

## Image registration using Hough transform and Phase correlation in Wavelet domain

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**Abstract:** This paper presents a method for registering images using phase correlation technique in fourier domain, hough transform and multi-resolution wavelet. To register images, source and input images are transformed to wavelet domain. An angular transition can be obtained by applying hough transform technique followed by phase correlation. Then we apply phase correlation technique to find x-axis and y-axis transition. We apply wavelet transform to reduce processing time and also use its coefficients as edge information instead of canny detector. With multi-resolution property of wavelet transform, registration time can be greatly reduced. After we get all transition parameters, we transform the input images according to these parameters. Then, we compose and blend all images into a new large image with details of all source images. From our experiment, we can find the accurate transition both x-y translation and angular transition with less error.

**Keywords:** image registration, phase correlation, hough, wavelet

### 1. INTRODUCTION

Image registration is the method to register or arrange small images and compose them to make one image that contains details of all input images. Images taken as inputs of process must have some overlapped area. Registered images help us to view overall details without swapping image many times. So that image analysis can be done easily. In satellite image analysis, image registration can be applied in process. Satellite images are captured in a row, continuous in the same interval of satellite orbit. Moreover, almost all images taken by satellite have some overlapped area. We will take a pair of images, register them to create new image then take other image to register again. This process is continued until all images have been registered. We get a new large image with details of all images in correct position. This image has more useful information in geography because it is a map of the region we have taken and can be applied in many analysis processes. This image may be a natural resource map, mineral map or military map.

This paper presents an image registration technique using hough transform and phase correlation in wavelet domain. Several algorithms have been proposed in the past both in spatial and frequency domain. Image registration in spatial domain uses techniques such as user's specified control points and searching algorithm. The first technique is simple but depends on user's ability to select good control points [6, 9, 10]. Selecting bad control points can cause errors in the registration process. The second technique gives better registration results and covers several cases of geometric transformation such as rotation, translation, sheering, and different point's of views [4, 6, 7, 11, 12]. However, it requires greater complexity and processing time since the algorithm has to search all the possible solutions before identifying the correct one.

Phase correlation is the most widely used technique for image registration in frequency domain [1, 2, 5, 8]. It applies the property of Fourier transform in order to find translation between images. The algorithm is easy to implement and requires less amount of processing time due to the benefit of fast Fourier transform (FFT) algorithm. However, phase correlation can be used to find only translation in x-y axis. Other techniques must be applied in advance to transform information from x-y axis to suitable space in order to find other types of geometric transform such as angular transition, zooming, sheering, or stretching. In the proposed algorithm, hough transform is used to transform line information in x-y

axis to hough space  $(\rho, \theta)$ . Therefore, rotation in x-y axis will be transformed to translation in hough space  $(\rho, \theta)$ . Then, phase correlation can be used to find angular transition in hough space  $(\rho, \theta)$ .

Recently, wavelet transform has become a widely used technique to compress image data. With its multi-resolution property, the resolution of the image can be reduced. This results in less amount of processing time. Therefore, in the proposed algorithm, we first transform our images into wavelet domain. Then, angular transition is calculated using hough transform and phase correlation. Generally, edge detection algorithm must be done before hough transform process to extract lines in the image. This process causes additional processing time. Since high frequency information corresponding to edges is already there in wavelet domain, we can directly use this information as input to hough transform. As a result, we can eliminate edge detection process thus additionally reduce the processing time. However edge detection using information in wavelet domain is not as good as edges from canny edge detector or sobel. There are some trivial details and noises in an image that can cause errors and affect our result. Therefore, we filter images using low pass filter to eliminate trivial details and noises in images. Only main lines and details remain in the image. After we obtain angular transition, the image is rotated using angular transition parameter. Finally, phase correlation is performed to obtain translation parameter in x-y axis.

### 2. OVERVIEW OF THE PROPOSED IMAGE REGISTRATION IN WAVELET DOMAIN

In image registration process, we work with a pair of images. We call the first image as "Source image" and the other one as "Input image". The work flow from our system is shown in Fig.1. We begin the process by filtering both images with low pass filter. Then, we transform images using wavelet transform. After wavelet transformation each image is separated into 4 components, LL LH HL and HH. In stead of using canny detector, we combine LH, HL and HH coefficients to obtain edge information. After that, we start finding angular transition parameter to compensate rotation of the camera. Hough transform method is performed to edge information of both images to transform angular transition in image space to translation transition in hough space. After we

obtain hough space of both image, we use phase correlation process to find angular transition between 2 images. We then use angular transition obtained from previous step to rotate LL part in wavelet components of the input image. After that we find x and y translation between images. We use only LL component from source and input images. Phase correlation is applied in this step. After we get all transitions, we compose source and input image together to obtain an output image. The detail of each process will be discussed in the next sections.

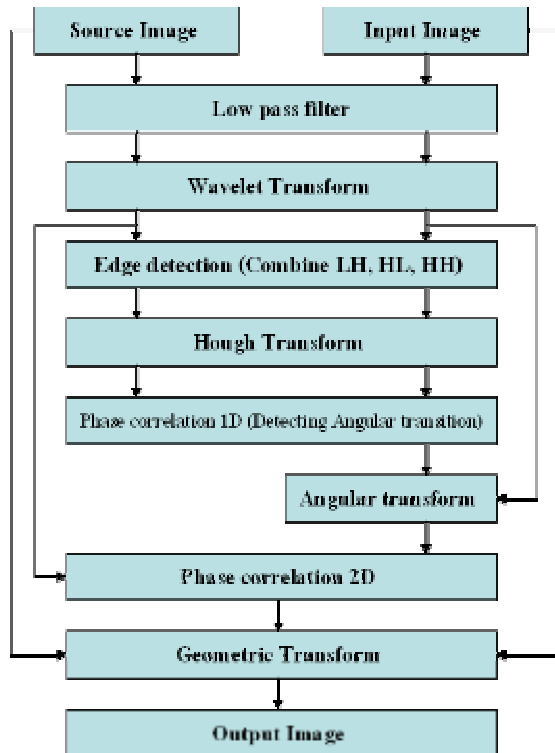


Fig.1 Block diagram presents our system flow

**2.1 Wavelet transform**

Wavelet transform is a method to separate an image into different frequency parts. After wavelet transform, image is separated as Fig. 2

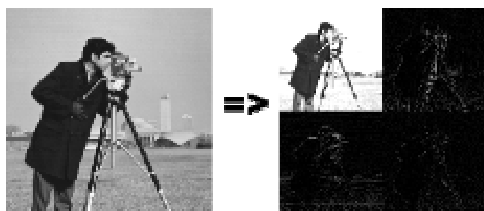


Fig. 2 Image is transformed with wavelet transform.

The top-left part of image is called LL part. The LL part of image is the lowest frequency. Most of this part is the details of original image. The bottom-right part is called HH, which is the highest frequency. This part consists of only the sharp edges in diagonal direction of original image. In LL part you

can see all details of original image with size 1/4 of original image. Wavelet transform is one of the techniques used to reduce resolution of the image needed to be processed. The smaller size of the image, the less processing time and processing space.

We can further apply wavelet transform to LL part and we can continue transform with LL part to get lower resolution of image, however error will increase accumulatively for each level of wavelet, see Fig. 3.

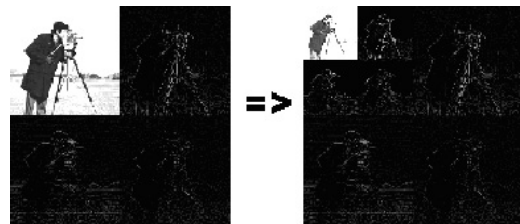


Fig. 3 Multi-level wavelet transform

**2.2 Edge detection**

Before applying hough transform, edge information must be obtained from the image. We may use canny detector or sobel in this step. However, we can take the advantage of wavelet property by combining LH, HL and HH part together to use as an edge detection process. But the result of edge detection process from wavelet is not as good as that of canny detector or sobel. There are many trivial points and lines in the image that may cause an error in hough transform process. We can improve quality of edge information from wavelet coefficients by pre-processing an image with low pass filter to filter out small points that disperse an image. If we select a suitable type of filter, we will get better quality edge image as shown in Fig. 4.



Fig. 4 (left) Edge image using canny detector, (center) Edge image using wavelet without filter, (right) Edge image using wavelet and low pass filter

**2.3 Hough transform [2]**

Hough transform is a technique to collect all lines hidden in an image. Hough transform can indicate how lines are in an image at any distance and any angle. Hough transform will collect lines from image in hough space  $(\rho, \theta)$ .  $\rho$  is a distance of the line from original point and  $\theta$  is an angle of the line. Each cell in hough space is the density of line at  $(\rho, \theta)$ . We can get  $\rho$  from

$$\rho = x \cos \theta + y \sin \theta \tag{1}$$

We can plot an image from hough space as show in Fig. 5.

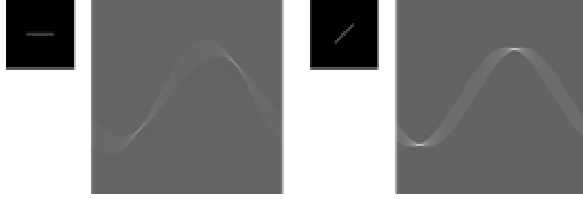


Fig. 5 Original image and hough image plotted from hough space. In x-axis is  $\theta$  and y-axis is  $\rho$ .  $\theta$  is changed from 1 to 360 degree

The brightest spots in figure are the dense points of hough space. Notice that, when  $\theta$  is changed, hough image will also change in x-axis. However, there are many trivial lines in hough image. These lines may disturb our results, therefore, we have to threshold hough information to reduce this problem.

### 2.4 Phase correlation [1]

Phase correlation is one of Fourier properties. From Fourier equation

$$\mathfrak{F}\{f(x, y)\} = \hat{f}(\omega_x, \omega_y) \quad (2)$$

$$\mathfrak{F}\{f(x + \Delta x, y + \Delta y)\} = \hat{f}(\omega_x, \omega_y) e^{j(\omega_x \Delta x + \omega_y \Delta y)} \quad (3)$$

We have image 1 that is shifted to any point and become image 2.

$$I_1(x + \Delta x, y + \Delta y) = I_2(x, y) \quad (4)$$

We apply equation (2) and (3) in (4).

$$\hat{I}_1(\omega_x, \omega_y) e^{j(\omega_x \Delta x + \omega_y \Delta y)} = \hat{I}_2(\omega_x, \omega_y) \quad (5)$$

$$\frac{\hat{I}_1(\omega_x, \omega_y)}{\hat{I}_2(\omega_x, \omega_y)} = e^{j(\omega_x \Delta x + \omega_y \Delta y)} \quad (6)$$

$e^{j(\omega_x \Delta x + \omega_y \Delta y)}$  is x-axis transition and y-axis transition from input images in Fourier domain. We can use inverse Fourier transform to get the x-y translation in spatial domain.

$$Corr(x, y) = \mathfrak{F}^{-1}\{e^{j(\omega_x \Delta x + \omega_y \Delta y)}\} = \delta(x - \Delta x, y - \Delta y) \quad (7)$$

We should select transition at the point where correlation is 1. However, we cannot get the perfect correlation because of non-overlapping area and noise in input images. Therefore, we select the point that correlation is maximum instead.

$$(x, y) = \arg \max_{(\bar{x}, \bar{y})} \{Corr(\bar{x}, \bar{y})\} \quad (8)$$

We use x and y as an x-axis and y-axis transition respectively. Notice that x and y never become negative but translation can be negative. If transition is negative x and y become back to another side because of Fourier transform and phase correlation. As a result, we have to determine both positive and negative. There is another problem. If input images have different contrast, less overlapping area, or large amount of noise, the result from phase correlation may become incorrect. Therefore, it is better to use another form of phase correlation, which incorporates magnitude of Fourier coefficients to reduce the effect. From (6) we adapt to

$$\overline{Corr}(\omega_x, \omega_y) = \frac{\hat{f}_1(\omega_x, \omega_y) \hat{f}_2^*(\omega_x, \omega_y)}{|f_1(\omega_x, \omega_y)| |f_2(\omega_x, \omega_y)|} = e^{j(\omega_x \Delta x + \omega_y \Delta y)} \quad (9)$$

Symbol  $\hat{f}_2^*(\omega_x, \omega_y)$  is complex conjugate of  $\hat{f}_2$ .

### 3. EXPERIMENT

We perform the experiment on 3 input images, cameraman and satellite images of Bangkok and Pentagon. Several resolution of the input images (64x64, 128x128, 256x256, 512x512 and 1024x1024) are used. We divide our experiment into 2 parts. In the first part, we test the translation effect in x-y axis. The second part is to find both angular transition and x-y translation. The x-y translation from 5 pixels to 100 pixels are divided into 10 steps and tested. To obtain angular transition, we tested the total of 36 steps with 10 degrees apart. We also compare accuracy and processing time of the proposed algorithm with that of [2] as shown in table 1 and 2 and Fig. 6 and 7.

The results show that the propose algorithm can correctly detect x-y translation with errors of  $\pm 2^{n-1}$ , where n is the number of levels of wavelet transform we applied to the image. This error occurs due to the reduction of the resolution in wavelet domain. However, the error does not affect the result much. In the case of angular transition, the results using phase correlation and hough transform with canny detector as an edge detection process causes average error about 0.6%. Using edge information from wavelet domain without preprocessing is resulted in average errors about 3.23%. However, after we apply low pass filter before wavelet transform, average error decreases to 0.8%. Even though the error is greater than the previous case, it does not obviously affect output image. Moreover, by using information from wavelet domain, we can reduce the processing time not only from the reduction in resolution of the processing information but also the elimination of edge detection process. The larger the resolution of the image, the more reduction in the processing time. This can help making real-time image registration and analysis of high resolution images possible.

Processing time in our system is less than [2] especially when image resolution is large, see Fig.6 and 7. Notice that if images are small (less than 200 x 200 pixels), there is no need to apply wavelet transform in the process because processing time may differ less than 5 seconds while error may obviously seen. If images are large, processing time in our system will reduce sharply while error may not affect our result image much. The example result images are shown in Fig.8.

Table 1 Translation error occurred in our system compare to [2] (pixel)

Translation	[2]	Proposed system
Even value	0	0
Odd value	0	$\pm 1$

Table 2 Average angular transition error occurred in our system compare [2]

Angular transition	[2]	Proposed system
Average error	0.67	0.8

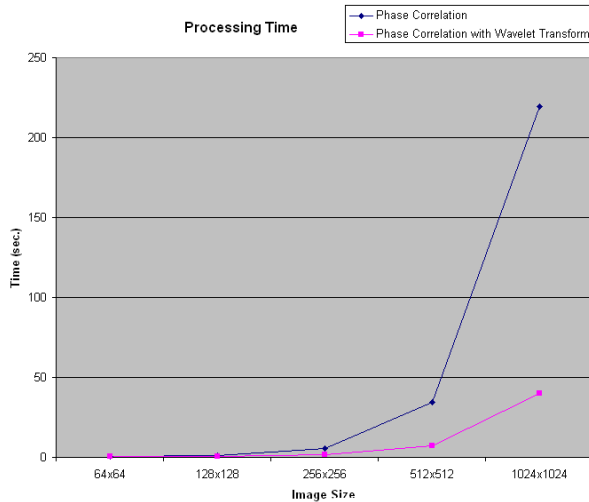


Fig. 6 Processing time of phase correlation technique with and without wavelet pre-processing

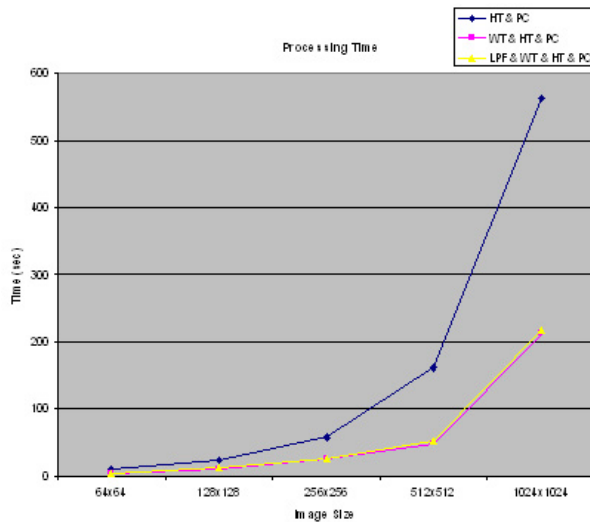


Fig. 7 Processing time of angular transition detection with and without wavelet and low pass filter



Fig.8 Example result images from our system

**4. CONCLUSION AND FUTURE WORK**

Image registration with phase correlation can find transition of two images correctly in case of integer translation. Wavelet transform can reduce the resolution of the image by 1/4 at each level. With lower resolution of the image, we can process with less time and space. Moreover, we can use information from wavelet transform in the place of edge detection process. However, some errors occur accumulatively at each level of wavelet. Therefore, we have to adjust some parameters to get the best registration result. Sub-pixel

registration is still required to accurately correct our result in case of real number transition. Even though edge information from wavelet transform is lower quality, we can improve the quality by filtering an image before applying wavelet transform to filter out trivial details that can cause errors. We may apply fast hough transform or other line collector method to further reduce processing time. For very low resolution of the image, there is no need to apply wavelet transform in the process because processing time may differ less than 5 seconds while error may obviously seen. However, for high resolution of the image which is usually the case for image registration, registration time is very high, thus registration in wavelet domain as proposed in this paper can help reduce the processing time sharply while error occurred is trivial and may not obvious.

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