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Dynamics of an Axially Moving Thermoelastic Beam-plate

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Key Words : Thermoelastic Beam-plate(열탄성 보-평판), Moving Beam(이동하는 보), Spectral Element Model(스펙트럴요소 모델)

ABSTRACT

For accurate prediction of the thermal shock-induced vibrations, this paper develops a spectral element model for axially moving thermoelastic beam-plates. The spectral element model is formulated from the frequency-dependent dynamic shape functions which satisfy the governing equations in the frequency-domain. Some numerical studies are conducted to evaluate the present spectral element model and also to investigate the vibration characteristics of an example axially moving beam-plate subjected to thermal loadings.

1. INTRODUCTION

Dynamic characteristics of various types of structural elements subjected to thermal loading (heat) become increasingly important during last half century due to their many applications in diverse engineering fields. When a sudden thermal loading is applied to a structure, a very rapid thermal process may occur to induce very rapid movements in the structure, thus causing the structure to vibrate. The thermally induced vibrations may be encountered, for example, in the high-speed modern aircrafts subjected to aerodynamic heating, the nuclear reactors in extremely high-temperature and temperature gradient environment, the high-speed propulsion units, and the galvanized steel sheets passing through a hot zinc tank.

The thermally induced vibration of beams subjected to suddenly applied heat flux distributed along its span was studied by Boley [1]. Since then, numerous studies have been conducted for various thermoelastic structures [2-5]. The existing previous studies on the thermally induced vibration have been focused mostly on the stationary thermoelastic structures. To the authors' best knowledge, the dynamics of axially moving thermoelastic structures has not been investigated yet. Furthermore, the spectral element method (SEM) [6] has not been applied to solve such moving thermoelastic structure problems. Thus, the purpose of this paper is to develop a spectral element model for axially moving thermoelastic beam-plates.

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2. DERIVATION OF GOVERNING EQUATIONS

2.1 Equations of motion

Consider a thin beam-plate is moving in the axial x (axial) direction at a moving speed of c. The beam-plate has the thickness h and width b. The material properties of the beam-plate are given by the Young's modulus E and Poisson's ratio ν. Assume that the beam-plate has a small amplitude vibration: $w(x,t)$ is the displacement of the mid-plane of the beam-plate in the z direction and $u(x,t)$ in the x direction. The equations of motion and the relevant boundary conditions of the beam-plate can be derived from the Hamilton's principle:

$$\int_{t_1}^{t_2} (\delta K - \delta P + \delta W)dt = 0 \tag{1}$$

The stain energy P and kinetic energy K are given by

$$\begin{aligned} P &= \frac{1}{2}\int_0^L \left(D w''^2 + \overline{EA}u'^2 + M_T w'' - N_T u'\right)dx \\ K &= \frac{\rho}{2}\int_0^L \left\{A(c+\dot{u})^2 + A(\dot{w}+cw')^2 + I\dot{w}'^2\right\}dx \end{aligned} \tag{2}$$

where L is the span between two simple supports, $A = bh$ $D = EI/(1-\nu^2)$, $I = bh^3/12$, and

$$\begin{aligned} \overline{EA} &= \frac{EA}{1-\nu^2} \\ M_T(x,t) &= \frac{E\alpha b}{1-\nu}\int_{-h/2}^{h/2} \Delta T(x,z,t)z\,dz \\ N_T(x,t) &= \frac{E\alpha b}{1-\nu}\int_{-h/2}^{h/2} \Delta T(x,z,t)dz \end{aligned} \tag{3}$$

where M_T and N_T are the thermal moment and the thermal (axial) force, respectively, and $\Delta T(x,z,t)$ is the difference between the absolute temperature $T(x,z,t)$ and the reference absolute temperature T_0. The virtual work is given by

$$\begin{aligned} \delta W = &\int_0^L \{p_x(x,t)\delta u(x,t) + p_z(x,t)\delta w(x,t)\}dx \\ &+ N_1(t)\delta u_1(t) + N_2(t)\delta u_2(t) + M_1(t)\delta\phi_1(t) \\ &+ M_2(t)\delta\phi_2(t) + V(t)_1\,\delta w_1(t) + V_2(t)\delta w_2(t) \end{aligned} \tag{4}$$

where $p_x(x,t)$ and $p_z(x,t)$ are the distributed loads acting on the beam-plate in the x and z directions, respectively. M_i, V_i and N_i (i=1, 2) represent the boundary moments, transverse shear forces and axial forces applied at $x = 0$ and $x = L$, respectively. The transverse displacements, the axial displacements and the slopes $\phi = \partial w/\partial x$ at the boundaries are defined by

$$\begin{aligned} w_1(t) &= w(0,t)\ , \quad w_2(t) = w(L,t) \\ \phi_1(t) &= w'(0,t)\ , \quad \phi_2(t) = w'(L,t) \\ u_1(t) &= u(0,t)\ , \quad u_2(t) = u(L,t) \end{aligned} \tag{5}$$

Substituting Eqs. (2) and (4) into Eq. (1) and integrating by parts yields the equations of motion as

$$\begin{aligned} &\overline{EA}u'' - \rho A\ddot{u} = -p_x(x,t) + N_T'/2 \\ &Dw'''' + \rho Ac^2 w'' + 2\rho Ac\dot{w}' - \rho I\ddot{w}'' + \rho A\ddot{w} = p_z(x,t) - M_T''/2 \end{aligned} \tag{6}$$

and the boundary conditions as

$$\begin{aligned} N(0,t) &= -N_1(t) \quad \text{or} \quad u(0,t) = u_1(t) \\ N(L,t) &= N_2(t) \quad \text{or} \quad u(L,t) = u_2(t) \\ M(0,t) &= -M_1(t) \quad \text{or} \quad \phi(0,t) = \phi_1(t) \\ M(L,t) &= M_2(t) \quad \text{or} \quad \phi(L,t) = \phi_1(t) \\ V(0,t) &= -V_1(t) \quad \text{or} \quad w(0,t) = w_1(t) \\ V(L,t) &= V_2(t) \quad \text{or} \quad w(L,t) = w_2(t) \end{aligned} \tag{7}$$

2.2 Heat conduction equation

The temperature field $T(x,z,t)$ or $\Delta T(x,z,t)$ is governed by the heat conduction equation, which can be derived from the law of energy conservation written as

$$\begin{aligned} &-k\left(T'' + T^{\circ\circ}\right) + \rho c_p cT' + \left(T_0\alpha^2 E_v + \rho c_p\right)\dot{T} \\ &\qquad + \left(T_0\alpha E/1-\nu\right)\left(\dot{u}' - z\dot{w}'' + w'\dot{w}'\right) = 0 \end{aligned} \tag{8}$$

where α is the coefficient of thermal expansion, c_p is the specific heat at the constant strain, k is the thermal conductivity of the medium, and $E_v = E(1+\nu)/(1\text{-}2\nu)(1\text{-}\nu)$. The symbol circle (°) denotes the derivative with respective to the coordinate. Assume that the beam-plate is subject to the thermal loads applied only on the top or bottom surface of the beam-plate. Due to the geometry of the beam-plate, one may assume the temperature as the function of only z and t to simplify Eq. (8) as

$$kT^{\circ\circ} - \left(T_0\alpha^2 E_v + \rho c_p\right)\dot{T} = 0 \tag{9}$$

Once the proper thermal boundary conditions are specified for a given problem, one can readily solve Eq. (8) for $T(z, t)$ and then apply the solutions into Eq. (6) to estimate the thermal-induced vibration of a beam-plate.

3. SPECTRAL ELEMENT FORMULATION

Based on the DFT theory, assume the solutions of Eq. (6) in the spectral forms [7] as

$$u(x,t) = \sum_{n=0}^{N-1} U_n(x)\, e^{i\omega_n t}, \quad w(x,t) = \sum_{n=0}^{N-1} W_n(x)\, e^{i\omega_n t} \tag{10}$$

where $U_n(x)$ and $W_n(x)$ (n = 0, 1,..., N–1) represent the spectral components of $u(x, t)$ and $w(x, t)$, respectively. Similarly, represent the external loads and thermal loads into the spectral forms. Substituting Eq. (10) and all other spectral representations into Eq. (6) gives

$$\begin{aligned} &\overline{EA}U_n'' + \rho A\omega_n^{\,2} U_n = F_{xn} \\ &DW_n'''' + \left(\rho Ac^2 + \rho I\omega_n^{\,2}\right)W_n'' + 2i\rho Ac\omega_n W_n' - \rho A\omega_n^{\,2} W_n = F_{zn} \end{aligned} \tag{11}$$

where

$$F_{xn}(x) = -P_{xn} + N_{Tn}'/2, \quad F_{zn}(x) = P_{zn} - M_{Tn}''/2 \tag{12}$$

The spectral element formulation begins with the governing equations without the external forces [6, 7]. Thus, the general solutions of the homogeneous differential equations reduced from Eq. (11) can be assumed as follows:

$$U_n(x) = A_n\, e^{\kappa_n x}, \qquad W_n(x) = B_n\, e^{\lambda_n x} \tag{13}$$

where κ_n and λ_n denote the wavenumbers for the axial and transverse vibration modes, respectively. Substituting Eq. (13) into the homogeneous differential equations reduced from Eq. (11) may yield two dispersion relations, from which two and four wavenumbers can be obtained for the axial and transverse vibration modes, respectively. By using these wavenumbers, the general solutions Eq. (13) can expressed in the forms as

$$U_n(x) = \left[\boldsymbol{E}_{Un}(x;\omega_n)\right]\{\boldsymbol{C}_n\}\ , \quad W_n(x) = \left[\boldsymbol{E}_{Wn}(x;\omega_n)\right]\{\boldsymbol{C}_n\} \tag{14}$$

where $\{\boldsymbol{C}_n\}$ is the (6 by 1) constant vector to be determined by boundary conditions.

Now, consider a finite beam-plate element of length L. The spectral nodal DOFs are defined by

$$\begin{aligned} U_{n1} &= U_{n1}(0),\ W_{n1} = W_{n1}(0),\ \Phi_{n1} = W_{n1}'(0) \\ U_{n2} &= U_{n2}(L),\ W_{n2} = W_{n2}(L),\ \Phi_{n2} = W_{n2}'(L) \end{aligned} \tag{15}$$

Applying Eq. (13) to Eq. (14) may yield a relationship between the spectral nodal DOFs vector $\{\boldsymbol{d}_n\}$ and the constant vector $\{\boldsymbol{C}_n\}$ as follows:

$$\{\boldsymbol{d}_n\} = \left[\boldsymbol{X}_n(\omega_n)\right]\{\boldsymbol{C}_n\} \tag{16}$$

where

$$\{\boldsymbol{d}_n\} = \{U_{n1} \quad W_{n1} \quad \Phi_{n1} \quad U_{n2} \quad W_{n2} \quad \Phi_{n2}\}^{\mathrm{T}} \tag{17}$$

One can eliminate the constant vector $\{\boldsymbol{C}_n\}$ from Eq. (14) by using Eq. (16) to obtain

$$\begin{aligned} U_n(x) &= \left[\boldsymbol{E}_{Un}\right]\left[\boldsymbol{X}_n\right]^{-1}\{\boldsymbol{d}_n\} \equiv \left[\boldsymbol{N}_{Un}(x;\omega_n)\right]\{\boldsymbol{d}_n\} \\ W_n(x) &= \left[\boldsymbol{E}_{Wn}\right]\left[\boldsymbol{X}_n\right]^{-1}\{\boldsymbol{d}_n\} \equiv \left[\boldsymbol{N}_{Wn}(x;\omega_n)\right]\{\boldsymbol{d}_n\} \end{aligned} \tag{18}$$

where $[\boldsymbol{N}_{Un}]$ and $[\boldsymbol{N}_{Wn}]$ are the dynamic (frequency-dependent) shape function matrices.

In the following, the variational approach [8] is used to formulate the spectral element matrix by using the displacement fields given by Eq. (18) as well as the temperature field which will be given in the next section. The weak form statements of the original governing equations, Eq. (11), are given by

$$\int_0^l \left(\overline{EA}U_n'' + \rho A\omega_n^2 U_n - F_{xn}\right)\delta U_n \, dx = 0$$

$$\int_0^l \left\{DW_n'''' + \left(\rho Ac^2 + \rho I\omega_n^2\right)W'' + 2i\rho Ac\omega_n W_n' - \rho A\omega_n^2 W_n - F_{zn}\right\}\delta W_n \, dx = 0 \qquad (19)$$

By substituting the loading terms of Eq. (12) into Eq. (18) and integrating by parts, one may obtain

$$[S_n(\omega)]\{d_n\} = \{f_n\} \qquad (20)$$

where $[S_n(\omega)]$ is the frequency-dependent spectral element matrix defined by

$$\begin{aligned}[S_n(\omega)] &= \int_0^l \left\{\overline{EA}[N_{Un}']^T[N_{Un}'] - \rho A\omega_n^2[N_{Un}]^T[N_{Un}]\right\}dx \\ &+ \int_0^l \left\{D[N_{Wn}'']^T[N_{Wn}''] - \left(\rho Ac^2 + \rho I\omega_n^2\right)[N_{Wn}']^T[N_{Wn}'] \right. \\ &\left. + i\rho Ac\omega_n\left([N_{Wn}]^T[N_{Wn}'] - [N_{Wn}']^T[N_{Wn}]\right) - \rho A\omega_n^2[N_{Wn}]^T[N_{Wn}]\right\}dx\end{aligned} \qquad (21)$$

and $\{f_n\}$ is the spectral nodal forces defined by

$$\begin{aligned}\{f_n\} &= \{N_{1n}\ V_{1n}\ M_{1n}\ N_{2n}\ V_{2n}\ M_{2n}\}^T \\ &+ \int_0^l P_{xn}(x)[N_{Un}]^T dx + \int_0^l P_{zn}(x)[N_{Wn}]^T dx \\ &- \frac{1}{2}\left[N_{Tn}(l)N_{Un}(l) - N_{Tn}(0)N_{Un}(0)\right]^T \\ &+ \frac{1}{2}\left[M_{Tn}(l)N_{Wn}'(l) - M_{Tn}(0)N_{Wn}'(0)\right]^T\end{aligned} \qquad (22)$$

All spectral elements can be assembled in a completely analogous way to that used in the conventional FEM.

The temperature filed is governed by Eq. (9) and it can be solved in the spectral form as

$$T(z,t) = \sum_{n=0}^{N-1} T_n(z)e^{i\omega_n t} \quad \text{with} \quad T_n(z) = B_{n1}e^{-\tau_n z} + B_{n2}e^{\tau_n z} \qquad (23)$$

and

$$\tau_n = \beta\sqrt{i\omega_n}\,\beta = (1+i)\beta\sqrt{0.5\omega_n} \qquad (24)$$

The constants B_{n1} and B_{n2} are determined by the thermal boundary conditions specified on the upper and lower surfaces of beam-plate. Once the spectral components of temperature T_n are computed from (23), the corresponding spectral components of the thermal moment M_T and the thermal force N_T in Eq. (22) can be readily computed from Eqs. (3) and (12).

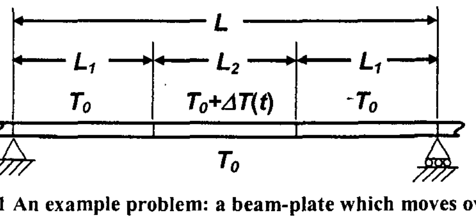

Fig. 1 An example problem: a beam-plate which moves over two simple supports

4. NUMERICAL RESULTS AND DISCUSSION

As an illustrative example problem, a beam-plate which is axially moving over the two simple supports of distance $L = 2\ m$ is considered. The beam-plate has the thickness $h = 5mm$, width $b = 0.5\ m$, Young's modulus $E = 73GPa$, Poisson's ratio $\nu = 0.33$, mass density $\rho = 2770\ kg/m^3$, thermal expansion coefficient $\alpha = 23.0\times10^{-6}/K$, thermal conductivity $k = 177\ W/mK$, and the specific heat $c_p = 875\ J/kg{\cdot}K$. As shown in Fig. 1, the temperature change is applied only on the middle part of the upper surface while the other parts are kept having the room temperature T_0.

Table 1. Natural frequencies(Hz) of the beam-plate obtained by the present SEM, FEM and the exact theory (Blevins [9])

Fluid Velocity (*m/s*)	Method	N	$\omega_1^{(w)}$	$\omega_2^{(w)}$	$\omega_3^{(w)}$	$\omega_4^{(w)}$	$\omega_5^{(w)}$	$\omega_{15}^{(u)}$
0	Exact[9]	·	3.083	12.329	27.743	49.321	77.066	679.78
	SEM	1	3.083	12.329	27.742	49.317	77.057	679.78
	FEM	10	3.083	12.331	27.758	49.403	77.371	680.48
		50	3.083	12.329	27.743	49.322	77.067	679.81
		100	3.083	12.329	27.743	49.322	77.067	679.79
8	SEM	1	2.248	11.790	27.277	48.889	76.649	679.78
	FEM	10	2.248	11.792	27.296	48.983	76.982	680.48
		50	2.248	11.790	27.279	48.893	76.660	679.81
		100	2.248	11.790	27.279	48.893	76.659	679.79
12.33	SEM	1	0.0	11.012	26.631	48.297	76.087	679.78
	FEM	10	0.0	11.015	26.654	48.402	76.446	680.48
		50	0.0	11.012	26.633	48.301	76.098	679.81
		100	0.0	11.012	26.633	48.301	76.097	679.79

Note: N = number of finite elements used in the analysis
(w) = transverse displacement (bending) mode
(u) = axial displacement mode

First, to verify the exactness of the present spectral element model, the natural frequencies of the beam-plate obtained by the present spectral element model (SEM), the finite element model (FEM), and the exact theory (only for stationary beam, *i.e.*, $c = 0\ m/s$) are compared in Table 1 for various moving speed of the beam-plate. The number of finite elements used in FEM is increased from ten to one hundred, while only one finite element used for the SEM results. Table 1 shows that the SEM results are identical to the exact results when $c = 0\ m/s$, and the FEM results certainly converge to the SEM results when $c \neq 0\ m/s$ as the number of finite elements used in FEM is increased. This may prove that the present spectral element model is exact. One more thing we can observe from Table 1 is that in general the magnitudes of natural frequencies (real parts of eigenfrequencies) decrease as the moving speed of beam-plate increased. The first natural frequency becomes zero first time at about $c = 12.33\ m/s$, at which the divergence instability occurs. T_0 investigate the thermal-induced vibrations of the beam-plate, the temperature on the middle part of the upper surface of beam-plate is suddenly elevated

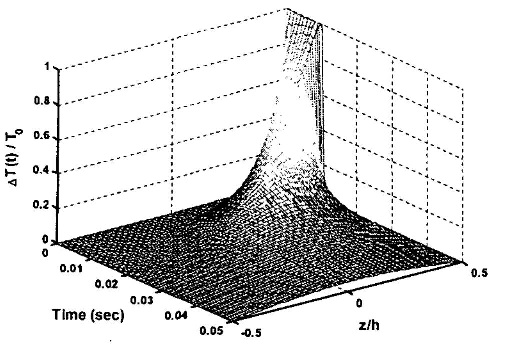

Fig. 2 Time history of the temperature distribution through the thickness of beam-plate.

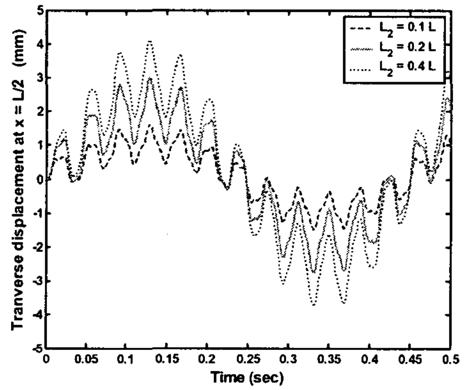

Fig. 3 The transverse displacements *vs.* the size of L_2 on which thermal loading is applied when $c = 4$ *m/s*.

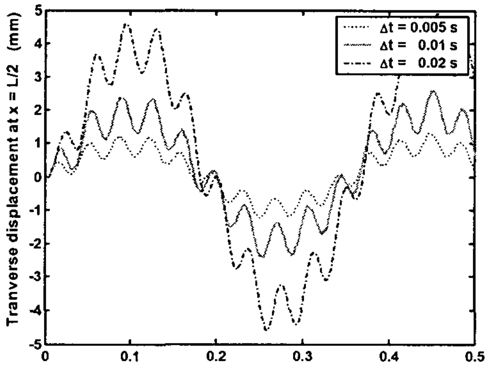

Fig. 4 Time responses of the transverse displacement *vs.*the duration of thermal loading Δt when $c = 4$ *m/s*.

so that $\Delta T = 20K$ and the elevated temperature is sustained for 0.01 seconds from $t = 0$, where $L_1 = L_3 = 0.8$ *m* and $L_2 = 0.4$ *m*. As the result, the corresponding time history of the temperature distribution through the beam-plate thickness is shown in Fig. 2. Figure 3 shows the time responses of the transverse displacements depending on the length of middle part (L_2) subjected to the sudden temperature change, when the moving speed of beam-plate is c = 4 *m/s*. The time responses in both axial and transverse displacements tend to increase as the length of middle part becomes larger. Figure 4 also shows the time responses of the transverse displacement depending on the duration of thermal loading Δt when the moving speed of beam-plate is $c = 4$ *m/s*. The transverse displacement tends to increase as the duration of thermal loading becomes larger.

5. CONCLUSIONS

In this paper, a spectral element model is developed for the axially moving beam-plate which is subjected to external thermal loadings. The spectral element model is formulated from the frequency-dependent dynamic shape functions which are the exact frequency-domain solutions of the governing equations. To evaluate the present spectral element model, the conventional finite element model is also formulated in this study. Numerical studies have been conducted to verify the high accuracy of the present spectral element model and also to investigate the thermal-induced vibrations of an axially moving beam-plate subjected to a sudden temperature change on the upper surface of the beam-plate.

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