

## Hystereresis Compensation in SMA Actuators Through Numerical Inverse Preisach Model Implementation

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**Abstract:** The aim of this paper is to compensate hysteresis phenomena in Shape Memory Alloy (SMA) actuators by using numerical inverse Preisach model. This is used to design a controller that correct hysteresis effects and improve accuracy for the displacement of SMA actuators. Firstly, hysteresis is identified by numerical Preisach model implementation. The geometrical interpretation from first order transition curves is used for hysteresis modeling. Secondly, the inverse Preisach model is formulated and incorporated in open-loop control system in order to obtain desired input- output relationship with hysteresis reducing. The experimental results for hysteresis compensation by using this method are also shown in this paper.

**Keywords:** Hysteresis, Preisach model, SMA actuator, compensation.

### 1. INTRODUCTION

SMA is metal, which exhibits the shape memory effect property. This occurs through a solid-state phase change, that is a molecular rearrangement. The two phases, which occur in shape memory alloys, are martensite and austenite. As higher temperatures, the material is in the austenite phase. As the temperature is lowered, material changes to martensite phase and grows until at sufficiently low temperatures. The appropriate current intensity can be used for this purpose. This unusual property is being applied to a wide variety of applications in a number of different fields, such as: aeronautics, surgical tools, muscle wires, ...

One of the problems in designing the controller for SMA actuators is hysteresis phenomenon. This is encountered in many applications that involve SMA, magnetic materials or piezoelectric actuators. Hysteresis is a complex nonlinearity phenomenon with properties of branching and nonlocal memory. These phenomena lead to inaccuracy and reduce the effectiveness of the control systems.

The model of hysteresis in electromagnetics, magnetostrictive, and piezoceramics actuators has been addressed in many reports [2], [7], [8], ..., but only a few in SMA actuators. Furthermore, the models are usually developed for physical based simulation rather than for control design.

Two distinct types of models have been proposed to capture the hysteretic characteristics. The first type of models is derived from the physics of hysteresis and combined with empirical factors to describe the models [14], [15], [16]... However, these models are not easy to be used in control systems. The second type of models are based on phenomenological nature and described phenomena mathematically [1], [2], [3], [4]... Among these, Preisach model is a well – known approach to model the hysteresis functions.

Preisach model is just a geometrical model and less meaning in general, but it is a convenient tool to analyze and compensate the hysteresis phenomena. Therefore, it is suitable for control applications.

In this paper, hysteresis in SMA actuators is identified by Preisach model. This model is then inverted and established a controller for hysteresis compensation.

The remainder of the paper is organized as follows. Section 2 provides an introduction to the Preisach model and its geometrical interpretation. Numerical Preisach model is

presented in section 3 along with the experimental results for the identification of hysteresis model. In section 4, the inverse model is incorporated in open – loop control scheme to compensate hysteresis phenomena for SMA actuators. Concluding remarks are provided in section 5.

### 2. HYSTERESIS IDENTIFICATION BY PREISACH MODEL

#### 2.1 Preisach model

The main assumption made in the Preisach model is that the system consists of a parallel summation of a continuum of weighted hysteresis operators  $\gamma_{\alpha\beta}$ . This is illustrated in Fig.1

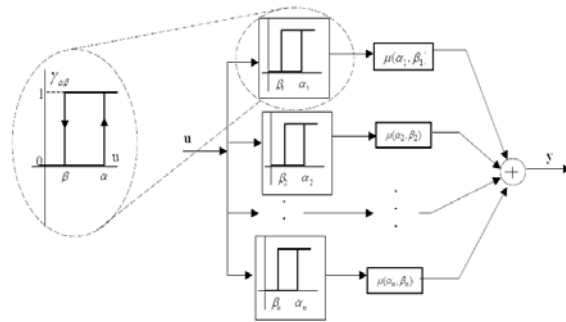


Fig.1 Preisach model schematic

As the input varies with time, each relays adjusts its output according to the current input value, and the weighted sum of these output provides the system output:

$$y(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) [\gamma_{\alpha\beta} u(t)] d\alpha d\beta \tag{1}$$

The collection of weights  $\mu(\alpha, \beta)$  describes the relative contribution of each relays in overall hysteresis.

#### 2.2 Geometrical interpretation of Preisach model

It is assumed that there is a one-to-one corresponding between operator  $\gamma_{\alpha\beta}$  and point  $(\alpha, \beta)$  on the half plan  $\alpha \geq \beta$ . This is also called the Preisach plan. The function  $\mu(\alpha, \beta)$  is assumed to be equal zero outside the triangle **T** in Fig.2.

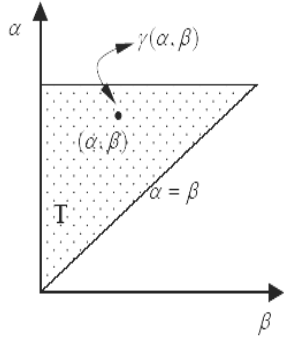


Fig.2 Preisach plan

To start the discussion, consider the case of hysteresis formation in SMA actuator when the driving input  $u(t)$  is current, and the resulting output is displacement  $y(t)$ . When the current to the SMA actuator is zero, the output of all operators is zero.

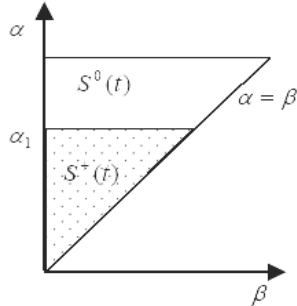


Fig.3 Division of the triangle due to a increasing input  $\alpha_1$

Now, we assume that the input is monotonically increased to a value  $\alpha_1$  as shown in Fig.3, the outputs of all hysteresis operators with  $\alpha$  switching value less than  $\alpha_1$  become equal to +1. Geometrically, it leads to the subdivision of the triangle **T** into two sets:  $S^+(t)$  consisting of point  $(\alpha, \beta)$  where the output is +1, and  $S^0(t)$  where the output is zero. This subdivision is made by the line  $\alpha = u(t)$  that moves upwards as the input is being increased.

Next, the input is monotonically decreased from  $\alpha_1$  to a value  $\beta_1$ . As the input is being decreased, the outputs of all hysteresis operators in the set  $S^+(t)$  with  $\beta$  switching values greater than  $\beta_1$  become equal to zero. This changes the previous subdivision of **T** into two sets again. The interface between  $S^+(t)$  and  $S^0(t)$  has now two links, the horizontal and vertical ones. As illustrated by Fig.4, vertical link moves from right to left as the current is monotonically decreased from  $\alpha_1$  to  $\beta_1$ . The vertical line is  $\beta = u(t)$ .

By generating this analysis with the case of increasing or decreasing of the input, the following conclusion can be reached. At any instant of time, the triangle **T** is subdivided into two sets:  $S^+(t)$  and  $S^0(t)$  corresponding the output of hysteresis operators is +1 or zero. The interface between  $S^+(t)$  and  $S^0(t)$  is the staircase line whose vertices have  $\alpha$  and  $\beta$  coordinates respectively with local maxima and minima of input at previous instant of time.

Thus, at any instant of time the integral in (1) can be expressed as

$$y(t) = \iint_{S^+(t)} \mu(\alpha, \beta) \gamma_{\alpha\beta} u(t) d\alpha d\beta - \iint_{S^0(t)} \mu(\alpha, \beta) \gamma_{\alpha\beta} u(t) d\alpha d\beta \quad (2)$$

as  $\gamma_{\alpha\beta} u(t) = 0, \forall (\alpha, \beta) \in S^0(t)$ , and  $\gamma_{\alpha\beta} u(t) = 1, \forall (\alpha, \beta) \in S^+(t)$

this expression becomes:

$$y(t) = \iint_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta \quad (3)$$

From the above expression, the output of Preisach model depends on the subdivision of triangle **T**. This subdivision is determined by an interface which depends on the past extremum values of input. This means that the Preisach model describes hysteresis nonlinearity with nonlocal memory.

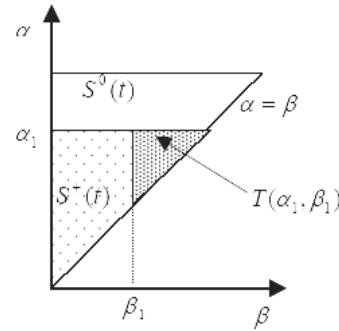


Fig.4 Division of the triangle due to a decreasing input  $\beta_1$

### 3. NUMERICAL PREISACH MODEL FOR HYSTERESIS OF SMART ACUATORS

To apply classical Preisach model of hysteresis for SMA actuators, it encounters many difficulties. It requires the evaluation of double integrals in (1). This is a time consuming procedure that make difficulties the use of Preisach model in practical applications. In this paper, the numerical Preisach model implementation is based on the geometrical interpretation method, which avoids double integrals in Preisach model formula. To do this, we define a new function:

$$F(\alpha_1, \beta_1) = \frac{1}{2} (y_{\alpha_1} - y_{\alpha_1\beta_1}) \quad (4)$$

where  $y_{\alpha_1}$  is the output at current value of  $\alpha_1$ , and  $y_{\alpha_1\beta_1}$  is the output after current has been decreased to  $\beta_1$  from its maximum value of  $\alpha_1$ .

From Fig.4, we can find the fact that the integral over the area  $T(\alpha_1, \beta_1)$  equals the difference in hysteresis outputs of current values of  $\alpha_1$  and  $\beta_1$

$$\iint_{T(\alpha_1, \beta_1)} \mu(\alpha, \beta) d\alpha d\beta = F(\alpha_1, \beta_1) \quad (5)$$

The set  $S^+(t)$  can be subdivided into n trapezoids  $Q_k$ , see Fig.5. At a result, we have

$$\iint_{S^+(t)} \mu(\alpha, \beta) d\alpha d\beta = \sum_{k=1}^{n(t)} \iint_{Q_k(t)} \mu(\alpha, \beta) d\alpha d\beta \quad (6)$$

where n trapezoids  $Q_k$  may change with time.

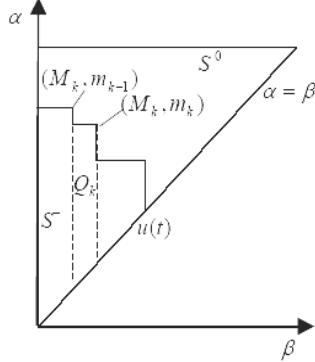


Fig.5 Triangle T for numerical implementation of Preisach model corresponding to a decreasing input current signal.

Each trapezoid  $Q_k$  can be represented as a difference of two triangles  $T(M_k, m_{k-1})$  and  $T(M_k, m_k)$ . Therefore, we can obtain the following expression

$$y(t) = 2 \sum_{k=1}^{n(t)} [F(M_k, m_{k-1}) - F(M_k, m_k)] \quad (7)$$

In the case the input is monotonically decreasing, in Fig.5, the final link of the interface is a vertical one, thus:  $m_n = u(t)$ . Consequently, the expression (7) can be written as

$$y(t) = 2 \sum_{k=1}^{n(t)-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + 2[F(M_n, m_{n-1}) - F(M_n, u(t))] \quad (8)$$

If the input is monotonically increasing, in Fig. 6, the final link of the interface is a horizontal one,  $m_n = M_n(t) = u(t)$ , and (7) becomes

$$y(t) = 2 \sum_{k=1}^{n(t)-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + 2F[u(t), m_{n-1}] \quad (9)$$

Using the above expressions, the output of the Preisach model can be calculated from an input, a set of first-order transition curves, and an input history that are all specified by the user.

To implement this algorithm, there are two main steps for hysteresis modeling. First, a square mesh covering the triangle T is created. The number of switching points in the Preisach plan, as shown in Fig.2, can be selected by the user. During this stage, a discrete set of first - order transition curves is entered. This set consists of mesh values of the function  $F(\alpha, \beta)$  which is obtained from experimental data.

At the second stage, an input history and current values of input are entered. Using these data, the alternating series of dominant input extrema  $(M_k, m_k)$ ,  $(M_k, m_{k-1})$  must be determined and updated at each instant of time. All terms in the formulate (8), (9) are computed from these values:  $M_k, m_k, m_{k-1}$ , and the mesh value  $F_{\alpha\beta}$ . This is done by first determining particular square (or triangle) cells to which points  $(M_k, m_k)$ ,  $(M_k, m_{k-1})$ , and  $(M_n, u(t))$  belong, and then by computing the value of  $F(\alpha, \beta)$  at these points by means of interpolation of mesh values of  $F(\alpha, \beta)$  at the vertices of these cells. After computing all values of  $F(\alpha, \beta)$ , the output value can be calculated in (8) or (9) for the case of increasing (or decreasing) of the input.

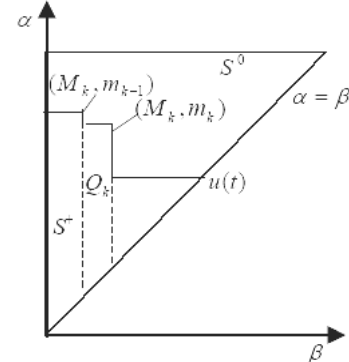


Fig.6 Triangle T for numerical implementation of Preisach model corresponding to an increasing input current signal.

### Experiment result

In the experiment setup for hysteresis modeling and compensation, a small tensile SMA actuator is used with some main specifications: heat current: ca.2V/0.85A, gen. force: 8N, displacement: ca.5mm. The displacement is measured by a potentiometer with high precision. This system is real-time controlled by using Advantech PCI-1711 Card with Realtime Windows Target Toolbox of Matlab.

There are 55 switching points are used in this program, as shown in Fig.7. The more switching points we used, the more accuracy of model we obtained. For each switching  $(\alpha, \beta)$  points, the mesh values of the function  $F(\alpha, \beta)$  in (4) are calculated by measuring the output displacement as the input current is increased to  $\alpha$  ( $f_\alpha$ ) and then decreased to  $\beta$  ( $f_{\alpha\beta}$ ).

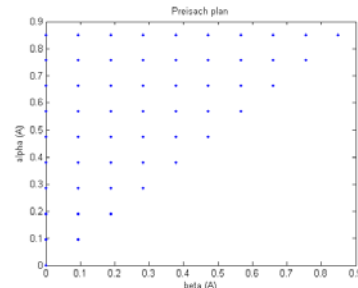


Fig.7 Number of switching points are used in the experiment

The input applied to SMA actuator is decaying sinusoid signal, shown in Fig. 8. During the real - time control, the dominant input maxima and minima is determined and updated at each instant of time. The maximum and minimum points are updated by comparing any new dominant extrema with previous extrema. During this work, the previous extrema can be deleted none, one or more point and added the new dominant extrema.

The induced output displacement from experiment is shown in Fig.9. The output of hysteresis model computed from (8) and (9) is shown in Fig.10. The experimentally measured output values agreed with Preisach model prediction are shown in Fig.11.

The discrepancy is due to the limited of switching points available in the corresponding region of the Preisach triangle. This error can be eliminated by increasing the number of points in Preisach plan, Fig.7. Fig.11 can be verified the consistence performance of the Preisach model in predicting the hysteresis nonlinearity phenomena in SMA actuators.

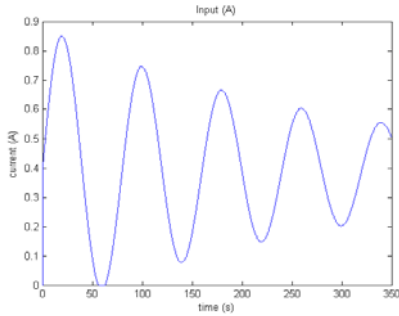


Fig. 8 Input current signal

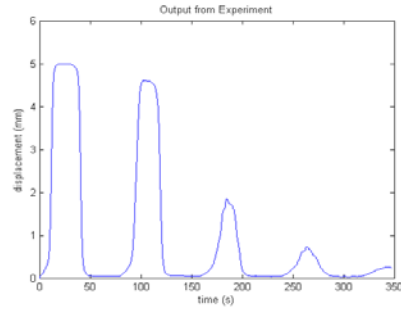


Fig.9 Output displacement from experiment.

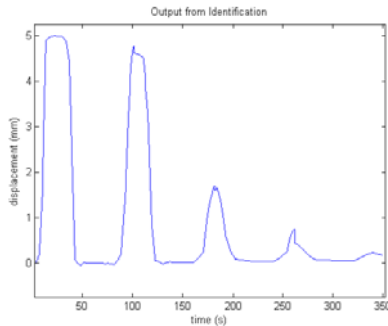


Fig.10 Output displacement from Preisach hysteresis modeling

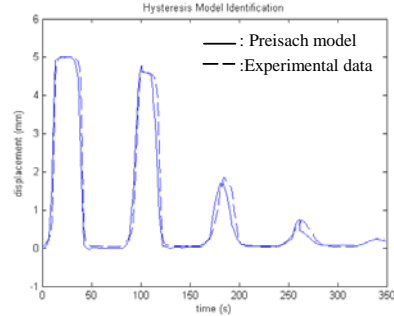


Fig.11 Comparison of the output displacement through experiment and Preisach hysteresis modeling.

#### 4. HYSTERESIS COMPENSATION BY INVERSE PREISACH MODEL

The inverse Preisach model is developed to correct the hysteresis effect of SMA actuators. In this section, the implementation of the inverse Preisach model is described first, and then its application in hysteresis compensation for control is proposed.

##### 4.1 Inversion of Preisach model

The inverse Preisach model determines the current that will result the desired output displacement. In the formulation of the inverse Preisach model, there are two different cases corresponding to the decreasing and increasing of the current input. To find the current input values  $u(t)$  for the desired output  $y_d(t)$ , the expressions in (8) and (9) are rewritten as follows

$$\text{Case of decreasing desired displacement output} \\ y_d(t) = \sum_{k=1}^{n-1} (y_{M_k m_k} - y_{M_k m_{k-1}}) + y_{M_n u(t)} - y_{M_n m_{n-1}} \quad (10)$$

Inverting above expression, the solution to the inverse model for this case is derived to be:

$$u(t) = \beta = G^{-1}(y_{M_n u(t)}) = \\ = G^{-1} \left[ y_d(t) + y_{M_n m_{n-1}} - \sum_{k=1}^{n-1} (y_{M_k m_k} - y_{M_k m_{k-1}}) \right] \quad (11)$$

Case of increasing desired displacement output

$$y_d(t) = \sum_{k=1}^{n-1} (y_{M_k m_k} - y_{M_k m_{k-1}}) + y_{u(t)} - y_{u(t) m_{n-1}} \quad (12)$$

and the input current value for the desired output

$$u(t) = \alpha = G^{-1}(y_{u(t)} - y_{u(t) m_{n-1}}) = \\ = G^{-1} \left[ y_d(t) - \sum_{k=1}^{n-1} (y_{M_k m_k} - y_{M_k m_{k-1}}) \right] \quad (13)$$

Furthermore, the input current can be computed from the inverting of expression (9) for this case

$$u(t) = \alpha = G^{-1}(F(u(t), m_{n-1})) = \\ = G^{-1} \left[ \frac{y_d(t)}{2} - \sum_{k=1}^{n(t)-1} (F(M_k, m_{k-1}) - F(M_k, m_k)) \right] \quad (14)$$

The input current value used in the inverse model is the desired displacement. This computation in (11), (13), or (14) consists of the following procedure: updating dominant input extrema  $(M_k, m_k)$  and  $(M_k, m_{k-1})$  at each instant of time, calculation  $F(\alpha, \beta)$  or  $y_{\alpha\beta}$  corresponding to the desired output displacement, and determination of the square or triangle in Preisach plan which the inverse solution may lie. The input current, the  $\alpha$  or  $\beta$  value, is computed from the expression (11),(13), or (14) by mean of interpolation. These calculations are performed at each instant of time, therefore this procedure can be implemented in real – time control.

##### 4.2 Open-loop compensation of hysteresis in SMA

In order to obtain high performance of the control systems, hysteresis phenomena must be compensated. Due to the inverse Preisach model can predict the current that would achieve the desired displacement of SMA actuators., Consequently, it is possible to incorporate this inverse model in open – loop control system to compensate hysteresis nonlinearity effect. This control strategy is shown in Fig. 12.

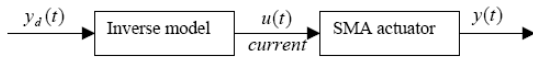


Fig.12 Open – loop compensation of hysteresis in SMA actuator

The desired output displacement is used as the input of hysteresis inverse model. The output of this inverse model is the current which is used to apply to SMA actuator. The performance of the compensation strategy is experimentally investigated and the result is shown in Fig.14.

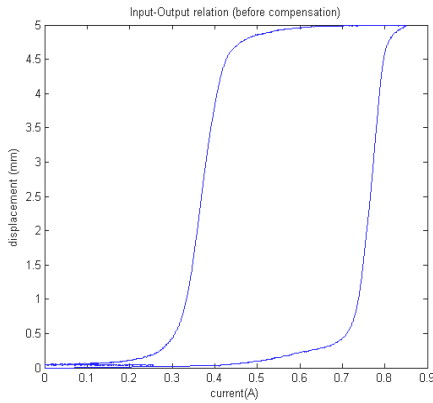


Fig. 13 Current – displacement curve from experiment before compensation

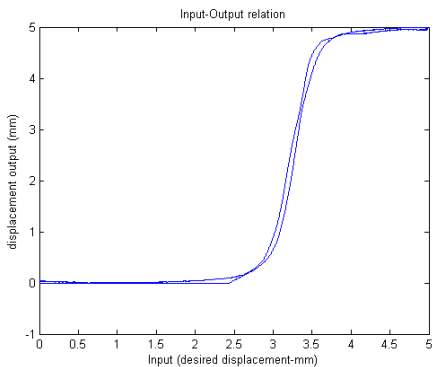


Fig 14 Hysteresis curve obtained from experiment after using hysteresis compensation strategy.

The hysteresis curve between input and output relation before compensation is shown in Fig.13. The experimental result for the displacement actuator after using hysteresis compensation strategy is shown in Fig.14. The output displacement is nearly close to the command signal. Consequently, the open-loop compensation strategy based on the inverse Preisach model provides the effective for reducing the hysteresis nonlinearity phenomena of SMA actuators.

**5. CONCLUSION**

In this paper, the Preisach model for hysteresis in SMA current – displacement response has been presented. The experiments show that the model is suitable for predicting hysteresis in SMA actuators. The numerical inverse Preisach model is successfully applied for hysteresis compensation and

the desired relationship between current and displacement can be obtained. This method is not only useful for SMA actuators but also other actuators with hysteresis phenomena.

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**REFERENCES**

- [1] I.D. Mayergoyz, “Mathematical models of hysteresis and their applications”, Elsevier Science Inc., 2003.
- [2] Samir Mittal and Chia-Hsiang Menq, “Hysteresis compensation in electromagnetic actuators through Preisach model inversion”, IEEE/ASME Transactions on Mechatronics, Vol.5, No.4, 2000.
- [3] H. Hu, R. Ben Mrad, “A discrete-time compensation algorithm for hysteresis in piezoceramic actuators”, Mechanical Systems and Signal Processing 18 (2004) 169-185.
- [4] Byung – Jun Choi, Yun – Jung Lee and Bong – Yeol Choi, “Fast Preisach modeling method for shape memory alloy actuators using major hysteresis loop”, Smart Materials and Structures, 13(2004) 1069-1080.
- [5] Mughees M. Khan and Dimitris C. Lagoudas, “Modeling of Shape memory Alloy pseudoelastic spring elements using Preisach model for passive vibration isolation”.
- [6] Oscar Alejos and Edward Delle Torre, “Improving numerical simulation of Preisach models for accuracy and speed”, IEEE Transactions on Magnetics, Vol.36, No.5, 2000.
- [7] Declan Hughes and John T Wen, “Preisach Modeling of piezoceramic and shape memory alloys hysteresis”, Smart Mater. Struct. 6(1997) 287-300.
- [8] Xiaobo Tan, John S Baras, “Modeling and control of a magnetostrictive actuators”, CDC02-REG0909.
- [9] Juan Manuel Cruz- Hernandez and Vincent Hayward, “On the linear compensation of hysteresis”, Proceeding of the 36<sup>th</sup> Conference on Decision & Control, San Diego, California USA, 1997.
- [10] Juan Manuel Cruz- Hernandez and Vincent Hayward, “Phase control approach to hysteresis reducing”, IEEE Transactions on Control Systems Technology, Vol.9, No.1, pp. 17-26.
- [11] Tadahiro Hasegawa, Sumiko Majima, “A control system to compensate the hysteresis by Preisach model on SMA actuator”, 1998 International Symposium on micro mechatronics and human science.
- [12] D. Davino, C. Natale , S. Pirozzi, C. Visone, “A fast compensation algorithm for real-time control of magnetostrictive actuators”, Journal of magnetism and Magnetic Materials.
- [13] Klaus Kuhnen, “Modeling, identification and compensation of complex hysteretic nonlinearities, a modified Prandtl Ishlinskii approach”, European Journal of Control, Vol.9, No.4, 2003, pp.407- 418.
- [14] Ralph C. Smith, Chad Bouton, Rick Zrostlik, “ Partial and full inverse compensation for hysteresis in smart material systems”.
- [15] Jame M. Nealis, Ralph C. Smith, “Nonlinear adaptive parameter estimation algorithms for hysteresis model of magnetostrictive actuators”, North Carolina State University.
- [16] L.A Whitehead, M.A Mossman, “Numerically efficient method for modeling elastomeric hysteresis”, The University

of British Columbia.

- [17] Ralph C. Smith, Zoubeida, Ounaies, Robert Wieman, "A model for rate – dependent hysteresis in piezoceramic materials operating at low frequencies", NASA, CR-2001-11062, ICASE Report No. 2001-26.
- [18] Patrick M. Sain, Michael K.Sain and B.F. Spencer, "Models for hysteresis and application to structural control", Proceedings American Control Conference, Page 16-20, 1997.
- [19] Robert, B.Gorbet, David W.L. Wang, "A dissipativity approach to stability of a shape memory alloy position control system", IEEE Transactions on Control Systems Technology.
- [20] Paiboon Nakmahachalasint, Khai D.T.Ngo, Loc Vu Quoc, "A static Hysteresis model for power ferrites", IEEE Transactions on Power electronics, Vol.17, No.4, 2002.