta_{1,k}^T Q \Delta_{1,k} + \Delta_{2,k}^T R \Delta_{2,k} \le I, \tag{4}$$

for all k and $Q = Q' > 0$, $R = R' > 0$.

Let

$$w_k{=}\Delta_{1,k}(E_1x_k + E_2u_k), \tag{5}$$
$$v_k{=}\Delta_{2,k}(E_1x_k + E_2u_k). \tag{6}$$

Then, the system represented by (2) and (3) is of the form:

$$x_{k+1} = Ax_k + Bu_k + Gw_k, \tag{7}$$
$$y_k = Cx_k + v_k. \tag{8}$$

The uncertainty w_k and v_k in (7) and (8) satisfies the following inequality called integral quadratic constraint:

$$\begin{aligned}
&\sum_{i=k-N}^{k-1}(w_i^TQw_i+v_i^TRv_i)\\
&=\sum_{i=k-N}^{k-1}\left\{\Big[\Delta_{1,i}(E_1x_i+E_2u_i)\Big]^TQ\Big[\Delta_{1,i}(E_1x_i+E_2u_i)\Big]\right.\\
&\qquad\left.+\Big[\Delta_{2,i}(E_1x_i+E_2u_i)\Big]^TR\Big[\Delta_{2,i}(E_1x_i+E_2u_i)\Big]\right\}\\
&=\sum_{i=k-N}^{k-1}(E_1x_i+E_2u_i)^T\Big[\Delta_{1,i}^TQ\Delta_{1,i}+\Delta_{2,i}^TR\Delta_{2,i}\Big](E_1x_i\\
&\qquad+E_2u_i)\\
&\le\sum_{i=k-N}^{k-1}(E_1x_i+E_2u_i)^T(E_1x_i+E_2u_i) \qquad (9)
\end{aligned}$$

where the last inequality is obtained from (4).
The system (7) and (8) will be represented in a batch form on the most recent time interval $[k-N,k]$ called the horizon. The horizon intial time $k-N$ will be denoted by k_N hereafter for simplicity. On the horizon $[k_N,k]$, the finite number of measurements is expressed in terms of the state x_k at the current time k as follows:

$$Y_{k-1} \;=\; \bar{C}_Nx_k+\bar{B}_NU_{k-1}+\bar{G}_NW_{k-1}+V_{k-1} \qquad (10)$$

where

$$Y_{k-1}\triangleq[y_{k_N}^T\;\; y_{k_N+1}^T\;\; \cdots\;\; y_{k-1}^T]^T, \qquad (11)$$
$$U_{k-1}\triangleq[u_{k_N}^T\;\; u_{k_N+1}^T\;\; \cdots\;\; u_{k-1}^T]^T, \qquad (12)$$
$$W_{k-1}\triangleq[w_{k_N}^T\;\; w_{k_N+1}^T\;\; \cdots\;\; w_{k-1}^T]^T, \qquad (13)$$
$$V_{k-1}\triangleq[v_{k_N}^T\;\; v_{k_N+1}^T\;\; \cdots\;\; v_{k-1}^T]^T, \qquad (14)$$

and $\bar{C}_N$, $\bar{B}_N$, $\bar{G}_N$ are obtained from

$$\bar{C}_i\triangleq\begin{bmatrix}CA^{-i}\\CA^{-i+1}\\CA^{-i+2}\\\vdots\\CA^{-1}\end{bmatrix}=\begin{bmatrix}\bar{C}_{i-1}\\C\end{bmatrix}A^{-1}, \qquad (15)$$

$$\begin{aligned}
\bar{B}_i&\triangleq-\begin{bmatrix}CA^{-1}B & CA^{-2}B & \cdots & CA^{-i}B\\0 & CA^{-1}B & \cdots & CA^{-i+1}B\\0 & 0 & \cdots & CA^{-i+2}B\\\vdots & \vdots & \vdots & \vdots\\0 & 0 & \cdots & CA^{-1}B\end{bmatrix}\\
&=\begin{bmatrix}\bar{B}_{i-1} & -\bar{C}_{i-1}A^{-1}B\\0 & -CA^{-1}B\end{bmatrix}, \qquad (16)
\end{aligned}$$

$$\begin{aligned}
\bar{G}_i&\triangleq-\begin{bmatrix}CA^{-1}G & CA^{-2}G & \cdots & CA^{-i}G\\0 & CA^{-1}G & \cdots & CA^{-i+1}G\\0 & 0 & \cdots & CA^{-i+2}G\\\vdots & \vdots & \vdots & \vdots\\0 & 0 & \cdots & CA^{-1}G\end{bmatrix}\\
&=\begin{bmatrix}\bar{G}_{i-1} & -\bar{C}_{i-1}A^{-1}G\\0 & -CA^{-1}G\end{bmatrix}, \qquad (17)
\end{aligned}$$

for $2\le i\le N$. $\bar{C}_1$, $\bar{B}_1$ and $\bar{G}_1$ are defined as

$$\begin{aligned}
\bar{C}_1&\triangleq CA^{-1},\\
\bar{B}_1&\triangleq -CA^{-1}B,\\
\bar{G}_1&\triangleq -CA^{-1}G.
\end{aligned}$$

The noise term $\bar{G}_NW_{k-1}+V_{k-1}$ in (10) can be shown to be zero-mean with covariance Ξ_N given by

$$\Xi_i\triangleq\bar{G}_i\big[\mathrm{diag}(\overbrace{Q\;\cdots\;Q}^{i})\big]\bar{G}_i^T+\big[\mathrm{diag}(\overbrace{R\;\cdots\;R}^{i})\big] \qquad (18)$$

With (13) and (14), the inequality (9) can be further represented by the following compact form:

$$\begin{aligned}
W_{k-1}^TQ_NW_{k-1}+V_{k-1}^TR_NV_{k-1}&\le X_{k-1}^TM_NX_{k-1}\\
&+X_{k-1}^TS_NU_{k-1}+U_{k-1}^TN_NU_{k-1} \qquad (19)
\end{aligned}$$

where

$$X_{k-1}\triangleq[x_{k_N}^T\;\; x_{k_N+1}^T\;\; \cdots\;\; x_{k-1}^T]^T, \qquad (20)$$
$$Q_i\triangleq\big[\mathrm{diag}(\overbrace{Q\;\cdots\;Q}^{i})\big], \qquad (21)$$
$$R_i\triangleq\big[\mathrm{diag}(\overbrace{R\;\cdots\;R}^{i})\big], \qquad (22)$$
$$M_i\triangleq\big[\mathrm{diag}(\overbrace{E_1^TE_1\;\cdots\;E_1^TE_1}^{i})\big], \qquad (23)$$
$$S_i\triangleq\big[2\cdot\mathrm{diag}(\overbrace{E_1^TE_2\;\cdots\;E_1^TE_2}^{i})\big], \qquad (24)$$
$$N_i\triangleq\big[\mathrm{diag}(\overbrace{E_2^TE_2\;\cdots\;E_2^TE_2}^{i})\big]. \qquad (25)$$

Let us define

$$\begin{aligned}
&J\big[x_k,W_{k-1}\big]\\
&\triangleq\Big[W_{k-1}^TQ_NW_{k-1}+(K_{k-1}-\bar{C}_Nx_k-\bar{G}_NW_{k-1})^T\\
&\times R_N(K_{k-1}-\bar{C}_Nx_k-\bar{G}_NW_{k-1})-X_{k-1}^TM_NX_{k-1}\\
&-X_{k-1}^TS_NU_{k-1}-U_{k-1}^TN_NU_{k-1}\Big], \qquad (26)
\end{aligned}$$

where

$$K_{k-1}=Y_{k-1}-\bar{B}_NU_{k-1}, \qquad (27)$$

and

$$X_{k-1}=\hat{A}_Nx_k+\hat{B}_NU_{k-1}+\hat{G}_NW_{k-1}, \qquad (28)$$

with

$$\hat{A}_i\triangleq\begin{bmatrix}A^{-i}\\A^{-i+1}\\A^{-i+2}\\\vdots\\A^{-1}\end{bmatrix}=\begin{bmatrix}\hat{A}_{i-1}\\I\end{bmatrix}A^{-1}, \qquad (29)$$

$$\hat{B}_i \triangleq -\begin{bmatrix} A^{-1}B & A^{-2}B & \cdots & A^{-i}B \\ 0 & A^{-1}B & \cdots & A^{-i+1}B \\ 0 & 0 & \cdots & A^{-i+2}B \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & A^{-1}B \end{bmatrix}$$
$$= \begin{bmatrix} \hat{B}_{i-1} & -\hat{A}_{i-1}A^{-1}B \\ 0 & -A^{-1}B \end{bmatrix}, \tag{30}$$

$$\hat{G}_i \triangleq -\begin{bmatrix} A^{-1}G & A^{-2}G & \cdots & A^{-i}G \\ 0 & A^{-1}G & \cdots & A^{-i+1}G \\ 0 & 0 & \cdots & A^{-i+2}G \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & A^{-1}G \end{bmatrix}$$
$$= \begin{bmatrix} \hat{G}_{i-1} & -\hat{A}_{i-1}A^{-1}G \\ 0 & -A^{-1}G \end{bmatrix}, \tag{31}$$

for $2 \le i \le N$. $\hat{A}_1$, $\hat{B}_1$ and $\hat{G}_1$ are defined as

$$\hat{A}_1 \triangleq A^{-1},$$
$$\hat{B}_1 \triangleq -A^{-1}B,$$
$$\hat{G}_1 \triangleq -A^{-1}G.$$

Arranging (26) in terms of W_{k-1} yields

$$\begin{aligned} &J\left[x_k, W_{k-1}\right] \\ &= W_{k-1}^T(Q_N + \bar{G}_N^T R_N \bar{G}_N - \hat{G}_N^T M_N \hat{G}_N)W_{k-1} - 2W_{k-1}^T \\ &\times \Big[\bar{G}_N^T R_N(K_{k-1} - \bar{C}_N x_k) + \hat{G}_N^T M_N(\hat{A}_N x_k + \hat{B}_N U_{k-1}) \\ &+ \hat{G}_N^T S_N U_{k-1}\Big] + \overline{W}_{k-1}, \end{aligned} \tag{32}$$

where $\overline{W}_{k-1}$ is the terms without W_{k-1} and is as follows:

$$\begin{aligned} \overline{W}_{k-1} =& (K_{k-1} - \bar{C}_N x_k)^T R_N (K_{k-1} - \bar{C}_N x_k) - (\hat{A}_N x_k \\ &+ \hat{B}_N U_{k-1})^T M_N (\hat{A}_N x_k + \hat{B}_N U_{k-1}) - (\hat{A}_N x_k \\ &+ \hat{B}_N U_{k-1})^T S_N U_{k-1} - U_{k-1}^T N_N U_{k-1} \\ =& x_k^T \Big[\bar{C}_N^T R_N \bar{C}_N - \hat{A}_N^T M_N \hat{A}_N\Big] x_k - x_k^T \Big[2\bar{C}_N^T R_N K_{k-1} \\ &+ 2\hat{A}_N^T M_N \hat{B}_N U_{k-1} + \hat{A}_N^T S_N U_{k-1}\Big] + \Big[K_{k-1}^T R_N K_{k-1} \\ &- (\hat{B}_N U_{k-1})^T M_N (\hat{B}_N U_{k-1}) - (\hat{B}_N U_{k-1})^T S_N U_{k-1} \\ &- U_{k-1}^T N_N U_{k-1}\Big] \\ =& x_k^T \Phi_1 x_k + 2x_k^T \Phi_2 + \Phi_3 \end{aligned} \tag{33}$$

with

$$\Phi_1 \triangleq \bar{C}_N^T R_N \bar{C}_N - \hat{A}_N^T M_N \hat{A}_N, \tag{34}$$
$$\begin{aligned} \Phi_2 \triangleq& (\bar{C}_N^T R_N \bar{B}_N - \hat{A}_N^T M_N \hat{B}_N - \frac{1}{2}\hat{A}_N^T S_N)U_{k-1} \\ &- \bar{C}_N^T R_N Y_{k-1}, \end{aligned} \tag{35}$$
$$\begin{aligned} \Phi_3 \triangleq& Y_{k-1}^T R_N Y_{k-1} - 2U_{k-1}^T \bar{B}_N^T R_N Y_{k-1} + U_{k-1}^T (\bar{B}_N^T R_N \bar{B}_N \\ &- \hat{B}_N^T M_N \hat{B}_N - \hat{B}_N^T S_N - N_N)U_{k-1}. \end{aligned} \tag{36}$$

Let us define the following quantities for brevity:

$$P \triangleq Q_N + \bar{G}_N^T R_N \bar{G}_N - \hat{G}_N^T M_N \hat{G}_N, \tag{37}$$

$$\begin{aligned} F \triangleq& \bar{G}_N^T R_N (K_{k-1} - \bar{C}_N x_k) + \hat{G}_N^T M_N (\hat{A}_N x_k + \hat{B}_N U_{k-1}) \\ &+ \hat{G}_N^T S_N U_{k-1}. \end{aligned} \tag{38}$$

Then, (32) can be rewritten as follows:

$$\begin{aligned} &J\left[x_k, W_{k-1}\right] \\ &= W_{k-1}^T P W_{k-1} - 2W_{k-1}^T F + x_k^T \Phi_1 x_k + 2x_k^T \Phi_2 + \Phi_3 \end{aligned} \tag{39}$$

Now we are in a position to find out the corresponding set $\chi_k\left[Y_\cdot, U_\cdot\right]$ of all possible states x_k at time k for the uncertain system represented by (7) and (8). If and only if there exists an uncertainty W_{k-1} such that

$$J\left[x_k, W_{k-1}\right] \le 0, \tag{40}$$

then $x_k \in \chi_k\left[Y_\cdot, U_\cdot\right]$.
Let consider the following minimization problem to obtain the set $\chi_k\left[Y_\cdot, U_\cdot\right]$

$$\min_{W_{k-1}} J\left[x_k, W_{k-1}\right]. \tag{41}$$

As in [9], [10], (41) can be written as

$$\min_{W_{k-1}} J\left[x_k, W_{k-1}\right] = (x_k - \hat{x}_{k|k})^T \Pi^{-1} (x_k - \hat{x}_{k|k}) - \rho_k \tag{42}$$

Recall that $x^T P x + 2x^T q$ is minimized at $x = -P^{-1}q$ and its optimal value is $-q^T P^{-1} q$.
Therefore,

$$\begin{aligned} J^* =& \min_{W_{k-1}} J\left[x_k, W_{k-1}\right] \\ =& \min_{W_{k-1}} \Big[W_{k-1}^T Q_N W_{k-1} + (K_{k-1} - \bar{C}_N x_k - \bar{G}_N W_{k-1})^T \\ &\times R_N (K_{k-1} - \bar{C}_N x_k - \bar{G}_N W_{k-1}) - X_{k-1}^T M_N X_{k-1} \\ &- X_{k-1}^T S_N U_{k-1} - U_{k-1}^T N_N U_{k-1}\Big] \\ =& -F^T P^{-1} F + \overline{W}_{k-1} \\ =& -\Big[\bar{G}_N^T R_N (K_{k-1} - \bar{C}_N x_k) + \hat{G}_N^T M_N (\hat{A}_N x_k + \hat{B}_N U_{k-1}) \\ &+ \hat{G}_N^T S_N U_{k-1}\Big]^T P^{-1} \Big[\bar{G}_N^T R_N (K_{k-1} - \bar{C}_N x_k) + \hat{G}_N^T M_N \\ &(\hat{A}_N x_k + \hat{B}_N U_{k-1}) + \hat{G}_N^T S_N U_{k-1}\Big] + \overline{W}_{k-1}, \end{aligned} \tag{43}$$

when

$$\begin{aligned} W_{k-1} =& \Big[Q_N + \bar{G}_N^T R_N \bar{G}_N - \hat{G}_N^T M_N \hat{G}_N\Big]^{-1} \Big[\bar{G}_N^T R_N (K_{k-1} \\ &- \bar{C}_N x_k) + \hat{G}_N^T M_N (\hat{A}_N x_k + \hat{B}_N U_{k-1}) + \hat{G}_N^T S_N U_{k-1}\Big]. \end{aligned} \tag{44}$$

Because F in (38) can be rewritten as follows:

$$\begin{aligned} F \triangleq& \bar{G}_N^T R_N (K_{k-1} - \bar{C}_N x_k) + \hat{G}_N^T M_N (\hat{A}_N x_k + \hat{B}_N U_{k-1}) \\ &+ \hat{G}_N^T S_N U_{k-1} \\ =& \Psi_1 x_k - (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1}) \end{aligned} \tag{45}$$

with

$$\Psi_1 \triangleq \hat{G}_N^T M_N \hat{A}_N - \bar{G}_N^T R_N \bar{C}_N, \tag{46}$$

$$\Psi_2 \triangleq G_N^T R_N, \tag{47}$$

$$\Psi_3 \triangleq \hat{G}_N^T M_N \hat{B}_N + \hat{G}_N^T S_N - \bar{G}_N^T R_N \bar{B}_N. \tag{48}$$

Therefore,

$$\begin{aligned} J^* =& -\left[\Psi_1 x_k - (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1})\right]^T P^{-1} \left[\Psi_1 x_k - (\Psi_2 Y_{k-1} \right. \\ & \left. + \Psi_3 U_{k-1})\right] + x_k^T \Phi_1 x_k + 2 x_k^T \Phi_2 + \Phi_3 \\ =& x_k^T \left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right] x_k + 2 x_k^T \left[\Psi_1^T P^{-1} (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1}) \right. \\ & \left. + \Phi_2\right] + \left[(\Psi_2 Y_{k-1} + \Psi_3 U_{k-1})^T P^{-1} (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1}) + \Phi_3\right] \\ =& (x_k - \hat{x}_{k|k})^T \left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right] (x_k - \hat{x}_{k|k}) - \rho_k \end{aligned} \tag{49}$$

where

$$\begin{aligned} \hat{x}_{k|k} =& -\left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right]^{-1} \left[\Psi_1^T P^{-1} (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1}) + \Phi_2\right] \\ =& -\left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right]^{-1} \left[\Psi_1^T P^{-1} \Psi_2 Y_{k-1} + \Psi_1^T P^{-1} \Psi_3 U_{k-1}) \right. \\ & \left. + (\bar{C}_N^T R_N \bar{B}_N - \hat{A}_N^T M_N \hat{B}_N - \frac{1}{2} \hat{A}_N^T S_N) U_{k-1} - \bar{C}_N^T R_N Y_{k-1}\right] \\ =& -\left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right]^{-1} \left[(\Psi_1^T P^{-1} \Psi_2 - \bar{C}_N^T R_N) Y_{k-1} + (\Psi_1^T \right. \\ & \left. \times P^{-1} \Psi_3 + \bar{C}_N^T R_N \bar{B}_N - \hat{A}_N^T M_N \hat{B}_N - \frac{1}{2} \hat{A}_N^T S_N) U_{k-1}\right] \\ =& H Y_{k-1} + L U_{k-1} \end{aligned} \tag{50}$$

with

$$H \triangleq -\left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right]^{-1} \left(\Psi_1^T P^{-1} \Psi_2 - \bar{C}_N^T R_N\right) \tag{51}$$

$$\begin{aligned} L \triangleq & -\left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right]^{-1} (\Psi_1^T P^{-1} \Psi_3 + \bar{C}_N^T R_N \bar{B}_N - \hat{A}_N^T M_N \hat{B}_N \\ & - \frac{1}{2} \hat{A}_N^T S_N) U_{k-1} \end{aligned} \tag{52}$$

and

$$\begin{aligned} \rho_k =& -\left[(\Psi_2 Y_{k-1} + \Psi_3 U_{k-1})^T P^{-1} (\Psi_2 Y_{k-1} + \Psi_3 U_{k-1}) + \Phi_3\right] \\ & + \hat{x}_{k|k}^T \left[\Phi_1 - \Psi_1^T P^{-1} \Psi_1\right] \hat{x}_{k|k}. \end{aligned} \tag{53}$$

Then the ellipsoid

$$\chi_k = \left\{ x_k : (x_k - \hat{x}_{k|k})^T \Sigma^{-1} (x_k - \hat{x}_{k|k}) \le \rho_k \right\}, \tag{54}$$

with

$$\Sigma^{-1} = \Phi_1 - \Psi_1^T P^{-1} \Psi_1 \tag{55}$$

and $\hat{x}_{k|k}$, ρ_k in (50) and (53) respectively, is the set of of all possible states at the current time consistent with given system input, output measurements and uncertainties satisfying the integral quadratic constraint.

3. Conclusion

In this paper, an integral quadratic constrained robust FIR filter is proposed for a class of linear discrete time uncertain systems. With the consideration of the system uncertainty, the proposed FIR filter has better performance than the existing FIR filter for the unstable systems.

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