Fault Diagnosis for Parameter Change Fault

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Abstract: In this paper we propose a new fault detection and isolation (FDI) method for those faults of parameter change type. First, we design a residual generator based on the δ-operator model of the plant by using the stable pseudo inverse system. Second, the parameter change is estimated by using the property of the block Hankel operator. Third, reliability with respect to stability is quantified. Fourth, the limitations for the meaningful diagnosis in our method are given. The numerical examples demonstrate the effectiveness of the proposed method.

Keywords: Fault diagnosis, Fault detection, Fault isolation, stability margin, δ-operator model

1. Introduction

There has been an increasing interest in theory and applications of fault diagnosis techniques since the attention to reliability, availability and safety of many real plants has been growing recently. The fault diagnosis problem has been studied since the late seventies [1], and many approaches have been proposed since then. They are divided into two main approaches from the viewpoint of the way to treat the faults. One deals with a fault as some additive signal [2][3][5], and reduces the fault diagnosis problem to a design problem of a filter that identifies the additive signal. The other deals with a fault as a variation of the plant model called as multiplicative fault [4][6], and reduces the fault diagnosis problem to an identification problem [7]. In the former case, a fault is easy to be dealt with since we can consider the fault as a specified disturbance, though it is not intuitive, and use some tools to estimate or observe it. The latter case is intuitive, but it requires a long time to detect and isolate the fault in the form of parameter change. It is therefore preferable that the FDI technique is intuitive and easy to handle.

From this standpoint, we propose a new FDI method with the two merits stated above. In this paper we deal with multiplicative faults, and estimate their variations. In Section 2, we consider an FDI problem for a δ-operator model system. This is because the model is convenient to handle the data used in our FDI technique, and it enables to maintain the meaning of multiplicative fault in the continuous time model as much as possible. In Section 3, we define the fault we deal with here, and give the general information about the fault diagnosis as preliminary. In Section 4, we propose a new design method for fault detection filter (residual generator) using a stable pseudo-inverse system based on decoupling theory, and obtain the residual. From the residual obtained, we estimate the parameter fault and evaluate the result in Section 5. In Section 6, we analyze the proposed method, give the limitations with respect to the system parameter change by the fault for the meaningful diagnosis in our method, and show the algorithm for fault diagnosis. In Section 7, we show a numerical example for illustration of our method, and give the conclusion in Section 8.

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2. Problem Statement

In this paper, we consider fault detection and diagnosis for the δ-operator model system as follows:

$$\delta x = Ax + Bu, \tag{1a}$$

$$y = Cx, \tag{1b}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, (p \geq m)$ and $y \in \mathbb{R}^p$ are the state, input and output vectors, respectively. We express the system after fault occurrence by:

$$\delta x = (A + \Delta_{Af})x + (B + \Delta_{Bf})u, \tag{2a}$$

$$y = Cx, \tag{2b}$$

where Δ_{Af}, Δ_{Bf} are parameter variations caused by faults. In addition, we assume that (A, B, C) is a minimal realization. We then transform the system (2) into the system with an additive fault f as follows:

$$\delta x = Ax + Bu + Bf \tag{3a}$$

$$y = Cx \tag{3b}$$

$$f = \Gamma_B(\Delta_{Af}x + \Delta_{Bf}u), \tag{3c}$$

where $\Gamma_B := (B^T B)^{-1} B^T$. Iserman *et.al.*[4] pointed out that the multiplicative fault of this type can be transformed to an additive fault. Thus we first detect and isolate the fault f, from which we next estimate the corresponding parameter change Δ_{Af}, Δ_{Bf}, and finally we evaluate the system reliability from the viewpoint of stability based on the estimated result.

3. Preliminary

We first describe the definition of the fault considered here and simply state a general fault diagnosis framework as a preliminary.

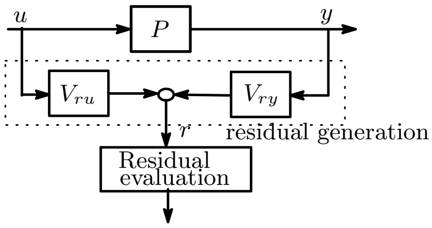

Fig. 1. Structure of model-based FDI system

3.1. Definition of the fault

The definition of fault is necessary to consider the fault diagnosis problem. Simani *et al.* [7] defined fault as an unpermitted deviation of at least one characteristic property of the system parameter from the acceptable, usual or standard condition. He also defined failure as a permanent interruption of a system's ability to perform a required function under specified operating conditions. Applying these definitions to our problem, fault means the occurrence of the signal f, and failure means the lack of stability caused by the parameter change Δ_{Af}, Δ_{Bf}. We use these definitions below.

3.2. Residual

In fault diagnosis, we need an indicator of fault occurrence, which is called residual. Naturally it is required to have the following property:

$$\begin{cases} r \neq 0 & \text{Fault} \\ r = 0 & \text{Not fault} \end{cases} \tag{4}$$

3.3. Framework of fault diagnosis

Fault diagnosis generally consists of the residual generation and its evaluation as shown in Fig.1, where P is a plant, $[V_{ru}, V_{ry}]$ is a residual generator.

First, we design a residual generator with the above property (4). Second, we evaluate the generated result. The details of the residual generation is described in Section 4, and that of the evaluation in Section 5.

4. A residual generator using stable pseudo-inverse system

Many residual generators are designed by using decoupling theory in order to isolate the fault occurrence position [2][5]. Since the transfer function from u to y, $G_{yu}(\delta)$, is equal to that from f to y, $G_{yf}(\delta)$, in (3), we can use a stable pseudo-inverse system [9][10] of $G_{yf}(\delta)$ in order to design a residual generator. Here the stable pseudo-inverse system $G^+(\delta)$ satisfies $Q(\delta) = G^+(\delta)G_{yf}(\delta) = G^+(\delta)G_{yu}(\delta)$, and $Q(\delta)$ is diagonal. By using the filter $G^+(\delta)$, we design a residual generator as follows:

$$\begin{aligned} r &= V_{ru}u + V_{ry}y \\ &= -Q(\delta)u + G^+(\delta)y \\ &= -Q(\delta)u + G^+(\delta)\left(G_{yu}(\delta)u + G_{yf}(\delta)f\right) \\ &= Q(\delta)f, \end{aligned} \tag{5}$$

where the residual generator $V_{ru}(\delta)$ and $V_{ry}(\delta)$ is selected by $-Q(\delta)$ and $G^+(\delta)$, respectively. Thus we can design a residual generator which detects the occurrence of the fault f and isolates the position (number) of f.

4.1. The condition of the residual generator design

We describe the feasibility condition for design of the residual generator mentioned above. According to the literature [9][10], it is reduced to the decoupling condition as follows:

Corollary 1: Assume that (A, B) is controllable, and (C, A) is observable. When the next decoupling matrix:

$$D := \begin{bmatrix} B_1^T (A^T)^{d_1-1} C^T \\ B_2^T (A^T)^{d_2-1} C^T \\ \vdots \\ B_m^T (A^T)^{d_m-1} C^T \end{bmatrix}$$

is *row-full rank*, then the residual generator $Q(\delta)$ for the system (1) can be designed.

Our final aim is not to estimate the fault signal f, but to evaluate the closed loop stability in order to avoid the system failure. Therefore, the stability of a part of the residual generator $G^+(\delta)$ is also important. Thus we need the following corollary [10].

Corollary 2: Let the system (A, B, C) be a minimal realization. Then the residual generator is stable if and only if the system (A, B, C) has no invariant zeros on imaginary axis.

The corollary assure the existence of the stable filter $G^+(\delta)$ under a certain condition. If the condition does not hold, we cannot design any stable residual filter $G^+(\delta)$, and consequently its output y becomes unbounded.

4.2. The design method

Describing the stable monic polynomial with dimension $d_i - 1$ as $\phi_i(\delta), i = 1, .., m$, the residual generator $G^+(\delta) = (A_r, B_r, C_r, D_r)$ is designed as follows:

$$A_r = A + B_r C \tag{6a}$$
$$C_r = D_r C \tag{6b}$$
$$N_\phi = \begin{bmatrix} B_1^T \phi_1(A^T) \\ B_2^T \phi_2(A^T) \\ \vdots \\ B_m^T \phi_m(A^T) \end{bmatrix} \tag{6c}$$
$$D_r = \Phi_0^T (D^\dagger)^T \tag{6d}$$
$$\Phi_0 = \operatorname*{diag}_{1 \le i \le m} \phi_i(0) \tag{6e}$$
$$B_r = -{N_\phi}^T (D^\dagger)^T \tag{6f}$$

where, $D^\dagger$ is the matrix satisfies the relation $DD^\dagger = I$.

5. Parameter change estimation and its evaluation

5.1. Estimation of parameter change

The residual generator obtained in Section 4 generates such a r that enables to isolate the signal f in the form of additive fault. In practice, however, we have $r = Q(\delta)f$, which is not the same as r. In addition, we deal with a multiplicative fault, so we have to estimate the parameter change caused by the fault. First, we estimate f from r and $Q(\delta)$, based on the following lemma:

Lemma 1: [11] Let (A, B, C) be a minimal realization, where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}$. Here $\mathcal{O}_k, \mathcal{C}_l$ are k-extended observability matrix, l-extended controllability matrix, respectively. Then the following relation holds.

$$\mathcal{H}_{kl} = \mathcal{O}_k \mathcal{C}_l \in \mathbb{R}^{kp\times lm} \tag{7}$$

Our aim is to estimate f from $r, Q(\delta)$. Here, we define

$$R := \begin{bmatrix} r(t) \\ \delta r(t) \\ \vdots \end{bmatrix}, \; F := \begin{bmatrix} \delta^{-1} f(t) \\ \delta^{-2} f(t) \\ \vdots \end{bmatrix}$$

then we have:

$$R = \mathcal{H}_{kl} F \tag{8}$$

Since $Q(\delta) := (A_q, B_q, C_q)$ can be designed as a minimal realization, $\mathcal{H}_{kl}$ is nonsingular in $k = l$. Therefore we obtain:

$$F = \mathcal{H}_{kl}^{-1} R \tag{9}$$

Recall $f = \Gamma_B(\Delta_{Af} x + \Delta_{Bf} u)$, and describe the data of f, x, u after the fault occurrence as

$$F = \begin{bmatrix} f(t) \\ \vdots \\ \delta^N f(t) \end{bmatrix}, X = \begin{bmatrix} x(t) \\ \vdots \\ \delta^N x(t) \end{bmatrix}$$

$$U = \begin{bmatrix} u(t) \\ \vdots \\ \delta^N u(t) \end{bmatrix}, Z = \begin{bmatrix} X \\ U \end{bmatrix}.$$

Then the estimate of parameter change caused by the fault is calculated as

$$\begin{bmatrix} \Delta_{Af} & \Delta_{Bf} \end{bmatrix} = Z^{\dagger} F^T, \tag{10}$$

where N is the data number used in estimation, and $(*)^{\dagger}$ denotes the pseudo inverse of $*$. Here the nonsingularity of Z depends on x, u, so the data used in the estimation need to be selected suitably.

5.2. Evaluation of the estimate

In this section, we evaluate the influence of the estimated parameter change on the closed-loop system from the viewpoint of stability. The stability margin is a useful criterion for this aim, so we compare the stability margin with the variation caused by the fault according to the following procedure:1) specify the system as the state feedback system, and calculate the stability margin, described as $b_{P,C}$ below, 2) after fault occurs, calculate the infinity norm of the variation, described as γ below, by using the estimated parameter change, 3) compare these two values obtained above $\sigma = \gamma / b_{P,C}$, and evaluate the system reliability. The closer to 1 this value σ is, the closer to failure the system is.

6. Analysis of the proposed method and the algorithm

In Section 4, and 5, we have described the design method of the residual generator which also isolates the position, and the evaluation technique from the viewpoint of stability. In this section, we analyze the proposed method. Recall the fault signal f we define in the eq.(3c):

$$f = \Gamma_B(\Delta_{Af} x + \Delta_{Bf} u), \tag{11}$$

where $\Gamma_B = (B^T B)^{-1} B^T$.

6.1. The restriction for the meaningful diagnosis

Next, we discuss the restriction of the parameter changes Δ_{Af}, Δ_{Bf} for accurate diagnosis. In the proposed method, the evaluation of the fault is made based on the parameter estimation and the measure of the stability. From the equation (11), the meaningful evaluation is implemented only when x, u are bounded. This means if the system (2) after the fault occurrence is stable, the meaningful evaluation is made. Thus the following corollary holds by the small gain theorem.

Corollary 3: [12] The meaningful evaluation is implemented if the condition (12) holds.

$$\|\Delta\|_\infty < b_{P,C} \tag{12}$$

$$b_{P,C} := \|\bar{E}(sI - A - BF) B \Gamma_B \bar{D}\|_\infty^{-1} \tag{13}$$

$$\bar{D} = \begin{bmatrix} I & I \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} I \\ F \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \Delta_{Af} & 0 \\ 0 & \Delta_{Bf} \end{bmatrix}$$

where F is a state feedback gain stabilizing $A + BF$.

Proof: See appendix A. ∎

6.2. Algorithm

We now show the algorithm of the proposed method.

Assume that the system (1) satisfies the conditions stated in Corollary 1, and Corollary 2; in addition, the parameter change caused by the fault satisfies the stability condition (12). Then the fault diagnosis system is constructed by the following procedure:

Step 1. Design the stable pseudo inverse system $G^+(\delta)$ according to Section 4.2.

Step 2. Construct the residual generator by selecting $V_{ru}(\delta) = -G^+(\delta) G_{yf}(\delta), V_{ry}(\delta) = G^+(\delta)$.

Step 3. Calculate the stability margin $b_{P,C}$ based on the eq.(13).

Step 4. Based on the relation (10) derived by Lemma 1, estimate parameter change by using the input, the state signal u, x and the residual r.

Step 5. Calculate the infinity norm of the variation γ based on the estimated result in Step 4.

Step 6. Display the ratio between $b_{P,C}$ and γ, and indicate the system reliability from the viewpoint of stability.

7. Numerical example

We show a numerical example in order to illustrate the proposed method. Here we consider the system (A, B, C) given by:

$$A = \begin{bmatrix} -0.9995 & 1.997 \\ 0 & -1.998 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0.999 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

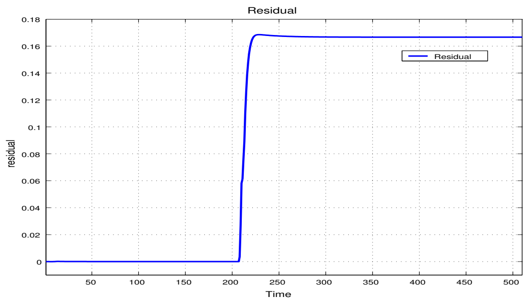

Fig. 2. Residual

This is a δ-model calculated based on the continuous model $A=[-1,2;0,-2]; B=[1;1]; C=[1,0]$; at the sampling time 0.001. Assume that the change of the system (A,B,C) is caused by the fault at 200 seconds. The parameter changes Δ_{Af}, Δ_{Bf} are selected by:

$$\Delta_{Af} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \Delta_{Bf} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

The residual generator is designed based on (A,B,C) mentioned in Section 4, and suitable state feedback controller is designed. The stability margin $b_{P,C}$ is calculated as $b_{P,C} = 1.037940$. The obtained residual r is shown in Fig.2. Although a slight delay caused by the dynamics of the residual generator is observed, it is a preferable response. The signal f is then calculated by using the relation (9), and from the calculated F, we estimate Δ_{Af}, Δ_{Bf} based on the relation (10). From the estimate, the infinity norm of the variation γ is calculated as $\gamma = 0.004074$. The ratio of the two values $\sigma = \gamma/b_{P,C}$ means the stability degree as well as the influence of the occurred fault on the closed loop system. Here we have $\sigma = 0.0039$. The closer to "1" this ratio is , the closer to be unstable the closed loop system is, therefore, we can see the fault does not influence on the closed loop stability so seriously. Thus we can grasp the reliability from the viewpoint of the closed loop stability just by observing this rate σ.

8. Conclusion

In this paper, we propose a new FDI method by reducing the FDI problem for a multiplicative fault to that for an additive fault. We then detect the fault by using residual generator consisting of a stable pseudo inverse system. The residual obtained is used to estimate the parameter change. The estimated fault is evaluated from the viewpoint of closed loop stability. The proposed method is effective where the fault system is stable. Numerical example illustrates the effectiveness of the proposed method.

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Appendix

A Proof of Corollary 3

Let a state feedback $u = Fx + v$ be applied for the system (1), and $A + BF$ be stable. When the faults occur for the system, then the system is represented by

$$\begin{aligned} \delta x &= (A + \Delta_{Af} + BF + \Delta_{Bf} F)x \\ &\quad + (B + \Delta_{Bf})v && (14a) \\ y &= Cx && (14b) \end{aligned}$$

We discuss the stability of the system (14). The system (14) is equivalent to the following system:

$$\begin{aligned} \delta x &= (A + BF)x + Bv + B\Gamma_B w && (15a) \\ y &= Cx && (15b) \\ w &= (\Delta_{Af} + \Delta_{Bf} F)x + \Delta_{Bf} v && (15c) \end{aligned}$$

For the discussion of the stability, set $v = 0$, then we have the following representation of the system as shown in Fig.3:

$$\begin{aligned} \delta x &= (A + BF)x + B\Gamma_B w && (16a) \\ y &= Cx && (16b) \\ w &= (\Delta_{Af} + \Delta_{Bf} F)x = \bar{D}\Delta\bar{E}x && (16c) \end{aligned}$$

Then applying small gain theorem to the system (16), we have the corollary.

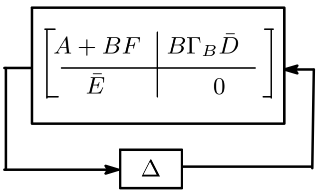

Fig. 3. The equivalent representation of the system (14)