Simultaneous Fault Isolation of Redundant Inertial Sensors based on the Reduced-Order Parity Vectors

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Abstract: We consider a fault detection and isolation problem for inertial navigation systems which use redundant inertial sensors. We propose a FDI method using average of multiple parity vectors which reduce false alarm and wrong isolation, and improve correct isolation. We suggest the number of redundant sensors required to isolate simultaneous faults. The performance of the proposed FDI algorithm is analyzed by Monte-Carlo simulation.

Keywords: fault detection and isolation, inertial sensors, parity equation

1. Introduction

Today all sorts of control, navigation and communication systems consist of various subsystems and thus the hardware and software structure of those systems are complicated. Therefore the importance of reliability of the whole systems has been increased. The reliability of the whole systems can be obtained by fault detection and isolation(FDI) method as well as the reliability of partial systems [1,2,3,4]. FDI methods have been studied from 1960 and have been proceeded on two approach such as hardware redundancy[5,6,7,8,9,10,11] and analytical redundancy [12,13,14].

Inertial navigation systems(INS) use three accelerometers and gyroscopes to calculate navigation information such as position, velocity and attitude. To obtain reliability and to enhance navigation accuracy, INS may use redundant sensors. A lot of studies on FDI for the redundant sensors have been performed so far. There are many papers for FDI such as look-up table[5], SE[5], GLT[6] and OPT[7] for hardware redundancy. These methods consist of three procedures such as parity equation generation, fault detection and isolation. The parity equation is obtained from residual or using vectors of null space of measurement matrix. And fault detection is performed by comparing the parity value with some threshold. These methods are adequate for large fault detection but not for small faults. The reason is that small threshold should be used for small fault detection and thus false alarm and wrong isolation probability increases because of effect of measurement noise. Also previous FDI methods do not consider simultaneous faults.

In this paper we propose a FDI method for redundant sensors for not only a single fault but also simultaneous faults. The proposed FDI method uses reduced-order parity vectors for single fault detection and isolation as well as double faults isolation. At least 7 redundant sensors are required for this FDI method. We analyze performance of the proposed isolation method for simultaneous faults through Monte-Carlo simulation.

2. Basic Concepts for the Proposed FDI Algorithm

In this section we define the problem under consideration in this paper and introduce some basic concepts which we use for the proposed FDI method.

Consider a typical measurement equation for redundant inertial sensors.

$$m(t) = Hx + f + \varepsilon \tag{1}$$

where

$m = \begin{bmatrix} m_1 & m_2 & \cdots & m_n \end{bmatrix}^T \in R^n$: the inertial sensor measurement

$H = \begin{bmatrix} h_1 & \cdots & h_n \end{bmatrix}^T$: n×3 measurement matrix with rank(H^T)=3

$x(t) \in R^3$: triad-solution(acceleration or angular rate)

$f(t) = \begin{bmatrix} f_1 & f_2 & \cdots & f_n \end{bmatrix}^T \in R^n$: fault vector

$\varepsilon(t) \sim N(0_n, \sigma I_n)$: a measurement noise vector, normal distribution(white noise)

A parity vector is obtained using a matrix V as follows:

$$p(t) = Vm(t) = Vf(t) + V\varepsilon(t) \tag{2}$$

where the matrix V satisfies

$$VH = 0 \ (V \in R^{(n-3)\times n}) \tag{3-1}$$

$$VV^T = I,\ V = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix},\ |v_i| = 1 \tag{3-2}$$

Problem Definition

Consider the measurement equation (1), where a single fault or double faults may occur. If some faults occur, detect and isolate the faults, even though double faults occurrence

The proposed FDI method consists of following two steps.

1) Determine whether a fault occurs or not.
2) When a fault occurs, determine whether it is a single fault or double fault, and isolate it. Isolation means to find a sensor which has fault.

Let's make the following assumption.

[Assumption 1]: We use n sensors, any 3 sensors among n are not on the same plane.

Gilmore [5] shows that if 6 inertial sensors are used with no three sensors on the same plane, it is possible to detect three simultaneous faults and isolate two simultaneous faults. So we assume that any three sensors are not on the same plane.

To explain the concept of the proposed FDI method for double faults, we make the following definitions.

In the case of excluding i^th sensor among n sensors.

$$m_{-i} = \begin{bmatrix} m_1 & m_2 & \cdots & m_{i-1} & m_{i+1} & \cdots & m_n \end{bmatrix}^T \tag{4}$$

Where m_{-i} is n-1 measurements which excludes the i[th] measurement(m_i).

H_{-i} : (n-1)×3 measurement matrix corresponding to m_{-i}.

V_{-i}: (n-4)×(n-1) parity matrix corresponding to H_{-i}.

$V_{-i}V_{-i}^T = I, V_{-i}H_{-i} = 0$

The parity vector is obtained from m_{-i} as follows:

$$p_{-i} = V_{-i}m_{-i} \tag{5}$$

<u>In the case of excluding i[th] and j[th] sensors among n sensors.</u>

$m_{-i,-j}$: n-2 measurements which excludes the i[th] measurement(m_i) and j[th] measurement(m_j).

$H_{-i,-j}$: (n-2)×3 measurement matrix corresponding to $m_{-i,-j}$.

$V_{-i,-j}$: (n-5)×(n-2) parity matrix corresponding to $H_{-i,-j}$.

$$V_{-i,-j}V_{-i,-j}^T = I, V_{-i,-j}H_{-i,-j} = 0$$

The parity vector is obtained from $m_{-i,-j}$ as follows:

$$p_{-i,-j} = V_{-i,-j}m_{-i,-j} \tag{6}$$

It is assumed that measurement noise $\varepsilon(t)$ is zero. We can know the following facts from the above definitions.

If a single fault at 1[st] sensor occurs, then $p_{-1}^T p_{-1} = 0$ and $p_{-i}^T p_{-i} > 0$ (i=2,…,n).

If simultaneous faults at 1[st] and 2[nd] sensor occur, then $p_{-1,-2}^T p_{-1,-2} = 0$ and $p_{-i,-j}^T p_{-i,-j} > 0$ (i,j=2,…,n :i≠j) for only n ≥ 7.

Where if n=6, then $V_{-i,-j}$ is 1×4 matrix and $p_{-i,-j}$ is scalar. So, $p_{-i,-j}$ may be zero for the specified faults. But if n ≥ 7, then $p_{-i,-j}$ is vector . So, $p_{-i,-j}^T p_{-i,-j} > 0$.

Actually, we use the following averaged parity vector because the measurement noise $\varepsilon(t)$ is not zero.

The averaged parity vector for q samples from $t = t_{k-q+1}$ to $t = t_k$ is defined as follows:

$$\overline{p} \equiv \frac{1}{q}\{p(t_{k-q+1}) + p(t_{k-q+2}) + \cdots + p(t_k)\} \tag{7}$$

If a fault doesn't occur, then the above averaged parity vector $\overline{p} = \frac{1}{q}V\overline{\varepsilon}$.

where $\overline{\varepsilon} \sim N(0_n, \frac{\sigma}{\sqrt{q}} I_n)$. So, $\overline{p}$ is effected less than p by measurement noise.

In this paper, we use the above concepts to obtain the proposed FDI method.

3. The proposed FDI method for double faults

In this section we consider a FDI method when double faults occur. The feature of the proposed FDI method is that when we decide a fault occurrence, then we try to find whether it is a single fault or double faults. The proposed FDI method is as Fig.1.

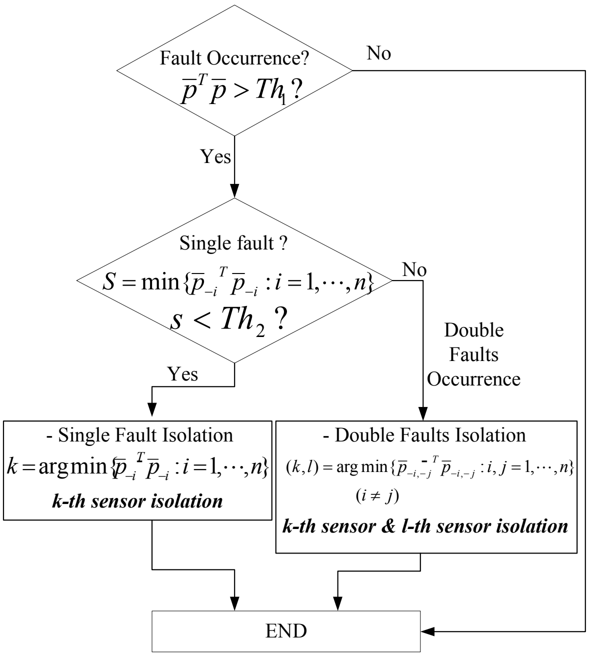


Fig.1 The proposed FDI algorithm

<u>The procedure of the proposed FDI algorithm</u>

Step1). Fault detection

Check whether there is a fault or not.

(8)

where H_1 denotes a fault hypothesis and H_0 no-fault hypothesis, and Th_1 is a threshold determined with false alarm from χ^2 distribution.

Stop if H_0 and go to step 2) if H_1.

Step2). Single or double fault detection

$$S = \min\{\overline{p}_{-i}^T\overline{p}_{-i} : i = 1,\cdots,n\} \tag{9}$$

If $S < Th_2$, then single fault occurs and go to step 3. Otherwise, then double faults occur and go to step 4. Where Th_2 is a threshold determined with false alarm from χ^2 distribution.

Step3). Single fault isolation

$$k = \arg\ \min\{\overline{p}_{-i}^T\overline{p}_{-i} : i = 1,\cdots,n\} \tag{10}$$

The k^{th} sensor is isolated.

Step4). Double faults isolation

$$(k,l) = \arg\ \min\{\overline{p}_{-i,-j}^T\overline{p}_{-i,-j} : i, j = 1,\cdots,n\ \ (i \neq j)\} \tag{11}$$

The k^{th} and l^{th} sensor are isolated.

4. Simulations

In this section Monte Carlo simulation is performed to analyze the performance of the proposed FDI method.

4.1 Simulation Conditions

Simulation conditions are as follows. We use 7 sensors and cone configuration as Fig.2. The measurement matrix H is described as follows

$$H = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0.4409 & 0.5528 & 0.7071 \\ -0.1573 & 0.6894 & 0.7071 \\ -0.6371 & 0.3068 & 0.7071 \\ -0.6371 & -0.3068 & 0.7071 \\ -0.1573 & -0.6894 & 0.7071 \\ -0.4409 & -0.5528 & 0.7071 \end{bmatrix} \quad (12)$$

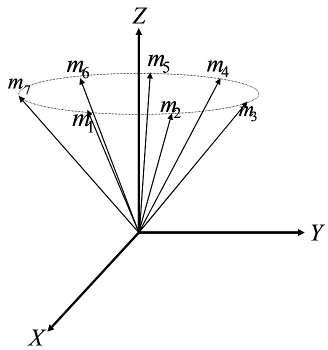


Fig.2 Cone configuration with 7 sensors

We use 100 samples for average. Suppose that sensors 1 and 7, f_1 and f_7, have faults.

The magnitudes of faults change counter clock-wise along circles with radius 2σ, 4σ and 6σ, respectively. Fig.3 shows the fault sizes of sensor 1 and 7 with respect to θ. Monte Carlo simulation is performed 300 times with above conditions.

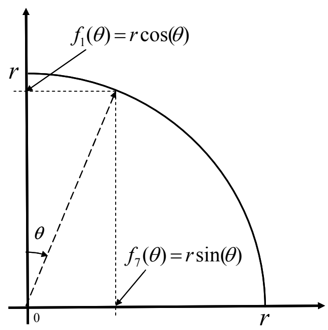
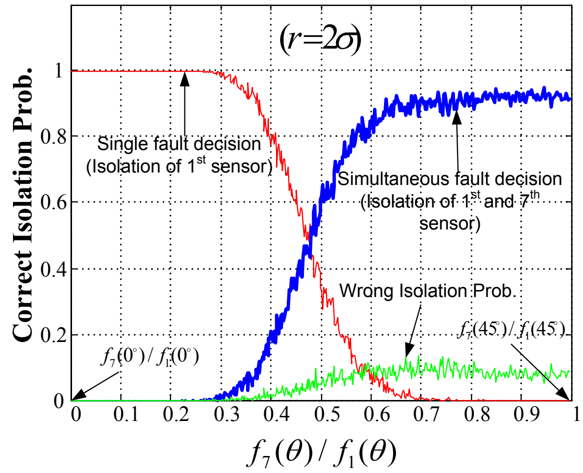


Fig3. Fault size of faulty sensors 1 and 7

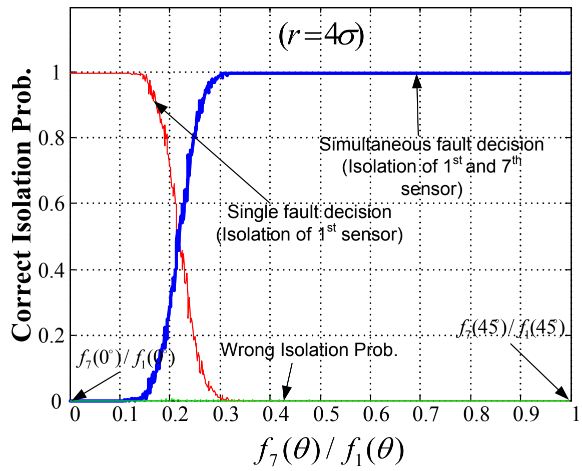
4.2 Simulation Results

In this simulation we analyze the performance of the proposed isolation method for 0°≤ θ ≤45°.

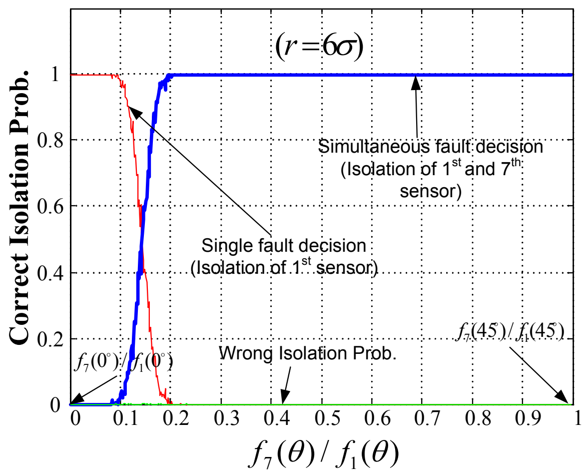
Fig.4 shows the correct isolation probability and wrong isolation probability of the proposed FDI method when radius of fault size is 2σ, 4σ and 6σ, respectively. We suppose that the 1st and 7th sensors have faults and the magnitude of the faults change as in Fig.3. Thin line shows the case of isolating only one sensor after deciding that only one sensor has fault. Bold line shows the case of isolating double sensors after deciding that two sensors have fault. When the magnitude of f_7 is small, the algorithm decide that there is only one fault f_1. When the magnitude of f_7 gets large, the algorithm decide that two faults occur. Also, we can know that if the radius is greater than 4σ, then wrong isolation probability is almost 0.



(a) Radius of 2σ



(b) Radius of 4σ



(c) Radius of 6σ

Fig.4 Correct isolation probability and wrong isolation probability in case of simultaneous faults occurrence

5. Conclusions

In this paper we propose a simultaneous fault detection and isolation method. Most of the existing FDI methods do not consider simultaneous faults. We propose an FDI method using averaged parity vector method which can be applied to simultaneous faults. We also propose decision rule to distinguish between a single fault and two simultaneous faults. We analyze the proposed simultaneous FDI method by Monte-Carlo simulation. The simulation results show that the performance of the proposed FDI method is good in the case of averaging 100 samples.

Acknowledgments

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