# An analysis on the Earth geoid surface variation effect for use of the tilt sensor in celestial navigation system.

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**Abstract**: The celestial navigation is one of alternatives to GPS system and can be used as a backup of GPS. In the celestial navigation system using more than two star trackers, the vehicle's ground position can be solved based on the star trackers' attitude information if the vehicle's local vertical or horizontal angle is given. In order to determine accurate ground position of flight vehicle, the high accurate local vertical angle measurement is one of the most important factors for navigation performance. In this paper, the Earth geophysical deflection was analyzed in the assumption of using the modern electrolyte tilt sensor as a local vertical sensor for celestial navigation system. According to the tilt sensor principle, the sensor measures the tilt angle from gravity direction which depends on the Earth geoid surface at a given position. In order to determine the local vertical angle from tilt sensor measurement, the relationship between the direction of gravity and the direction of the Earth center should be analyzed. Using a precision orbit determination software which includes the JGM-3 Earth geoid model, the direction of the Earth center and the direction of gravity are extracted and analyzed. Appling vector inner product and cross product to the both extracted vectors, the magnitude and phase of deflection angle between the direction of gravity and the direction of the Earth center are achieved successfully. And the result shows that the angle differences vary as a function of latitude and altitude. The maximum 0.094° angle difference occurs at 45° latitude in case of 1000 Km altitude condition.

**Keywords: Celestial navigation, JGM-3, geoid, tilt sensor, local vertical, gravity**.

## 1. INTRODUCTION

The celestial navigation is one of alternatives to GPS in the event of GPS loss or denial. Beside of the great success and widespread use of GPS, the vulnerabilities of the system are widely acknowledged. [1]

In the celestial navigation system using star tracker, the measurement of the direction of two or more stars with respect to a vehicle-fixed coordinate system provides an instantaneous determination of the vehicle's attitude with respect to the celestial reference frame. If there is also a determination of the direction of the local vertical with respect to the same coordinate system, the vehicle's attitude with respect to the local horizon system can be obtained. The determination of the local vertical is not trivial for a moving vehicle and in general will require corrections for coriolis forces and geophysical deflection. The local vertical provides the direction orthogonal to the geoid and, appropriately corrected, toward the center of the Earth. [1]

In this paper, the Earth geophysical deflection was analyzed in the assumption of using the modern electrolyte tilt sensor as a local vertical measurement sensor for celestial navigation system. According to the tilt sensor principle, the sensor measures the tilt angle from gravity direction which is orthogonal to the Earth geoid surface at a given position. In order to determine the direction of center of the Earth from local gravity direction, the relationship between the direction of gravity and the direction of the Earth center was analyzed.

## 2. ANALYSIS METHOD

The gravitational potential can be expressed in Eq. (1) with a point mass term and two perturbation model terms divided into zonal and tesseral harmonic.

$$U(r,l,L) = \frac{GM}{r} + U_Z(r,l) + U_t(r,l,L), \qquad (1)$$

Where r, l, L is radius, latitude and longitude respectively.

The zonal and tesseral harmonic parts express the Earth geoid surface variation as a function of latitude and longitude. In Eq. (1), the GM/r term means the potential of the Earth as a point mass. And the output resulted from Eq. (1) means the total potential including the point mass term and the effects of zonal and tesseral gravitational perturbation. Hence the direction of the Earth center can be obtained by GM/r term only and the direction of actual gravity can be obtained with the total potential resulted from Eq. (1).

The circular orbit, 90° inclination angle, and $\Omega = 0$ is selected to cover the entire Earth surface. And the simulation was performed in 10, 100, 200, 500, 1000Km altitude cases during one day to investigate the effect of altitude change.

The orbit determination software program which was used for this analysis was evaluated and proven with NASA POE (Precision Orbit Ephemeris) generated by global laser tracking. The position and velocity accuracy was estimated about 16 ~ 7m and 0.0157 ~ 0.0074 $m \cdot s^{-1}$ RMS respectively.[2] Since the orbit determination software includes the JGM-3 of the Earth geoid model, the direction of the Earth center and the direction of gravity are extracted as realistic as possible.

## 3. DATA ANALYSIS

The two vectors extracted during simulation are the center of the Earth vector and the actual gravity vector. To investigate the magnitude of differences between both vectors, vector inner product is used.

### 3.1 The magnitude of deflection angle.

Fig. 1 shows the data come from vector inner product and it means magnitude of deflection angle between the center of the Earth and gravity vector as a function of latitude in several altitude cases.

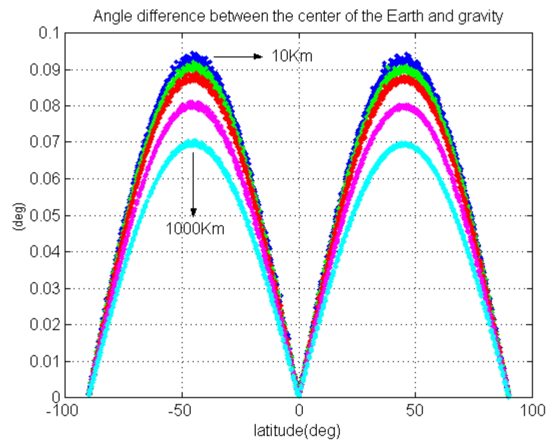


Fig. 1 The magnitudes of deflection angle between the center of the Earth and gravity vector.

In Fig.1 the magnitudes of deflection angle shows a kind of sinusoidal pattern as a function of latitude. The maximum value occurs at 45° latitude about 0.094° and 0.07° in 10Km and 100Km altitude cases respectively. The maximum deflection angle is decreased according to increasing altitude. And each data has certain range of variation at the same latitude point and this is caused by the local geoid surface variations. Fig. 2 represents a magnification of 1000Km data at 45° latitude point and shows certain range of variation in 1000Km case.

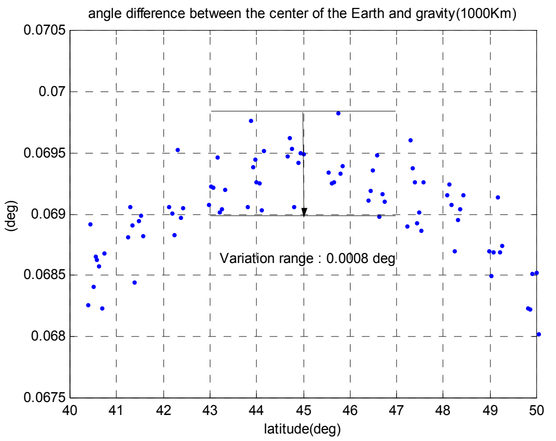


Fig. 2 The range of variation at 45° latitude (1000Km).

Table 1 Summary of deflection angle.(arc-second)

| | 10Km | 100Km | 200Km | 500Km | 1000Km |
|---|---|---|---|---|---|
| Max value (at 45° lat) | 338.4 | 331.2 | 320.4 | 291.6 | 252 |
| Range of Variation (at 45° lat) | 14.4 | 7.2 | 5.04 | 3.6 | 2.88 |

The range of variation decreases according to increasing altitude. The maximum deflection angle and range of variation at 45° latitude position in each altitude cases are summarized in table 1.

Fig. 3 is a diagram expression of the deflection angle at 45° latitude position with several altitude conditions. The origin of the diagram represents the direction of the Earth center and the radial direction means the gravity direction. The radial distance of the diagram represents the amount of deflection angle in each altitude cases. The diagram includes only three altitude data such as 1000, 500 and 10Km. Each data at different altitude has outer circle and inner circle, outer circle represents the maximum deflection angle and the distance between outer circle and inner circle represents the range of variation.

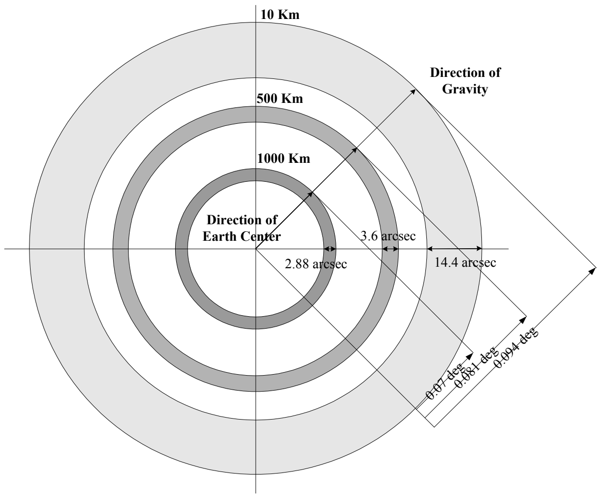


Fig. 3 A diagram expression of the deflection angle at 45° latitude.

### 3.2 The phase of deflection angle.

The magnitude of deflection angle informs the absolute error angle between the direction of the center of the Earth and the direction of gravity. However it does not tell the phase of the deflection angle. In other words, the radial distance of circles in Fig. 3 informs the absolute deflection angle but not specify exact direction of the actual gravity vector. In order to find the actual gravity vector, vector cross product method was applied.

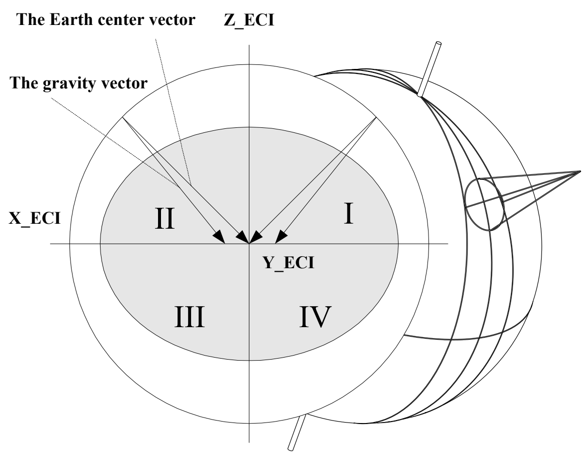


Fig. 4 The selected orbit and geometry.

Since the simulation orbit is selected as the $\Omega = 0$, the orbit plane normal is parallel to the ECI Y axis. After cross product, the result vector was compared with ECI Y axis using vector inner product again.

Fig. 4 shows the relationship with selected orbit plane and the ECI coordinate frame. The ECI Y axis is the direction of out of paper. If the gravity vector and the Earth center vector are on the $XZ_{ECI}$ plane, the output vector resulted from cross product will be coincident to ECI +Y or –Y axis. Also Fig. 4 represents that the output vector of cross product performed in upper-right corner(I) becomes 180° out of phase compared with the output vector performed in upper-left(II) corner. It is because the gravity vector aligns to the Earth center vector like as Fig. 4 geometrically. And the situation of the southern hemisphere (III, IV) is similar to the northern hemisphere(I, II). The Eq. (2) shows actual order of vector cross product performed.

$$output = gravity\_vector \times Earth\_center\_vector \qquad (2)$$

After cross product, the output vector was compared with ECI Y axis to investigate the amount of differences using vector inner product again. Fig. 5 shows the difference angle between output vector of cross product and the ECI Y axis in 1000Km altitude case.

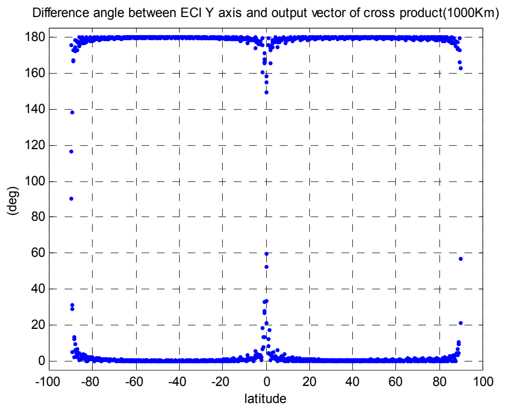


Fig. 5 The difference angle between ECI Y axis and the output vector of cross product. (1000Km)

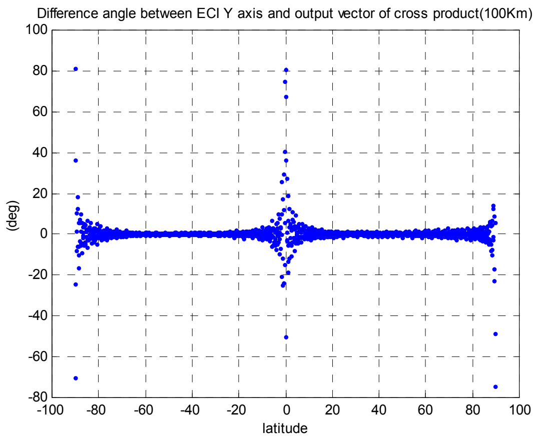


Fig. 6 The difference angle between ECI Y axis and the output vector of cross product. (100Km) (after phase conversion)

In Fig. 5, most of the data values are nearly 0° or 180° except at Poles and Equator. In Poles and Equator, the difference angle values are distributed randomly. And that means the actual gravity vector aligns almost toward the north-south direction with small deviation along every latitudes except around of Poles and Equator. According to the data, around the Poles and Equator, the phase of deflection angle between gravity vector and the center of the Earth vector is diverse.

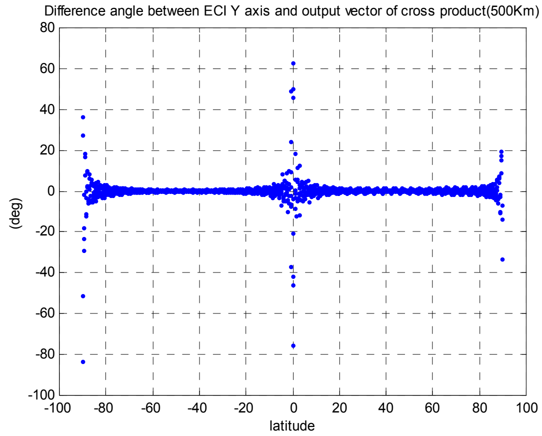


Fig. 7 The difference angle between ECI Y axis and the output vector of cross product. (500Km) (after phase conversion)

The data value of 180° means the output vector of cross product aligns with ECI –Y axis. Therefore it was converted into near 0° value by subtracting -180° for proper data analysis. Fig. 6 shows the result data in 100Km case and Fig. 7 shows the results data in 500Km cases after 180° phase conversion. Fig. 8 is a magnification of Fig. 7 at 45° latitude position and shows +/- 1° range maximum at 1000Km.

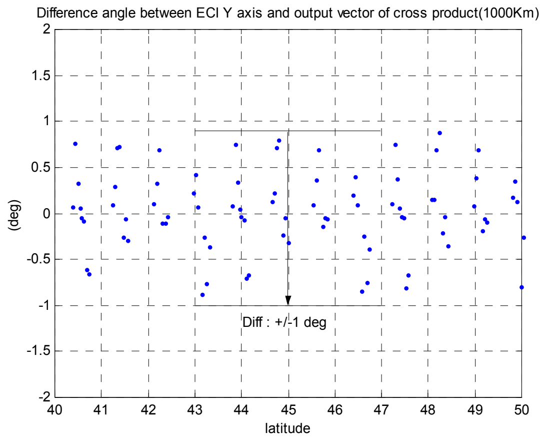


Fig. 8 Zoom in at 45° latitude of 1000Km case.

In order to visualize, it is possible to draw the Fig. 9 with data of magnitude and phase of deflection angle. Fig. 9 also represents the ground distance error based on the magnitude and phase of deflection angle in 1000Km altitude case. The magnitude (maximum absolute angle) between the gravity and the center of the Earth vector shows 252arcsecond (0.07°) at 1000Km case in Table 1. And the phase of deflection angle shows +/- 1° ranges in Fig 8 at the same altitude. According to the magnitudes and phase of deflection angle, we can draw the diagram such as Fig 9 which represents the ground track errors. The origin is the direction of the Earth center and the outer circle indicates possible points of intersection between gravity vector and the Earth surface viewing at 1000Km altitude with $0.07^o$ magnitude of deflection angle. And based on the phase of deflection angle is +/- $1^o$ ranges, the possible points of intersection could be limited upper circular arc and lower circular arc such as figure. The range of upper circular arc represents the deflection angle occurred at II and IV region as mentioned in Fig. 4. And also the lower circular arc represents the deflection angle occurred at I and III region.

In the figure, it is found that major deflection angle is toward the north-south direction at every latitudes except the around of Poles and equator. And also figure shows, if a proper compensation is applied, the ground track error minimize such as inner circle boundary.

If the north-south component in gravity vector is compensated by subtracting amounts equal to $0.07^o$ of magnitude angle, the outer circle (12.2Km) could be limited into inner circle boundary (21.3m). In other sides, it shows that a certain compensation method could be applied to the gravity vector (measured by tilt sensor) to find out the vector of the Earth center more accurately.

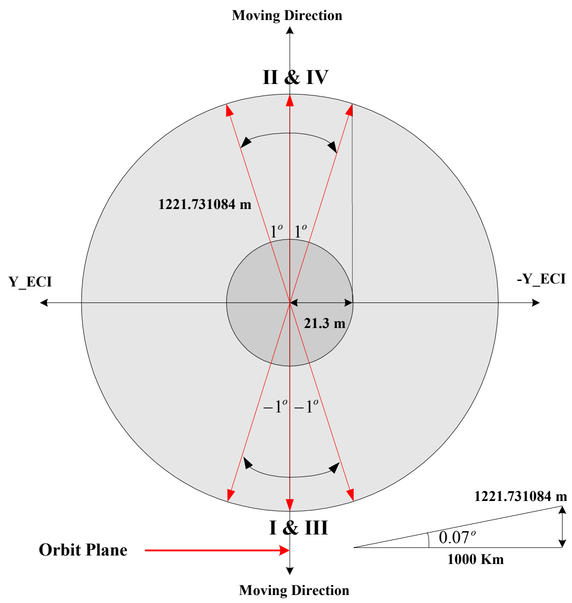


Fig. 9 The deflection angle vs. ground distance error (1000Km)

In 10Km altitude case, the same diagram as Fig. 9 could be drawn. The diameter of outer circle changes to 16.4m based on the $0.094^o$ magnitude of deflection angle. And the distribution of phase of deflection angle becomes larger as +/- $2^o$ range. The results show no more compensation is required in 10Km altitude case. And in summary, the ground track error increase according to increasing of altitude.

In the Poles and Equators the situation is much different. Based on the Fig. 1 and Fig. 5, they show the magnitude of deflection angle in the around of Poles and equator are very small (maximum $0.002^o$) and the phase of deflection angle are very diverse. Based on the magnitude of deflection angle, the ground track error is limited in the circle of 34.9m diameter without any compensation. The phase of deflection angle is very diverse but the magnitude of deflection angle is small enough to avoid certain compensation.

The analysis shows that a proper method of compensation is required for certain latitude and altitude range for high accurate determination of the direction of the Earth center. In the future, a proper method of compensation will be studied in the altitude and latitude point of view.

## 4. TILT SENSOR

There are several kinds of tilt sensors in the operational principle. Semiconductor based MEMS tilt sensor operates based on natural gas convection.[3][4] A conventional pendulum tilt sensor is introduced and has very high accuracy with optical detection technology.[5] In this paper, the electrolytic tilt sensor is assumed to be used as a local vertical sensor in celestial navigation system. The performance of modern electrolytic tilt sensor shows about more than several arc-second resolutions and several hundred ms time response. And it's performance has been greatly improved in condition of the wide temperature range using a compensation equations and also has been shorten in time response.[6] Besides of theses performance improvements, still there are several factors to be considered severely. One of theses, the electrolytic tilt sensor could not distinguish the actual tilt from effects of acceleration forced on the measurement axis. And the performance of sensor depends on the several conditions such as temperature and acceleration when the measurements are performed if no compensation is applied.

In spite of several weaknesses, the electrolytic tilt sensor has been investigated and it has great possibility to be used as a local vertical sensor because of its high resolution with simple mechanism and relative low price compared to other high resolution sensor such as accelerometers.

## 5. CONCLUSION

In the assumption of using the electrolytic tilt sensor as a vertical sensor for celestial navigation, the Earth geophysical deflection was studied and analyzed. The magnitude and phase of deflection angle between gravity vector and the Earth center vector was extracted and analyzed using precision orbit determination software successfully. In the future, a proper compensation method will be studied to obtain high accurate determination of the direction of the Earth center at any positions.

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