

## Switching rules based on fuzzy energy regions for a switching control of underactuated robot systems

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**Abstract:** One of control methods for underactuated manipulators is known as a switching control which selects a partially-stable controller using a prespecified switching rule. A switching computed torque control with a fuzzy energy region method was proposed. In this approach, some partly stable controllers are designed by the computed torque method, and a switching rule is based on fuzzy energy regions. Design parameters related to boundary curves of fuzzy energy regions are optimized offline by a genetic algorithm (GA). In this paper, we discuss on parameters obtained by GA. The effectiveness of the switching fuzzy energy method is demonstrated with some simulations.

**Keywords:** Switching control, Fuzzy energy based switching, Underactuated robot, Genetic algorithm, computed torque control

### 1. Introduction

Control of underactuated manipulators is an attractive research theme in robotics because of complex behaviors and difficulty of control [1], [2], [3], [4]. Since underactuated manipulators have some passive joints, their energy efficiency can be better than full actuated manipulators.

As a control method for underactuated systems, authors have already proposed a switching control, in which some partly stable controllers were designed by computed torque method and the switching law was obtained as the index of controller directly by fuzzy reasoning [5]. The switching control is proposed to design simply a control law without any complex variable transformation for underactuated manipulators. The switching law is a key-point to obtain successful results in this method. A logic based switching method using energy regions [6] is also proposed for a nonholonomic system without drift term.

In this paper, we propose a logic based switching mechanism using fuzzy energy region. Boundary curves to separate an energy region into some energy subregions are constructed by fuzzy reasoning. Fuzzy related parameters and control gains of the partly stable controllers are optimized by genetic algorithm (GA). We prepare some cost functions depending on a settling time. The present method is applied to an underactuated system with drift term such as an 2-DOF planar manipulator which has only one active joint. The effectiveness of the present method is illustrated with some simulations.

### 2. Underactuated Manipulator

Figure 1 shows a two-link underactuated manipulator, in which the second joint consists of a passive joint. Here,  $\tau_1$  denotes the applying torque of 1st joint,  $\theta_i$  denotes the angle of  $i$ th link,  $m_i$  denotes the mass of  $i$ th link,  $l_{gi}$  denotes the distance from the joint to the center of mass of  $i$ th link,  $I_i$  denotes the moment of inertia of  $i$ th link, and  $\mu_i$  denotes the coefficient of viscous friction. The dynamical model of the underactuated manipulator is given as follows:

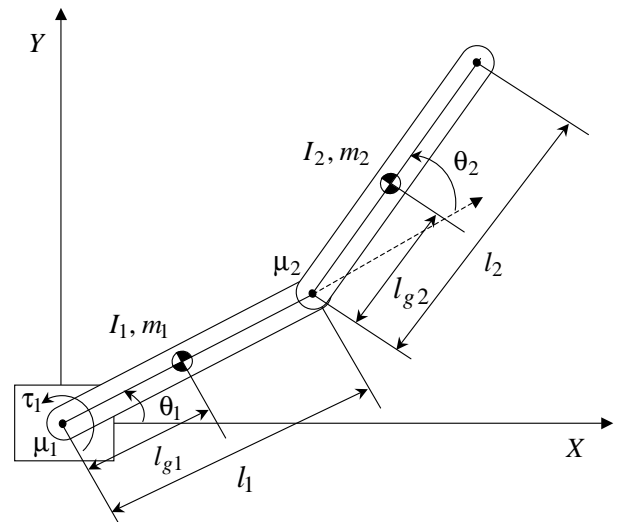


Fig. 1. Model of 2-link underactuated manipulator

$$M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) = \tau \tag{1}$$

where

$$\begin{aligned} \theta &= [\theta_1 \ \theta_2]^T \\ \tau &= [\tau_1 \ 0]^T \\ M(\theta) &= \begin{bmatrix} M_{11}(\theta) & M_{12}(\theta) \\ M_{12}(\theta) & M_{22}(\theta) \end{bmatrix} \\ M_{11}(\theta) &= (m_1 l_{g2}^2 + m_2 l_1^2 + I_1) + (m_2 l_{g2}^2 + I_2) \\ &\quad + 2m_2 l_1 l_{g2} \cos \theta_2 \\ M_{12}(\theta) &= (m_2 l_{g2}^2 + I_2) + m_2 l_1 l_{g2} \cos \theta_2 \\ M_{22}(\theta) &= m_2 l_{g2}^2 + I_2 \\ h(\theta, \dot{\theta}) &= [h_1(\theta, \dot{\theta}) \ h_2(\theta, \dot{\theta})]^T \\ h_1(\theta, \dot{\theta}) &= -(m_2 l_1 l_{g2})(2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \sin \theta_2 + \mu_1 \dot{\theta}_1 \\ h_2(\theta, \dot{\theta}) &= m_2 l_1 l_{g2} \dot{\theta}_1^2 \sin \theta_2 + \mu_2 \dot{\theta}_2 \end{aligned}$$

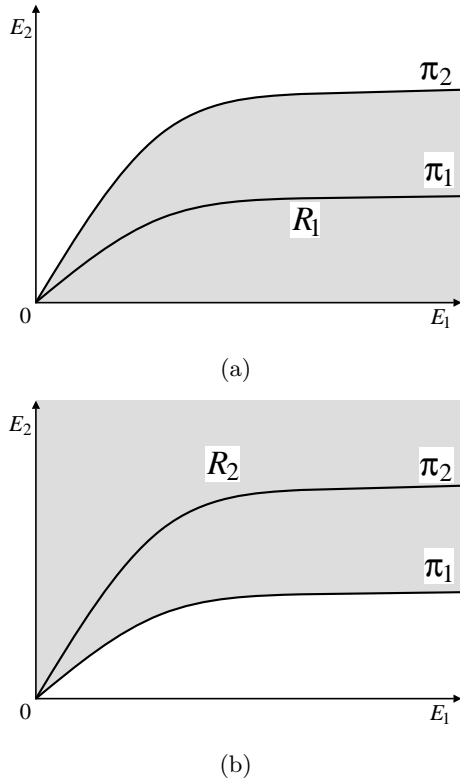


Fig. 2. Subregions of energy

### 3. Fuzzy Region Based Switching Control

#### 3.1. Partly stable controller

Equation (1) can be described by

$$\ddot{\theta}_1 = -\frac{M_{22}(\theta)}{D}h_1(\theta, \dot{\theta}) + \frac{M_{12}(\theta)}{D}h_2(\theta, \dot{\theta}) + \frac{M_{22}(\theta)}{D}\tau_1 \quad (2)$$

$$\ddot{\theta}_2 = \frac{M_{12}(\theta)}{D}h_1(\theta, \dot{\theta}) - \frac{M_{11}(\theta)}{D}h_2(\theta, \dot{\theta}) - \frac{M_{12}(\theta)}{D}\tau_1 \quad (3)$$

where

$$D = M_{11}(\theta)M_{22}(\theta) - M_{12}^2(\theta)$$

Here, it is found that we can design partly stable controllers for link 1 and link 2 using the computed torque method. The controller 1 to stabilize the link 1 is given by

$$\tau_1 = \frac{D}{M_{22}(\theta)} \left( \ddot{\theta}_1^* + \frac{M_{22}(\theta)}{D}h_1(\theta, \dot{\theta}) - \frac{M_{12}(\theta)}{D}h_2(\theta, \dot{\theta}) \right) \quad (4)$$

$$\ddot{\theta}_1^* = \ddot{\theta}_{d1} + K_{v1}(\dot{\theta}_{d1} - \dot{\theta}_1) + K_{p1}(\theta_{d1} - \theta_1)$$

and the controller 2 to stabilize the link 2 is given by

$$\tau_2 = -\frac{D}{M_{12}(\theta)} \left( \ddot{\theta}_2^* - \frac{M_{12}(\theta)}{D}h_1(\theta, \dot{\theta}) + \frac{M_{11}(\theta)}{D}h_2(\theta, \dot{\theta}) \right) \quad (5)$$

$$\ddot{\theta}_2^* = \ddot{\theta}_{d2} + K_{v2}(\dot{\theta}_{d2} - \dot{\theta}_2) + K_{p2}(\theta_{d2} - \theta_2)$$

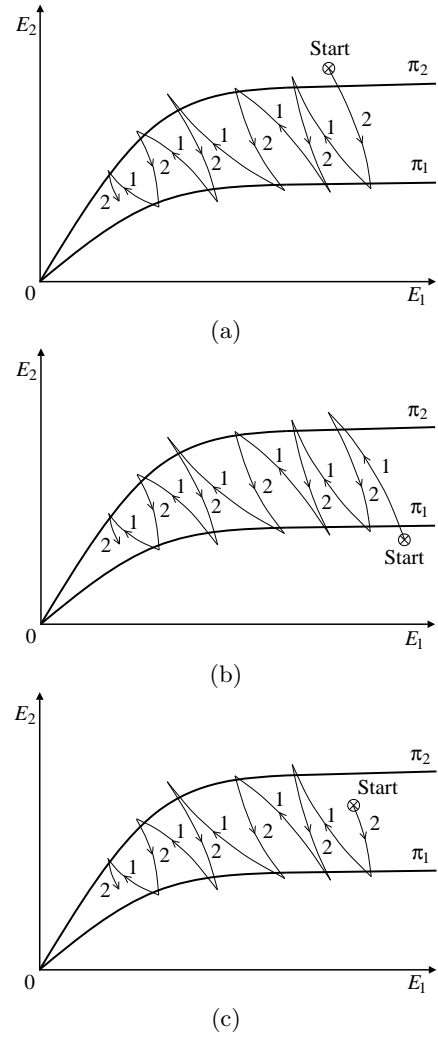


Fig. 3. Ideal convergence situations of switching method with two controllers

where the desired vector of  $\theta$  is defined as  $\theta_d = [\theta_{d1} \ \theta_{d2}]^T$ , in which the proportional gain of the controller  $i$  is  $K_{pi}$  and the derivative gain of the controller  $i$  is  $K_{vi}$ .

#### 3.2. Logic based switching method

Energy of each link is defined by

$$E_i \triangleq e_i^2 + \dot{e}_i^2, \quad i = 1, 2 \quad (6)$$

with

$$e_i = \theta_{di} - \theta_i$$

$$\dot{e}_i = \dot{\theta}_{di} - \dot{\theta}_i$$

The energy plane is composed of  $E_i$  as shown in Fig. 2. In Fig. 2,  $\pi_i$  is a boundary curve which determines the subregion of energy to use a partly stable controller, and is plotted by an exponential curve. The region  $R_i$  with gray shadow is the subregion to which the controller  $i$  is applied.

Ideal responses of energy are illustrated with Fig. 3, when assuming that  $E_i$  is decreased monotonously while the controller  $i$  is selected. In Fig. 3(a), the initial controller is the controller 2, because the initial point of energy locates at the subregion  $R_2$ . Even though  $E_2$  is decreased,  $E_1$  is

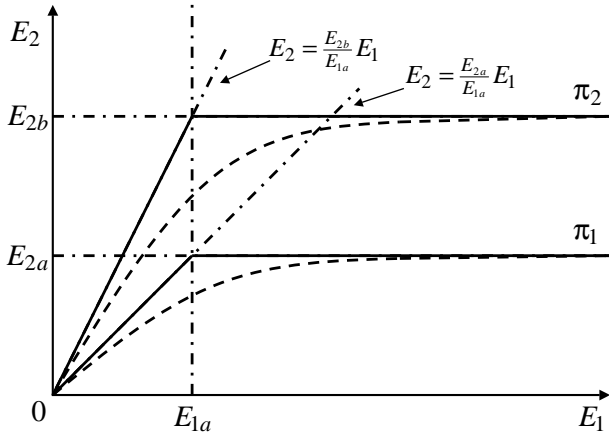


Fig. 4. Approximation of regions for a logical switching

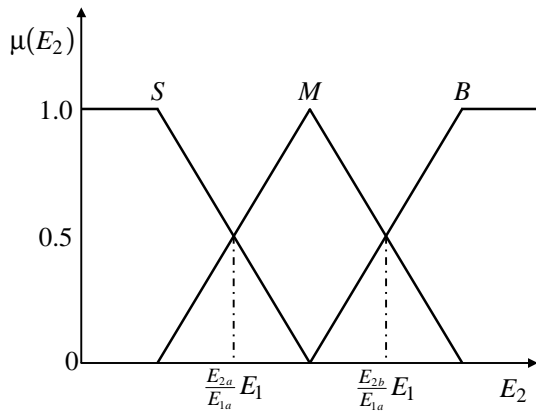


Fig. 5. Membership functions for  $E_1 \leq E_{1a}$

increased until the controller 1 will be selected, depending on the amount of current energy. When the controller 1 is selected, energy responses become the opposite situation. Consequently, each energy converges to zero with switching partly stable controllers. Fig. 3(b) is illustrated with the case when the initial point of energy locates at the subregion  $R_1$ . Fig. 3(c) is illustrated with the case when the initial point of energy locates at the overlapped region between  $R_1$  and  $R_2$ . It is assumed that the controller having the biggest index number has to be selected in the overlapped case. In this example, the controller 2 is selected.

### 3.3. Fuzzy energy region method

If a boundary curve comprises an exponential function, we can suitably design it with the amplitude and the time constant of a step-response. It is difficult to design such parameters of the function in advance, because we can't theoretically analyze them depending on the switching control. Therefore, we propose a fuzzy energy region based switching control method. At first, boundary curves are approximated by several straight-lines as shown in Fig. 4. After these approximations, fuzzy sets for  $E_2$  can be defined for  $E_1 \leq E_{1a}$  and  $E_1 > E_{1a}$  cases as shown in Fig. 5 and Fig. 6.  $E_{1a}$ ,  $E_{2a}$  and  $E_{2b}$  are the design parameters of fuzzy sets. In order to realize ideal energy responses, fuzzy rules are given as follows:

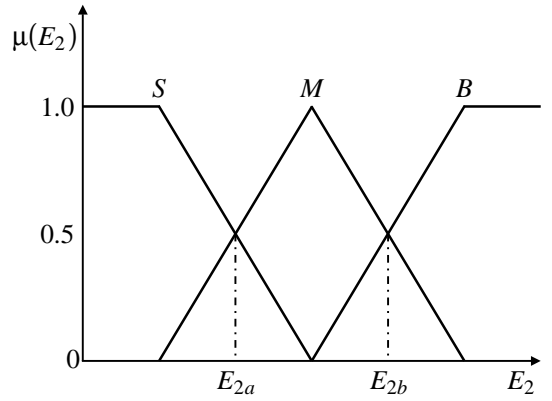


Fig. 6. Membership functions for  $E_1 > E_{1a}$

Table 1. GA operations and methods

GA operations	Method
Selection for crossover	Tournament strategy with 3 individuals
Crossover	Uniform crossover with probability 0.6
Probability of mutation	1/224
Alternation	Elite strategy with 10 individuals

- Rule 1 : If  $E_2 = S$  Then  $s_1 = 1$   
 Rule 2 : If  $E_2 = M$  and  $\phi_{t-1} = 1$  Then  $s_2 = 1$   
 Rule 3 : If  $E_2 = M$  and  $\phi_{t-1} = 2$  Then  $s_3 = 2$   
 Rule 4 : If  $E_2 = B$  Then  $s_4 = 2$

Note that, a parameter  $\phi_{t-1}$  which means the index of controller for one-step delay, is introduced, because one-step delayed controller must be retained in the overlapped energy region according to ideal energy response.  $s_i$  is the index of controller that must be used in the fuzzy rule  $i$ .

The advantage of the present method is to set design parameters roughly, comparing to the logic based switching method, because the boundary curves have fuzziness to use the present fuzzy reasoning.

## 4. Optimizing Fuzzy Energy Regions by GA

The present method has the same difficulty to design parameters in advance as the logic based switching method. Here, we discuss about the design parameters of fuzzy rules using GA. These parameters are  $E_{1a}$ ,  $E_{2a}$ ,  $E_{2b}$ ,  $K_{p1}$ ,  $K_{v1}$ ,  $K_{p2}$  and  $K_{v2}$ . Each parameter is encoded by 32 [bit], then the size of an individual is 224 [bit]. The searching domain of  $E_{1a}$ ,  $E_{2a}$  and  $E_{2b}$  is set from 0.1 to 15. The searching domain of  $K_{pi}$  and  $K_{vi}$ ,  $i = 1, 2$  is set from 0.01 to 100. Each parameter is decoded using gray code. The size of a population is 100. The maximum number of generations is 1000. GA operations used here are shown in Table 1.

A cost function is given by

$$f_c = \sum_{i=1}^2 \sum_{j=C_s}^{3000} \sum_{k=1}^2 E_k^i(j) \quad (7)$$

where  $i$  is the index of simulation trials,  $j$  is the index of dis-

Table 2. Setting parameters of simulations

Conditions	Setting value
Simulation time	30 [s]
Sampling interval	0.01 [s]
Mass of each link	$m_1 = 0.582$ [kg], $m_2 = 0.079$ [kg]
Length of each link	$l_1 = 0.4$ [m], $l_2 = 0.22$ [m]
Distance between center of gravity and each joint	$l_{g1} = 0.2$ [m] $l_{g2} = 0.11$ [m]
Coefficient of viscous friction of each joint	$\mu_1 = 0$ [Ns/m <sup>2</sup> ] $\mu_2 = 0.02$ [Ns/m <sup>2</sup> ]
Desired state vector	$[0 \ 0 \ 0 \ 0]^T$
1st initial state vector	$[0 \ \pi/4 \ 0 \ 0]^T$
2nd initial state vector	$[\pi \ \pi/6 \ 0 \ 0]^T$

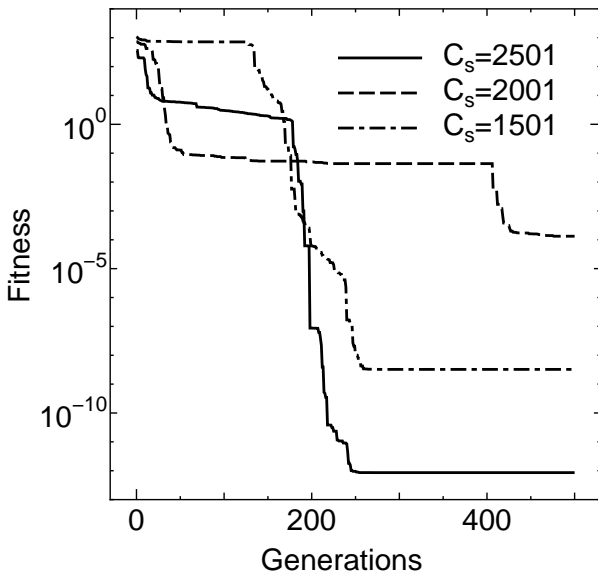


Fig. 7. Optimizing history of cost functions

crete times,  $k$  is the index of energy of each link and  $C_s$  is the starting index of discrete time to evaluate the response of an underactuated manipulator. Simulation conditions to train fuzzy parameters are shown in Table 2. Note that, the fitness function is not evaluated during a transition segment due to the dynamic characteristics of an underactuated system.

The training history in cost function is shown in Fig. 7. It is found from Fig. 7 that the case of  $C_s = 2501$  is smaller than other cases. Obtained parameters are shown in Table 3. Simulation results of each case using 1st training condition are shown in Figs. 8~10. Although cost functions are significantly different each other as shown in Fig. 7, whole response results are converged to the desired value. It is found from these results that the switching rule based on fuzzy energy regions has a certain robustness against for the design parameters of boundary curve and for the gains of partly stable controllers. In other words, we can design these parameters roughly using the present method.

Next, the obtained parameters in each  $C_s$  are applied to the case of untrained initial state vectors such as  $\theta(0) = [0 \ -\pi/4 \ 0 \ 0]^T$ . The responses of link angles of each  $C_s$  are

Table 3. Obtained design parameters of fuzzy energy region method

	$C_s = 2501$	$C_s = 2001$	$C_s = 1501$
$E_{1a}$	11.5097	12.6989	14.0461
$E_{2a}$	0.2906	0.2978	1.0417
$E_{2b}$	12.4336	14.9812	8.1119
$K_{p1}$	10.5593	5.2048	10.4885
$K_{v1}$	11.5647	5.7413	11.0411
$K_{p2}$	5.2400	2.6096	24.2865
$K_{v2}$	56.3331	50.7481	4.0119

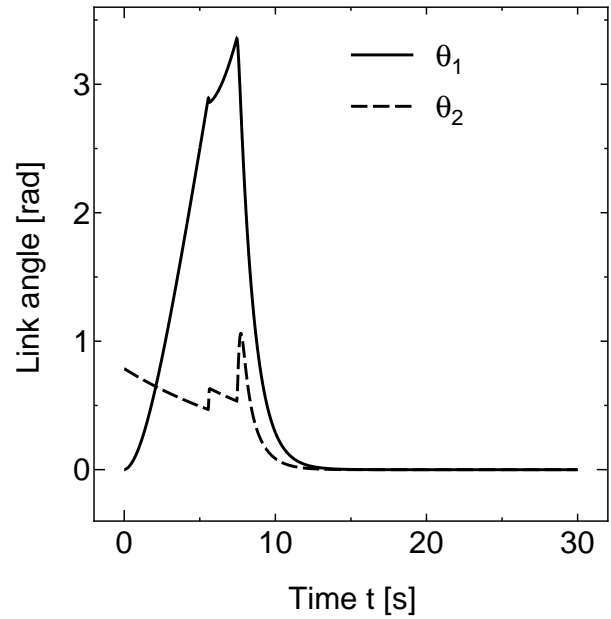


Fig. 8. Results of 1st training data by  $C_s = 2501$

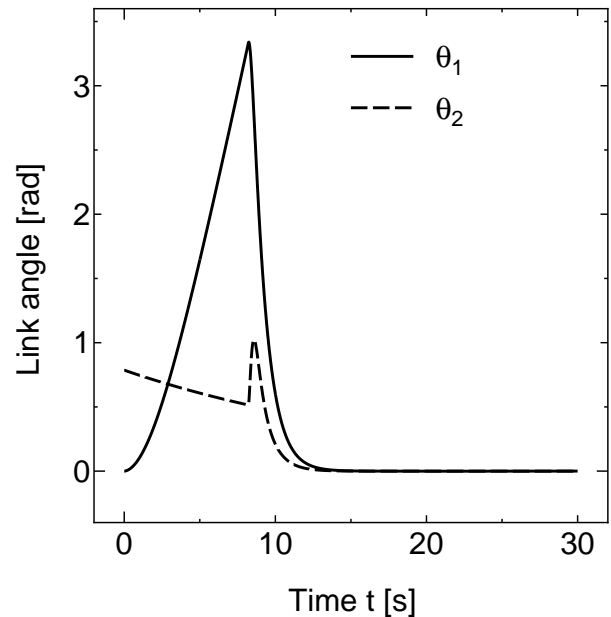


Fig. 9. Results of 1st training data by  $C_s = 2001$

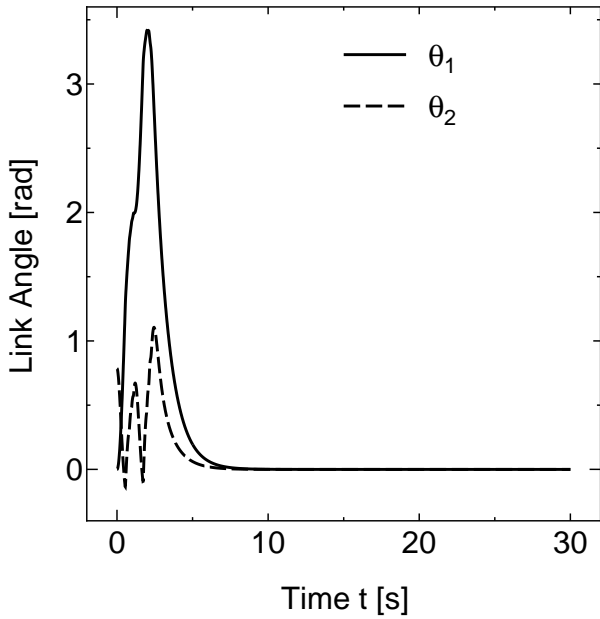


Fig. 10. Results of 1st training data by  $C_s = 1501$

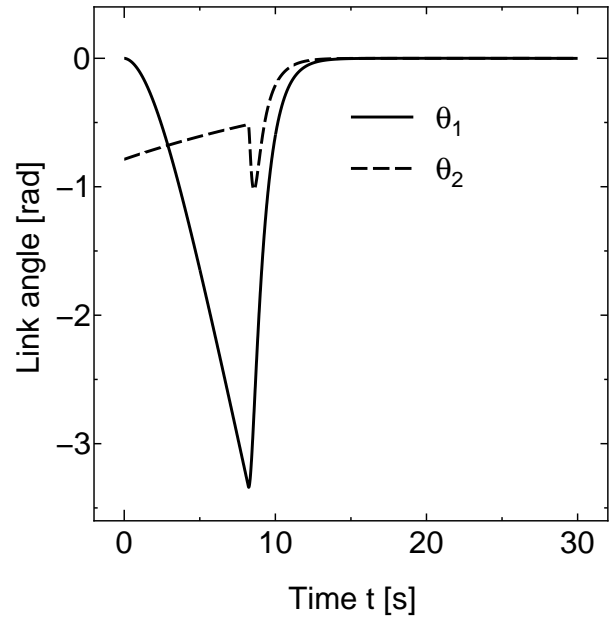


Fig. 12. Results of untrained data by  $C_s = 2001$

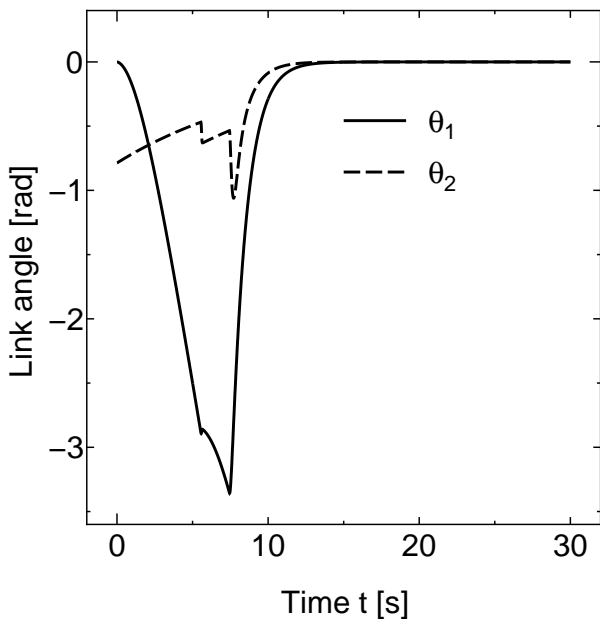


Fig. 11. Results of untrained data by  $C_s = 2501$

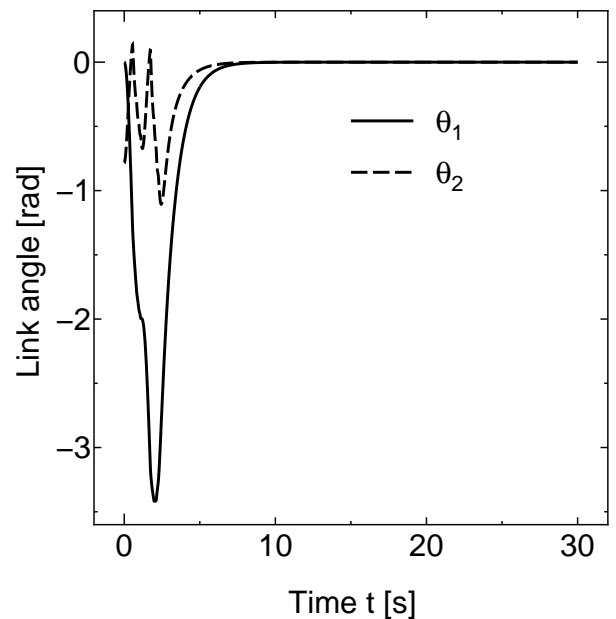


Fig. 13. Results of untrained data by  $C_s = 1501$

shown by Figs. 11~13. Despite of the untrained condition, all response results are converged to the desired value. From these results, it is considered that the present method finds out a certain attractor. The difference of cost function reflects on the time to find out the attractor, because the peak of response in Fig. 13 appears in more left-hand side than these in Fig. 11 and Fig. 12, while the peaks in Fig. 11 and 12 are almost same.

### 5. Conclusions

We have proposed a logic based switching method using fuzzy energy region, in which fuzzy design parameters and gains of partly stable controllers were trained by genetic al-

gorithm. A cost function was tried to use in optimizing parameters. The proposed method performed well in whole simulations. Using the proposed method, we can find out a certain attractor of underactuated robot systems. In future works, we consider the relationship between the attractor and the boundary curve of energy region.

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