Complete Identification of Isotropic Configurations of a Caster Wheeled Mobile Robot with Nonredundant/Redundant Actuation

Sungbok Kim and Byungkwon Moon

Department of Digital and Information Eng., Hankuk University of Foreign Studies, Korea
(Tel: +82-31-330-4091; E-mail: sbkim@hufs.ac.kr)

Abstract: In this paper, we present a complete isotropy analysis of a caster wheeled omnidirectional mobile robot(COMR) with nonredundant/redundant actuation. The motivation of this work is that the omnidirectional mobility loses significance in motion control unless the isotropy characteristics is maintained well. First, with the characteristic length introduced, the kinematic model of a COMR is obtained based on the orthogonal decomposition of the wheel velocities. Second, a general form of the isotropy conditions of a COMR is given in terms of physically meaningful vector quantities which describe the wheel configurations. Third, for all possible nonredundant and redundant actuation sets, the algebraic expressions of the isotropy conditions are derived to completely identify the isotropic configurations of a COMR. Fourth, the number of the isotropic configurations and the characteristic length required for the isotropy are discussed.

Keywords: Omnidirectional mobility, caster wheel, isotropy analysis, isotropic configuration

1. INTRODUCTION

The omnidirectional mobility of a mobile robot is required to navigate in daily life environment which is restricted in space and cluttered with obstacles. Several omnidirectional wheel mechanisms have been proposed, including universal wheels, Swedish wheels, orthogonal wheels, ball wheels, and so on. Recently, caster wheels were employed as a practical and effective means to develop an omnidirectional mobile robot at Stanford University, which was later commercialized by Nomadic Technologies as XR4000 [1]. Since caster wheels do without small peripheral rollers or support structure, a caster wheeled omnidirectional mobile robot or a COMR can maintain good performance at varying payload or ground condition. However, the omnidirectional mobility of a COMR cannot gain significance in motion control, unless the isotropy characteristics is maintained well.

There have been several works on the kinematic issues of a COMR. For a general form of wheeled mobile robots, a systematic procedure for kinematic modeling was developed [2]. In view of the minimal admissible actuation, it was shown that at least four joints out of two caster wheels should be actuated to avoid the singularity [3]. For some specific actuation sets, the global isotropy characteristics over the entire configuration was considered to obtain the optimal design parameters of the mechanism [4]. For representative actuation sets, the algebraic conditions for the (local) isotropy was derived to identify the isotropic configurations [5]. On the other hand, for an omnidirectional mobile robot employing Swedish wheels, the isotropy analysis was made but the results are incomplete and need further elaboration [6].

The purpose of this paper is to completely identifies the isotropic configurations of a COMR with nonredundant/redundant actuation. This paper is organized as follows. With the characteristic length introduced [6], Section 2 obtains the kinematic model based on the orthogonal decomposition of the wheel velocities. Section 3 gives a general form of the isotropy conditions in terms of physically meaningful vector quantities describing the wheel configurations. For all possible nonredundant and redundant actuation sets, Section 4 and 5 derive the algebraic expressions of the isotropy conditions to identify the isotropic configurations. Section 6 discusses the number of isotropic configurations and the characteristic length required for the isotropy. Finally, the conclusion is made in Section 7.

2. KINEMATIC MODEL

Consider a COMR with three caster wheels attached to a regular triangular platform moving on the xy plane, as shown in Fig. 1.

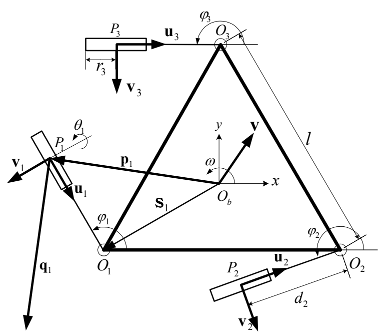

Fig. 2 A caster wheeled omnidirectional mobile robot.

Let l be the side length of the platform with the center O_b, and three vertices, O_i, $i=1,2,3$. For the i th caster wheel with the center P_i, $i=1,2,3$, we define the following. Let d_i and r_i be the length of the steering link and the radius of the wheel, respectively. Let θ_i and φ_i be the angles of the rotating and the steering joints, respectively. Let $\mathbf{u}_i$ and $\mathbf{v}_i$ be two orthogonal unit vectors along the steering link and the wheel axis, respectively, such that

$$\mathbf{u}_i=\begin{bmatrix} -\cos\varphi_i \\ -\sin\varphi_i \end{bmatrix}, \quad \mathbf{v}_i=\begin{bmatrix} -\sin\varphi_i \\ \cos\varphi_i \end{bmatrix} \tag{1}$$

Note that

$$\mathbf{u}_i\ \mathbf{u}_i^t + \mathbf{v}_i\ \mathbf{v}_i^t = \mathbf{I}_2 \tag{2}$$

$$\sum \mathbf{u}_i = \mathbf{0} \Leftrightarrow \sum \mathbf{v}_i = \mathbf{0} \tag{3}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero vector.

Let $\mathbf{p}_i$ be the vector from O_b to P_i, and $\mathbf{q}_i$ be the rotation of $\mathbf{p}_i$ by 90^o counterclockwise. Note that

$$\sum \mathbf{q}_i = 0 \Leftrightarrow \sum \mathbf{p}_i = 0 \tag{4}$$

$$\sum_1^3 \mathbf{p}_i = 0 \Leftrightarrow \sum_1^3 \mathbf{u}_i = 0 \tag{5}$$

Let $\mathbf{v}$ and ω be the linear and the angular velocities at O_b of the platform, respectively. For the i^{th} caster wheel, $i=1,2,3$, the linear velocity at the point of contact with the ground can be expressed by

$$\mathbf{v} + \omega\ \mathbf{q}_i = r_i \dot{\theta}_i \mathbf{u}_i + d_i \dot{\varphi}_i \mathbf{v}_i,\quad i=1,2,3 \tag{6}$$

Premultiplying (6) by $\mathbf{u}_i^t$ and $\mathbf{v}_i^t$, we have

$$\mathbf{u}_i^t \mathbf{v} + \mathbf{u}_i^t \mathbf{q}_i\ \omega = r_i\ \dot{\theta}_i,\quad i=1,2,3 \tag{7}$$

$$\mathbf{v}_i^t \mathbf{v} + \mathbf{v}_i^t \mathbf{q}_i\ \omega = d_i\ \dot{\varphi}_i,\quad i=1,2,3 \tag{8}$$

Assume that n $(3 \le n \le 6)$ joints of a COMR are actuated. With the characteristic length, $L(>0)$, introduced [6], the kinematics of a COMR can be written as

$$\mathbf{A}\ \dot{\mathbf{x}} = \mathbf{B}\ \dot{\Theta} \tag{9}$$

where $\dot{\mathbf{x}} = [\ \mathbf{v}\ L\omega\ ]^t \in \mathrm{R}^{3\times1}$ is the task velocity vector, and $\dot{\Theta} \in \mathrm{R}^{n\times1}$ is the joint velocity vector, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{g}_1^t & \frac{1}{L}\ \mathbf{g}_1^t\ \mathbf{h}_1 \\ \vdots & \vdots \\ \mathbf{g}_n^t & \frac{1}{L}\ \mathbf{g}_n^t\ \mathbf{h}_n \end{bmatrix} \in \mathrm{R}^{n\times3} \tag{10}$$

$$\mathbf{B} = \begin{bmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{bmatrix} \in \mathrm{R}^{n\times n} \tag{11}$$

are the Jacobian matrices. In (10), $\mathbf{g}_k$, $k=1,\cdots,n$, corresponds to either $\mathbf{u}_i$ or $\mathbf{v}_i$, $i=1,2,3$, while $\mathbf{h}_k$, $k=1,\cdots,n$, corresponds to $\mathbf{q}_i$, $i=1,2,3$. In (11), c_k, $k=1,\cdots,n$, corresponds to either r_i or d_i, $i=1,2,3$. It should be mentioned that the introduction of the characteristic length L makes all three columns of $\mathbf{A}$ to be consistent in physical unit.

The expressions of $\mathbf{g}_k^t\ \mathbf{h}_k$, $k=1,\cdots,n$, can be simplified as follows. In the case of the rotating joint for which $\mathbf{g}_k = \mathbf{u}_i$ and $\mathbf{h}_k = \mathbf{q}_i$, $i=1,2,3$,

$$\mathbf{g}_k^t\ \mathbf{h}_k = \mathbf{u}_i^t\ \mathbf{q}_i = \mathbf{v}_i^t\ \mathbf{p}_i \tag{12}$$

And, in the case of the steering joint for which $\mathbf{g}_k = \mathbf{v}_i$ and $\mathbf{h}_k = \mathbf{q}_i$, $i=1,2,3$,

$$\mathbf{g}_k^t\ \mathbf{h}_k = \mathbf{v}_i^t\ \mathbf{q}_i = -\ \mathbf{u}_i^t\ \mathbf{p}_i \tag{13}$$

It is worthwhile to mention that our kinematic modeling of a COMR does not involve matrix inversion, unlike the transfer method in [4]. For a given task velocity, the instantaneous motion of the wheel is decomposed into two orthogonal components: the instantaneous motion of the rotating joint and the instantaneous motion of the steering joint. The resulting kinematic model allows us to perform a geometric and intuitive analysis on the isotropy of a COMR.

3. ISOTROPY CONDITIONS

Based on (9), the isotropy conditions of a COMR can be stated as

$$\mathbf{A}^t\ \mathbf{A} \propto \mathbf{I}_3 \tag{14}$$

$$\mathbf{B} \propto \mathbf{I}_6 \tag{15}$$

From (11) and (15), the isotropy condition on $\mathbf{B}$ is obtained by

$$c_k = d > 0,\quad k=1,\cdots,n \tag{16}$$

(16) indicates that three caster wheels should be identical to have the steering link length equal to the wheel radius.

From (10) and (14), the isotropy condition on $\mathbf{A}$ is obtained by

$$\mathbf{A}^t\ \mathbf{A} = \frac{n}{2}\ \mathbf{I}_3 \tag{17}$$

which leads to the following three conditions:

$$\begin{aligned} &\mathrm{C1}: && \sum_1^n \mathbf{g}_k\ \mathbf{g}_k^t = \frac{n}{2}\ \mathbf{I}_2 \in \mathrm{R}^{2\times2} \\ &\mathrm{C2}: && \sum_1^n (\ \mathbf{g}_k^t\ \mathbf{h}_k\ )\ \mathbf{g}_k = 0 \in \mathrm{R}^{2\times1} \\ &\mathrm{C3}: && \frac{1}{L^2}\sum_1^n (\ \mathbf{g}_k^t\ \mathbf{h}_k)^2 = \frac{n}{2} \in \mathrm{R}^{1\times1} \end{aligned} \tag{18}$$

In general, C1 and C2 correspond to three and two scalar constraints, respectively, which are imposed on three steering joint angles, φ_k, $k=1,2,3$. Thus, the isotropy of a COMR can occur only at specific values of φ_k, $k=1,2,3$, called isotropic configurations, for which C1 and C2 are satisfied simultaneously. For a given isotropic configuration, C3, corresponding to one scalar constraint, determines the characteristic length required for the isotropy, denoted by L_{iso}.

In what follows, it is assumed that a COMR has three identical caster wheels having the steering link length equal to the wheel radius.

4. ISOTROPY ANALYSIS FOR NONREDUNDANT ACTUATION

4.1 Nonredundant Actuation Sets

A COMR with nonredundant acuation can have three actuated joints $(n=3)$, each of which can be either rotating or steering one. According to the number of active wheels and the combination of actuated joints, all possible nonredundant actuation sets, Θ, can be divided into three groups, denoted by NAG I, II, and III, as listed in Table 1.

Table 1 Three nonredundant actuation groups.

Number of actuated joints	Number of active wheels	Actuation set	Nonredundant actuation group
$n=3$	3	$\Theta=\{\theta_1,\theta_2,\theta_3\}$	NAG I
		$\Theta=\{\varphi_1,\varphi_2,\varphi_3\}$	
		$\Theta=\{\varphi_1,\theta_2,\theta_3\}$	NAG II
		$\Theta=\{\varphi_1,\varphi_2,\theta_3\}$	
	2	$\Theta=\{\theta_1,\varphi_1,\theta_2\}$	NAG III
		$\Theta=\{\theta_1,\varphi_1,\varphi_2\}$	

4.2 Isotropy Analysis for NAG I

Consider $\Theta=\{\theta_1,\theta_2,\theta_3\}$ where three rotating joints of three caster wheels are actuated, for which $[\,\mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\,]=[\,\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3\,]$.

Under the condition of C1, we have

$$\sum_{k=1}^{3}\mathbf{u}_k\ \mathbf{u}_k^{\,t}\ =\ 1.5\ \mathbf{I}_2 \tag{19}$$

which is

$$\begin{aligned} c_1^{\,2}+c_2^{\,2}+c_3^{\,2} &= 1.5\\ c_1s_1+c_2s_2+c_3s_3 &= 0.0 \end{aligned} \tag{20}$$

where $c_k=\cos(\varphi_k)$ and $s_k=\sin(\varphi_k)$, $k=1,2,3$.

There are eight different distributions of $\{\ \mathbf{u}_k, k=1,2,3\ \}$ satisfying (20), which can be divided into two distinctive groups characterized, respectively, by

$$\varphi_2=\varphi_1+\frac{2}{3}\pi,\ \ \varphi_1-\frac{\pi}{3},\ \ \varphi_3=\varphi_1+\frac{\pi}{3},\ \ \varphi_1-\frac{2}{3}\pi \tag{21}$$

$$\varphi_2=\varphi_1+\frac{\pi}{3},\ \ \varphi_1-\frac{2}{3}\pi,\ \ \varphi_3=\varphi_1+\frac{2}{3}\pi,\ \ \varphi_1-\frac{\pi}{3} \tag{22}$$

as shown in Fig. 2. The first group of four distributions, characterized by (21), is common in that $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ lie on three sides of a regular triangle in counterclockwise order, as shown in Fig. 2a). On the other hand, the second group of four distributions, characterized by (22), is common in that $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ lie on three sides of a regular triangle in clockwise order, as shown in Fig. 2b).

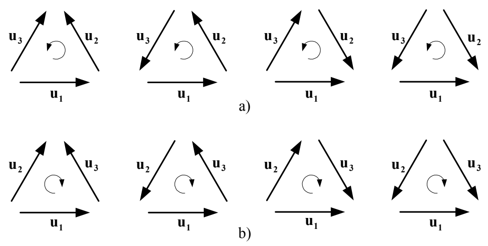

Fig. 2 Two distinctive groups of the distributions of $\{\ \mathbf{u}_k, k=1,2,3\ \}$: a) counterclockwise order and b) clockwise order.

Under the condition of **C2**, we have

$$\sum_{1}^{3}(\ \mathbf{u}_k^{\,t}\ \mathbf{q}_k\ )\ \mathbf{u}_k\ =\ \sum_{1}^{3}(\ \mathbf{v}_k^{\,t}\ \mathbf{p}_k\ )\ \mathbf{u}_k\ =\ 0 \tag{23}$$

which is equivalent to

$$\sum_{1}^{3}(\ \mathbf{v}_k^{\,t}\ \mathbf{p}_k\ )\ \mathbf{v}_k\ =\ \sum_{1}^{3}\alpha_k\,\mathbf{v}_k\ =\ 0 \tag{24}$$

where $\alpha_k=\mathbf{v}_k^{\,t}\ \mathbf{p}_k$, $k=1,2,3$, is the projection of $\mathbf{p}_k$ onto $\mathbf{v}_k$. For the first group of four distributions characterized by (21), it can be shown that [5]

$$|\alpha_1|\ =\ |\alpha_2|\ =\ |\alpha_3|\ =\ \alpha \tag{25}$$

$$\alpha_1\,\mathbf{v}_1\ +\ \alpha_2\,\mathbf{v}_2\ +\ \alpha_3\,\mathbf{v}_3\ =\ 0 \tag{26}$$

Since C1 places two scalar constraints, given by (20), but C2 does not place additional constraint on three variables, φ_1, φ_2 and φ_3, there exist infinite number of isotropic configurations in general. Fig. 3a) illustrates an isotropic configuration of a COMR, where three steering links form a regular triangle centered at the platform center, which is inscribed by a circle of radius α.

On the other hand, it can be shown that the second group of four distributions characterized by (22) cannot satisfy (24), so that the isotropy of $\mathbf{A}$ cannot be achieved.

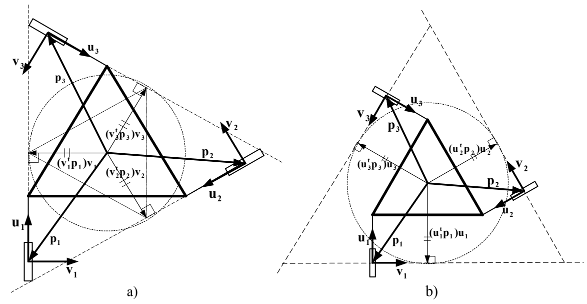

Fig. 3 Isotropic configurations for NAG I: a) $\Theta=\{\theta_1,\theta_2,\theta_3\}$ and b) $\Theta=\{\varphi_1,\varphi_2,\varphi_3\}$.

Similar analysis to the above can be made for $\Theta=\{\varphi_1,\varphi_2,\varphi_3\}$ where three steering joints of three caster wheels are actuated. Fig. 3b) illustrates an isotropic configuration of a COMR, where three wheel axes form a regular triangle centered at the platform center, which is inscribed by a circle of radius $\beta(=|\ \mathbf{u}_1^{\,t}\ \mathbf{p}_1\ |=|\ \mathbf{u}_2^{\,t}\ \mathbf{p}_2\ |=|\ \mathbf{u}_3^{\,t}\ \mathbf{p}_3\ |)$.

4.3 Isotropy Analysis for NAG II

Consider $\Theta=\{\varphi_1,\theta_2,\theta_3\}$ where one steering and two rotating joints of three caster wheels are actuated, for which $[\,\mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\,]=[\,\mathbf{v}_1\ \mathbf{u}_2\ \mathbf{u}_3\,]$.

First, under C1, we have

$$\mathbf{v}_1\ \mathbf{v}_1^{\,t}\ +\ \mathbf{u}_2\ \mathbf{u}_2^{\,t}\ +\ \mathbf{u}_3\ \mathbf{u}_3^{\,t}\ =\ 1.5\ \mathbf{I}_2 \tag{27}$$

Next, under C2, we have

$$(\ \mathbf{v}_1^{\,t}\ \mathbf{q}_1)\ \mathbf{v}_1+(\ \mathbf{u}_2^{\,t}\ \mathbf{q}_2)\ \mathbf{u}_2+(\ \mathbf{u}_3^{\,t}\ \mathbf{q}_3)\ \mathbf{u}_3=0 \tag{28}$$

or

$$(\ \mathbf{u}_1^{\,t}\ \mathbf{p}_1)\ \mathbf{u}_1+(\ \mathbf{v}_2^{\,t}\ \mathbf{p}_2)\ \mathbf{v}_2+(\ \mathbf{v}_3^{\,t}\ \mathbf{p}_3)\ \mathbf{v}_3=0 \tag{29}$$

With (27) being held, it can be shown that (29) cannot be satisfied unless d is equal to zero [5]. This tells that the isotropy of $\mathbf{A}$ can be achieved only when caster wheels reduce to conventional wheels without steering link. Fig. 4a) illustrates an isotropic configuration of a conventional wheeled mobile robot.

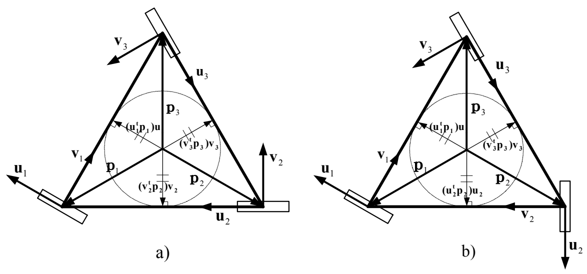

Fig. 4 With $d=0$, isotropic configurations for NAG II: a) $\Theta=\{\varphi_1,\theta_2,\theta_3\}$ and b) $\Theta=\{\varphi_1,\varphi_2,\theta_3\}$.

Similar analysis to the above can be made for $\Theta=\{\varphi_1,\varphi_2,\theta_3\}$ where two steering and one rotating joints of three caster wheels are actuated. Fig. 4b) illustrates an isotropic configuration of a conventional wheeled mobile robot.

4.4 Isotropy Analysis for NAG III

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2\}$ where both rotating and steering joints of one caster wheel and the rotating joint of another caster wheel are actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ ]$.

First, under **C1**, we have

$$\mathbf{u}_1\ \mathbf{u}_1^t + \mathbf{v}_1\ \mathbf{v}_1^t + \mathbf{u}_2\ \mathbf{u}_2^t = 1.5\ \mathbf{I}_2 \tag{30}$$

which is

$$c_2^2 = 0.5, \quad c_2 s_2 = 0.0 \tag{31}$$

There does not exist φ_2 satisfying (31), and the isotropy of $\mathbf{A}$ cannot be achieved at all.

Similar analysis to the above can be made for $\Theta = \{\theta_1, \varphi_1, \varphi_2\}$ where both rotating and steering joints of one caster wheel and the steering joint of another caster wheel are actuated.

5. ISOTROPY ANALYSIS FOR REDUNDANT ACTUATION

5.1 Redundant Actuation Sets

A COMR with redundant acuation can have four, five and six actuated joints $(n = 4,5,6)$, each of which can be either rotating or steering one. According to the number and combination of actuated joints and the number of active wheels, all possible redundant actuation sets, Θ, can be divided into five groups, denoted by RAG I, II, III, IV, and V, as listed in Table 2.

Table 2 Five redundant actuation groups.

Number of actuated joints	Number of active wheels	Actuation set	Redundant actuation group
$n=4$	2	$\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2\}$	RAG I
	3	$\Theta = \{\theta_1, \varphi_1, \theta_2, \theta_3\}$	RAG II
		$\Theta = \{\theta_1, \varphi_1, \varphi_2, \varphi_3\}$	
		$\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_3\}$	RAG III
$n=5$	3	$\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3\}$	RAG IV
		$\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \varphi_3\}$	
$n=6$	3	$\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3\}$	RAG V

5.2 Isotropy Analysis for RAG I

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2\}$ where both rotating and steering joints of two caster wheels are actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ \mathbf{g}_4\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ \mathbf{v}_2\ ]$.

First, under **C1**, we have

$$\sum_1^2 (\ \mathbf{u}_k\ \mathbf{u}_k^t + \mathbf{v}_k\ \mathbf{v}_k^t\ ) = 2\ \mathbf{I}_2 \tag{32}$$

which always holds. Next, under **C2**, we have

$$\sum_1^2 \{\ (\ \mathbf{u}_k^t\ \mathbf{q}_k)\ \mathbf{u}_k + (\ \mathbf{v}_k^t\ \mathbf{q}_k)\ \mathbf{v}_k \} = \sum_1^2 \mathbf{q}_k = 0 \tag{33}$$

or

$$\mathbf{p}_1 + \mathbf{p}_2 = 0 \tag{34}$$

which yields

$$\varphi_1 = \arcsin\left(\frac{1}{2\sqrt{3}}\frac{l}{d}\right), \quad \varphi_2 = \pi - \varphi_1 \tag{35}$$

Since C1 places no constraint and C2 places two scalar constraints, given by (34), on two variables, φ_1 and φ_2, in general, there exist multiple isotropic configurations independently of φ_3. Fig. 5 illustrates an isotropic configuration, where the steering links of two caster wheels are symmetric with respect to y-axis, with the centers of two caster wheels and the center of the platform lying on the line of $y = \frac{l}{2\sqrt{3}}$. Note that the isotropic configuration does not exist if the steering link length is less than $\frac{l}{2\sqrt{3}}$.

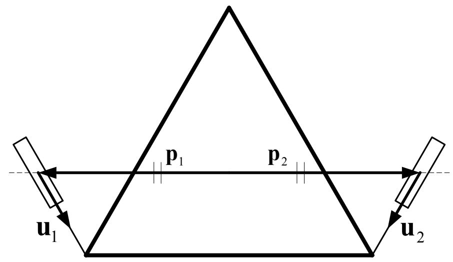

Fig. 5 Isotropic configuration for $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2\}$ belonging to RAG I.

5.3 Isotropy Analysis for RAG II

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2, \theta_3\}$ where both rotating and steering joints of one caster wheel and two rotating joints of two caster wheels are actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ \mathbf{g}_4\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ \mathbf{u}_3\ ]$.

First, under **C1**, we have

$$\mathbf{u}_2\ \mathbf{u}_2^t + \mathbf{u}_3\ \mathbf{u}_3^t = \mathbf{I}_2 \tag{36}$$

which is

$$c_2 s_2 + c_3 s_3 = 0.0 \tag{37}$$

yielding

$$\varphi_3 = \varphi_2 \pm \frac{\pi}{2} \tag{38}$$

Note that $\mathbf{u}_2$ and $\mathbf{u}_3$ are perpendicular to each other, and so are $\mathbf{v}_2$ and $\mathbf{v}_3$. Next, under C2, we have

$$\mathbf{q}_1 + (\ \mathbf{u}_2^t\ \mathbf{q}_2)\ \mathbf{u}_2 + (\ \mathbf{u}_3^t\ \mathbf{q}_3)\ \mathbf{u}_3 = 0 \tag{39}$$

or

$$\mathbf{p}_1 + (\ \mathbf{v}_2^t\ \mathbf{p}_2)\ \mathbf{v}_2 + (\ \mathbf{v}_3^t\ \mathbf{p}_3)\ \mathbf{v}_3 = 0 \tag{40}$$

Since C1 places one scalar constraint, given by (37), and C2 places two scalar constraints, given by (40), on three variables, φ_1, φ_2 and φ_3, there exist multiple isotropic configurations in general. Fig. 6a) illustrates an isotropic configuration, where the steering links of two caster wheels with actuated rotating joint are perpendicular to each other and the center of the other caster wheel with actuated rotating and steering joints is located in such a way as to satisfy (40).

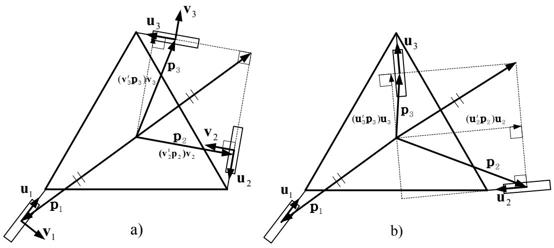

Fig. 6 Isotropic configurations for RAG II:
a) $\Theta = \{\theta_1, \varphi_1, \theta_2, \theta_3\}$ and b) $\Theta = \{\theta_1, \varphi_1, \varphi_2, \varphi_3\}$.

Similar analysis to the above can be made for $\Theta = \{\theta_1, \varphi_1, \varphi_2, \varphi_3\}$ where both rotating and steering joints of one caster wheel and two steering joints of two caster wheels are actuated. Fig. 6b) illustrates an isotropic configuration, where the steering links of two caster wheels with actuated steering joint are perpendicular to each other and the center of the other caster wheel with actuated rotating and steering joints is located in such a way as to satisfy

$$\mathbf{p}_1 + (\mathbf{u}_2^t \mathbf{p}_2)\mathbf{u}_2 + (\mathbf{u}_3^t \mathbf{p}_3)\mathbf{u}_3 = 0 \tag{41}$$

5.4 Isotropy Analysis for RAG III

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_3\}$ where both rotating and steering joints of one caster wheel and one rotating and one steering joints of two caster wheels are actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ \mathbf{g}_4\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ \mathbf{v}_3\ ]$.

First, under **C1**, we have

$$\mathbf{u}_2\ \mathbf{u}_2^t + \mathbf{v}_3\ \mathbf{v}_3^t = \mathbf{I}_2 \tag{42}$$

which is

$$c_2 s_2 - c_3 s_3 = 0.0 \tag{43}$$

yielding

$$\varphi_3 = \varphi_2 \tag{44}$$

Next, under **C2**, we have

$$\mathbf{q}_1 + (\mathbf{u}_2^t \mathbf{q}_2)\mathbf{u}_2 + (\mathbf{v}_3^t \mathbf{q}_3)\mathbf{v}_3 = 0 \tag{45}$$

or

$$\mathbf{p}_1 + (\mathbf{v}_2^t \mathbf{p}_2)\mathbf{v}_2 + (\mathbf{u}_3^t \mathbf{p}_3)\mathbf{u}_3 = 0 \tag{46}$$

Since C1 places one scalar constraint, given by (43), and C2 places two scalar constraints, given by (46), on three variables, φ_1, φ_2 and φ_3, there exist multiple isotropic configurations in general. Fig. 7 illustrates an isotropic configuration, where the steering links of one caster wheel with actuated rotating joint and another caster wheel with actuated steering joint are parallel to each other and the center of the other caster wheel with actuated rotating and steering joints is located in such a way as to satisfy (46).

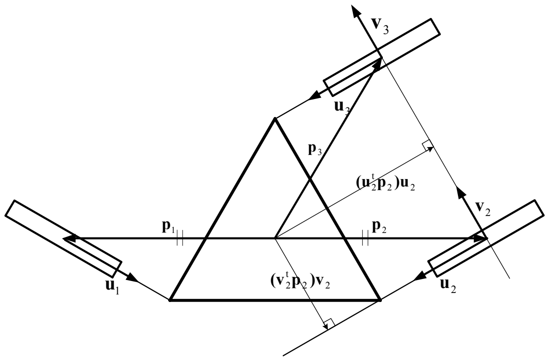

Fig. 7 Isotropic configuration for $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_3\}$ belonging to RAG IV.

5.5 Isotropy Analysis for RAG IV

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3\}$ where both rotating and steering joints of two caster wheels and the rotating joint of one caster wheel are actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ \mathbf{g}_4\ \mathbf{g}_5\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ \mathbf{v}_2\ \mathbf{u}_3\ ]$.

First, under **C1**, we have

$$\mathbf{u}_3\ \mathbf{u}_3^t = 0.5\ \mathbf{I}_2 \tag{47}$$

which is

$$c_3^2 = 0.5, \quad c_3 s_3 = 0.0 \tag{48}$$

There does not exist φ_3 satisfying (48) and the isotropy of A cannot be achieved at all.

Similar analysis to the above can be made for $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \varphi_3\}$ where both rotating and steering joints of two caster wheels and the steering joint of one caster wheel are actuated.

5.6 Isotropy Analysis for RAG V

Consider $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3\}$ where both rotating and steering joints of three caster wheels are fully actuated, for which $[\ \mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3\ \mathbf{g}_4\ \mathbf{g}_5\ \mathbf{g}_6\ ] = [\ \mathbf{u}_1\ \mathbf{v}_1\ \mathbf{u}_2\ \mathbf{v}_2\ \mathbf{u}_3\ \mathbf{v}_3\ ]$.

First, C1 holds always. Next, under C2, we have

$$\sum_1^3 \mathbf{p}_k = 0 \tag{49}$$

which yields

$$\varphi_2 = \varphi_1 \pm \frac{2\pi}{3}, \quad \varphi_3 = \varphi_1 \mp \frac{2\pi}{3} \tag{50}$$

Since C1 places no constraints and C2 places two scalar constraints, given by (49), on three variables, φ_1, φ_2 and φ_3, there exist infinite number of isotropic configurations in general. Fig. 8 illustrates an isotropic configuration of a COMR. where the centers of three caster wheels are symmetric with respect to the center of the platform.

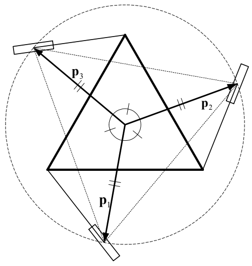

Fig. 8 Isotropic configuration for $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3\}$ belonging to RAG V.

6. SOME DISCUSSIONS

6.1 Number of Isotropic Configurations

Depending on the selection of actuated joints, the number of isotropic configurations which satisfy C1 and C2 can be either none, multiple(finite), or infinite. Table 3 lists the nonredundant and redundant actuation sets resulting in more than one isotropic configuration. From Table 3, the following observations can be made. When the actuation of three caster wheels are homogeneous, including $\Theta = \{\theta_1, \theta_2, \theta_3\}$, $\{\varphi_1, \varphi_2, \varphi_3\}$, and $\{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3\}$, there exist infinite number of isotropic configurations. When the number of actuated joints are redundant, including $\Theta = \{\theta_1, \varphi_1, \theta_2, \theta_3\}$, $\{\theta_1, \varphi_1, \varphi_2, \varphi_3\}$, $\{\theta_1, \varphi_1, \theta_2, \varphi_3\}$, and $\{\theta_1, \varphi_1, \theta_2, \varphi_2\}$, there exist multiple isotropic configurations. The only two exceptions are $\Theta = \{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3\}$ and $\{\theta_1, \varphi_1, \theta_2, \varphi_2, \varphi_3\}$. It should be mentioned that both homogeneity in wheel actuation and redundancy in joint actuation play a significant role for enhancing the isotropy of a COMR.

6.2 Isotropic Characteristic Length

As described in Section 3, the isotropy of a COMR can be achieved under three conditions, C1, C2, and C3. Once an isotropic configuration is identified under C1 and C2, the characteristic length required for the isotropy, L_{iso}, can be determined under C3.

As an example, let us consider the case of $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Under C3, we have

$$\frac{1}{L^2}\sum_{1}^{3}(\mathbf{u}_k^t\mathbf{q}_k)^2 = \frac{1}{L^2}\sum_{1}^{3}(\mathbf{v}_k^t\mathbf{p}_k)^2 = 1.5 \tag{51}$$

With (21) being held, from (51), the characteristic length of an isotropic COMR is obtained by

$$L_{iso} = \left(|\mathbf{v}_1^t\mathbf{p}_1| = |\mathbf{v}_2^t\mathbf{p}_2| = |\mathbf{v}_3^t\mathbf{p}_3|\right) \tag{52}$$

For all actuation sets with more than one isotropic configuration, Table 3 also lists the resulting isotropic characteristic length L_{iso}. Note that the isotropy of a COMR cannot be achieved unless $L = L_{iso}$.

7. CONCLUSION

This paper presented a complete isotropy analysis of a caster wheeled omnidirectional mobile robot with nonredundant and redundant actuation. All possible actuation sets with different number and combination of rotating and steering joints were considered. First, with the characteristic length introduced, the kinematic model was obtained. Second, a general forms of the isotropy conditions was given in terms of physically meaning vector quantities. Third, for all possible nonredundant and redundant actuation sets, the algebraic expressions of the isotropy conditions were derived to identify the isotropic configurations. Fourth, the number of the isotropic configurations and the characteristic length for the isotropy were discussed. The isotropy analysis made in this paper deserves special attention, in that the omnidirectional mobility does without the isotropy characteristics in motion control.

REFERENCES

[1] R. Holmberg, "Design and Development for Powered-Caster Holonomic Mobile Robot," Ph. D. Thesis, Dept. of Mechanical Eng. Stanford University, 2000.

[2] P. F. Muir and C. P. Neuman, "Kinematic Modeling of Wheeled Mobile Robots," Jour. of Robotic Systems, vol. 4, no. 2, pp. 281-340, 1987.

[3] G. Campion, G. Bastin, and B. D`Andrea Novel, "Structural Properties and Classification of Kinematic and Dynamic Models of Wheeled Mobile Robots," IEEE Trans. on Robots and Automations, vol. 12, no. 1, pp. 47-62, 1996.

[4] W. Kim, D. Kim, B. Yi, B. You, and S. Yang, "Design of an Omni-directional Mobile Robot with 3 Caster Wheels," Trans. on Control, Automation, and Systems Engineering, vol. 3, no. 4, pp. 210-216, 2001.

[5] S. Kim and H. Kim, "Isotropy Analysis of Caster Wheeled Omnidirectional Mobile Robot," Proc. of Int. Conf. on Robotics and Automation, pp. 3093-3098, 2004.

[6] S. K. Saha, J. Angeles, and J. Darcovich, "The Design of Kinematically Isotropic Rolling Robots with Omnidirectional Wheels," Mechanism and Machine Theory, vol. 30, no. 8, pp. 1127-1137, 1995.

Table 3 All actuation sets resulting in more than one isotropic configuration.

Actuation group	Actuation set Θ	Number of isotropic configurations	Isotropic characteristic length L_{iso}
NAG I	$\{\theta_1, \theta_2, \theta_3\}$	infinite	$(\lvert\mathbf{v}_1^t\mathbf{p}_1\rvert = \lvert\mathbf{v}_2^t\mathbf{p}_2\rvert = \lvert\mathbf{v}_3^t\mathbf{p}_3\rvert)$
	$\{\varphi_1, \varphi_2, \varphi_3\}$	infinite	$(\lvert\mathbf{u}_1^t\mathbf{p}_1\rvert = \lvert\mathbf{u}_2^t\mathbf{p}_2\rvert = \lvert\mathbf{u}_3^t\mathbf{p}_3\rvert)$
RAG I	$\{\theta_1, \varphi_1, \theta_2, \varphi_2\}$	multiple	$(\lVert\mathbf{p}_1\rVert = \lVert\mathbf{p}_2\rVert)$
RAG II	$\{\theta_1, \varphi_1, \theta_2, \theta_3\}$	multiple	$(\lVert\mathbf{p}_1\rVert = \sqrt{(\mathbf{v}_2^t\mathbf{p}_2)^2 + (\mathbf{v}_3^t\mathbf{p}_3)^2})$
	$\{\theta_1, \varphi_1, \varphi_2, \varphi_3\}$	multiple	$(\lVert\mathbf{p}_1\rVert = \sqrt{(\mathbf{u}_2^t\mathbf{p}_2)^2 + (\mathbf{u}_3^t\mathbf{p}_3)^2})$
RAG III	$\{\theta_1, \varphi_1, \theta_2, \varphi_3\}$	multiple	$(\lVert\mathbf{p}_1\rVert = \sqrt{(\mathbf{v}_2^t\mathbf{p}_2)^2 + (\mathbf{u}_3^t\mathbf{p}_3)^2})$
RAG V	$\{\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3\}$	infinite	$(\lVert\mathbf{p}_1\rVert = \lVert\mathbf{p}_2\rVert = \lVert\mathbf{p}_3\rVert)$