THE APPLICATION OF THEORY OF CONSTRAINT IN SCHEDULING

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ABSTRACT : This study was undertaken to develop a comprehensive scheduling method which applied the core concept(DBR) of TOC to PERT, and to combine Monte Carlo Simulation to revise the uncertainties of activities then to eliminate project duration uncertainty. Most of the project duration overlooks the fact that in spite of minimizing the project duration, the uncertainty of constrained resources still puts the reliability of project duration in jeopardy. For the contractor, however, the most important thing is to comply the project scheduling with the planning to reduce the uncertainty of the project activities, operational interaction and project duration. In order to demonstrate that the model can be used in construction project, the scheduling of a steel-structure project was used as a case study to verify the validity of this model.

Key words : uncertainty, Theory of Constraints, project schedule.

1. INTRODUCTION

Construction project is normally classified as the project has some individual and unique characteristics of manufacturing. Because the elements of construction problems are uncertain, and the nonlinear relationships among the elements make the situation of construction project is complex. Especially, from the viewpoint of time-control, the complicate relationships among the project activities amplify the uncertainty of project duration, and the delivery of project cannot reach the requirements as planned or as contract. Most of scheduling methods emphasize how to cut down the project duration and how to optimize the arrangement of project activities. But, for the contractor, the most importance is how to process the project smoothly. That means for completing the project as planned under the constraints like the contract budget in construction phase, the major concern of contractor for the scheduling is how to reduce the uncertainties of project activities, the relationships of activities and project duration.

One of the traditional scheduling methods is Program Evaluation and Review Technique (PERT), which is widely used but is also usually claimed to describe not very well the characteristics of repetitive activities in construction project. Mostly, because PERT is designed to evaluate the duration of individual activity in three-time base, but not really reflect the duration uncertainty that concerns with other paths and make the influence to the total schedule of project. In result, the deficiencies to extract the information of uncertainty of activity duration from the relationship of repetitive activities affect the quality of scheduling. In Taiwan, the past literature focuses on the reduction of project duration, but the reliability of planned duration in construction project is still very low. Because most of the scheduling of projects mainly care about resource assignment and resource leveling, but disregard constraint resource of uncertainty is really affect the variation of total project's duration, even the project's duration was reduced. Actually, due to the constraints of contracts, the real concern in jobsite for contractor is how to control the variation of project's duration, but not to reduce the project's duration.

In 1986, Dr. Goldratt proposed the Theory of Constraint (TOC) which can be used in the concept of scheduling management [1]. Basically, in the production management of TOC, the principle of Drum-Buffer-Rope (DBR) is facilitating to build the scheduling of system. The slowest or less productivity member (drum) in the system should be properly identify and setup at first, and the protection (buffer) which avoids the drum to affect the total schedule, the driving (rope) which expedites or improves the drum to meet the main purposes of a system should be carefully arranged.

This study tried to develop a comprehensive scheduling method which applied the core concept (DBR) of TOC to PERT, and to combine Monte Carlo Simulation (MCS) to revise the uncertainties of activities, then to demonstrate how the uncertainty of project schedule was reduced. The main purposes of this study are (1) to discuss the application of TOC in the schedule of construction project, and (2) to propose a new concept to reduce the uncertainty in the schedule of construction project. In order to demonstrate that the model can be used in construction project, the scheduling of a steel-structure project was used as a case study to verify the validity of this model.

2. THEORY OF CONSTRAINT

In 1980, Optimized Production Technology (OPT) was devised and applied, together with the concept of DBR, Theory of Constraint was then brought up by Goldratt in 1986. It was first introduced as Optimal Production Timetable which was a combination of OPT software and the concept of production, then Optimal Production Technology, and finally, Theory of Constraint [2].

Goldratt attributed the project constraint, not the techniques, to the main issue of overdue projects. The main consideration is that each system or organization should have its goal, so the factor handicaps the system reach its goal is constrain. According to TOC, each system has at least one constraint; otherwise, it is possible for the system to produce unlimitedly. Therefore, the constraints should be managed to improve the efficiency of production in a system. Drum-Buffer-Rope (DBR) is used to solve the scheduling problem of the production management in the constraint theory. After comparing the traditional manufacturing procedure, Just in Time (JIT) and TOC scheduling, Cook (1994) indicated that TOC scheduling or DBR scheduling contained less variance of procedure time than traditional scheduling [3]. In addition, Blackstone (1997) showed the delivery time achievement rate rose from 80-85% to 97% after using TOC [4]. Meaning the uncertainty of the manufacturing procedure time can be minimized definitely.

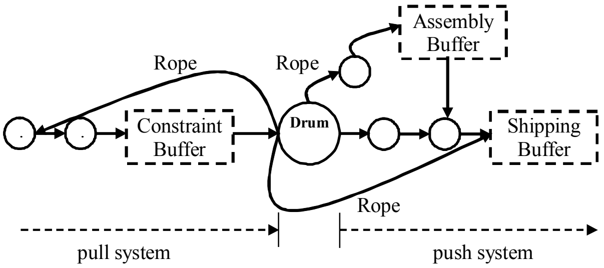

Fig. 1 DBR concept [5]

As shown in Fig. 1, if the output of the factory depends on certain equipment capacity, this resource is called Capacity Constraints Resources (CCR) or bottleneck. To focus on fully protecting and utilizing the constraint resource or bottleneck, the decision maker has to arrange a proper production schedule drum. Also, for the bottleneck not to be affected by the variables during production procedure, an appropriate buffer should be given to avoid shortness of material. Meanwhile, the adjustment of rope also improves the start time of the bottleneck. This is the principle of the Drum-Buffer-Rope concept.

3. SCHEDULING IN DBR MODEL

3.1 Scheduling in DBR Model Structure

This study discussed the application of Theory of Constraint in the traditional construction project scheduling utilizing PERT. It is also to be incorporated with the MCS to revise the activity uncertainty and the uncertainties between activities so that the construction project scheduling can be more reliable.

The analysis steps of DBR schedule model are as shown in Fig. 2.

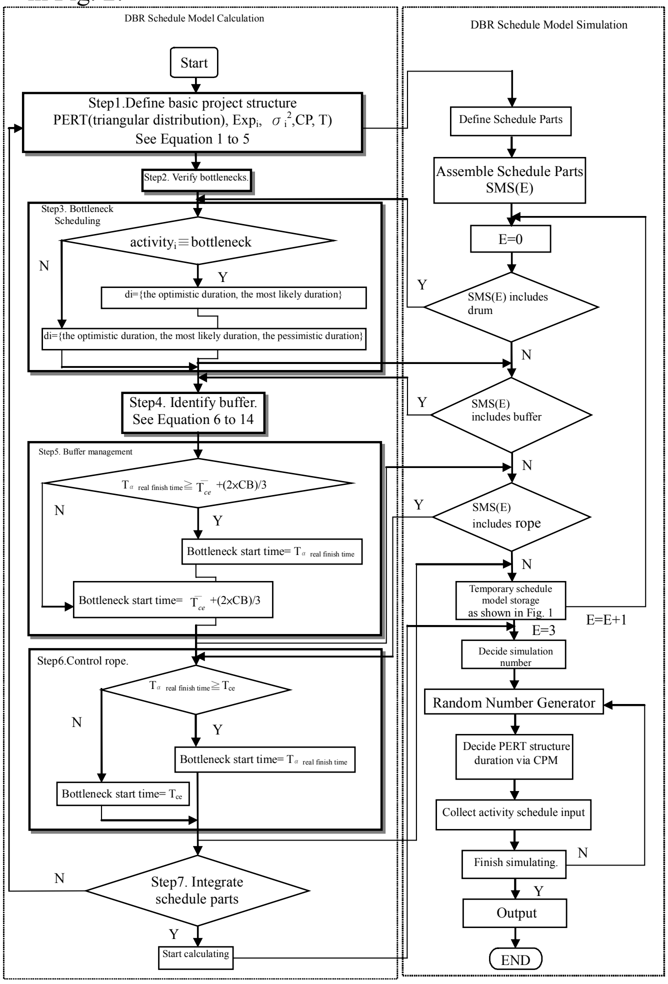

Fig. 2 Flowchart for calculation and simulation of DBR Schedule Model applied in construction project scheduling

Step1. Defining basic project network:

Based on the activities relationship and three-time values (the optimistic, the most likely, and the pessimistic) in the project, define the project network that is in line with project activities orders and a logical probability. Using equation 1 to 5 to figure out the expected activity duration, activity duration variables, the project completion date and the critical path (CP) [6][7]. In equation 3, to work out the expected project completion date (T_e), duration of activity (d_i) must be replaced by expected activity duration (Exp_i). In equation 5, lately and early finishing date is based on backward passing processing and forward passing processing. According to network, project manager will be able to build the drum, buffer and rope to satisfy the actual requirements under various activity conditions.

Step2. Verifying bottlenecks:

Reliability of schedule planning depends on the activity uncertainty and the related activity uncertainty. The bottleneck activity in a construction project is the one with the most impact on the project duration. To effectively control this bottleneck activity is to qualify the schedule planning. Bottleneck verification consists of three parts：

1. According to CPM, the critical path affects the completion date. The critical path activity is the one with zero total float (TF).

2. The number of the critical path can be more than one. Each critical path may affect the completion date probability distribution. The activity to be chosen as the bottleneck is the one with the highest three-time values distribution along the path with maximum completion date variables.

3. In case of possible multiple bottlenecks on the network, the activity closest to the activities assembly node should be considered because it has the most influence on the duration.

$$Exp_i = \frac{o_i + 4m_i + p_i}{6} \tag{1}$$

$$\sigma_i^2 = \left(\frac{p_i - o_i}{6} \right)^2 \tag{2}$$

$$T = \max \{ t_i + d_i \};\ i=1,2\ldots,n \tag{3}$$

$$t_j - t_i - d_i \geq 0, \forall_j \in S_i \tag{4}$$

$$TF_i = LF_i - EF_i \tag{5}$$

where

Exp_i: expected activity duration of activity i
o_i: optimistic activity duration of activity i
m_i: most likely activity duration of activity i
p_i: pessimistic activity duration of activity i
σ_i: activity duration variable of activity i
T: project duration
$t_{i,j}$: starting date of activity i and j
d_i: duration of activity i
n: total number of activities
S_i: set of successors of activity i
TF_i: total float of activity i
LF_i: lately finishing date of activity i
EF_i: early finishing date of activity i

Step3. Bottleneck Scheduling:

In the OPT software, the drum is determined by backward scheduling from customer orders. Other activity schedules are based on the drum scheduling to obtain the expected duration of the total processing time [8]. In other words, the entire production schedule is subject to bottleneck scheduling. Therefore the most likely time is the last completion date of the bottleneck activity. Estimation of other components, such as buffer or rope, is also based on bottleneck scheduling. Bottleneck scheduling minimizes the bottleneck activity time uncertainty and forecast the postponement or advance of non-bottleneck activities.

Step4. Identifying buffer:

Identification of buffer is to provide a shield so that variable activity won't affect bottleneck activity and its successors. Buffer time decreases proportionally when the activity completion date falls behind the expected schedule.

1. Constraint Buffer (CB): Equation 6 is used to calculate CB time that is placed in front of the bottleneck to provide a shield so that bottleneck resource can work per the planned schedule. Besides Chua (2001) states that bottleneck buffer is to be placed in front of the critical path activity to minimize resource limit and maximize duration reliability. T_{cp} and T_{ce} are separately bottleneck predecessor activity pessimistic time and expected time, which work in equation 3 to gauge the predecessor activity duration.

2. Assembly Buffer (AB): Equation 7 is used to calculat e AB time. It is to ensure bottleneck not to be delayed by other activities postponement when it is assembled with other activities. In the traditional CPM/PERT scheduling, there is assembly node duration variance resulting in the increase of project duration uncertainty. AB is therefore added to the assembly node where bottleneck and non-bottleneck are merged. T_{ap} and T_{ae} represent respectively pessimistic time and expected time of AB predecessor activity, which are placed them in equation 3 to work out the predecessor activity duration.

3. Shipping Buffer (SB): Equation 8 is used to calculate SB time that is to be set up in the shipping area. The purpose of SB is to prevent order delivery from being influenced by the processing variables. The production process in the traditional CPM/PET scheduling uses push system. Once the bottleneck duration is extended, the postponed duration will cause breach of the contract. That's why there is SB after bottleneck. T_{sp} and T_{se} represent respectively pessimistic time and expected time of SB predecessor activity, which are placed in equation 3 to work out the predecessor activity duration.

$$CB = \left(\frac{T_{cp} - T_{ce}}{2} \right); c = 1,2,\ldots,\alpha; \forall \alpha \in n \tag{6}$$

$$AB = \left(\frac{T_{ap} - T_{ae}}{2} \right); a = 1,2,\ldots,\beta; \forall \beta \in n \tag{7}$$

$$SB = \left(\frac{T_{sp} - T_{se}}{2} \right); s = 1,2,\ldots,\gamma; \forall \gamma \in n \tag{8}$$

where

CB: constraint buffer (day)
AB: assembly buffer (day)
SB: shipping buffer (day)
α: predecessor activity quantity of constraint buffer
β: predecessor activity quantity of assembly buffer
γ: predecessor activity quantity of shipping buffer
T_{cp}: pessimistic time of all predecessor activities of bottleneck buffer
T_{ce}: expected time of all predecessor activities of bottleneck buffer
T_{ap}: pessimistic time of all predecessor activities of assembly buffer
T_{ae}: expected time of all predecessor activities of assembly buffer
T_{sp}: pessimistic time of all predecessor activities of shipping buffer
T_{se}: expected time of all predecessor activities of shipping buffer

The above three buffer time equations are similar but the amounts of buffer predecessor activities are different, then the buffer time varies. Equations 9 to 11 consider the expected time of buffer predecessor activities to project expected time ratio to allocate each activity buffer time. T_e is based on equation 15 to calculate project expected time, noting that buffer time comes from each activity; equations 12 to 14 subtract extra buffer time from each activity. The subtracted extra buffer time will then be placed in the buffer zone with which the project uncertainty can be managed. Buffer management monitors the project schedule status as well as promoting the reliability of scheduling.

$$Buffertime_c = \left(\frac{p_c}{T_e} \times CB \right); c = 1,2,\ldots,\alpha; \forall \alpha \in n \tag{9}$$

$$Buffertime_a = \left(\frac{p_a}{T_e} \times AB \right); a = 1,2,\ldots,\beta; \forall \beta \in n \tag{10}$$

$$Buffertime_s = \left(\frac{p_s}{T_e} \times SB \right); s = 1,2,\cdots,\gamma; \forall \gamma \in n \tag{11}$$

$$\bar{p}_c = p_c - Buffertime_c \tag{12}$$

$$\bar{p}_a = p_a - Buffertime_a \tag{13}$$

$$\bar{p}_s = p_s - Buffertime_s \tag{14}$$

$$T_e = max\{t_i + Exp_i\};\ \mathrm{i}=1,2\ldots,\mathrm{n} \tag{15}$$

where

$\mathrm{Buffertime}_c$: buffer time of activity c before bottleneck buffer

$\mathrm{Buffertime}_a$: buffer time of activity a before assembly buffer

$\mathrm{Buffertime}_s$: buffer time of activity s before shipping buffer

T_e: project expected time

p_c: pessimistic time of c activity before constraint buffer

p_a: pessimistic time of a activity before assembly buffer

p_s: pessimistic time of s activity before shipping buffer

$\bar{p}_c$: pessimistic time without buffer time of activity c before bottleneck buffer

$\bar{p}_a$: pessimistic time without buffer time of activity a before assembly buffer

$\bar{p}_s$: pessimistic time without buffer time of activity s before shipping buffer

t_i: starting date of activity i and j

n: total number of activities

Step 5. Managing Buffer:

Schragenheim (1991) divides the buffer time zone into overlooking, warning and overworking three sections [10]. Based on equations 6 to 8 the size of each section is allocated equally. As shown in equation 16, take constraint buffer for example. If predecessor activity duration is larger than the expected time of predecessor activity minus buffer time ($\bar{T}_{ce}$), the time of warning zone should be added, that means appropriate actions should be taken. Such as the increase of people, machines and time should be considered to assure the constrain affects start is estimated.

$$\bar{T}_{ce} = \max\left\{ t_i + \frac{o_i + 4m_i + \bar{p}_c}{6} \right\} \tag{16}$$

Step6. Controlling rope:

Rope is traced back as the length of time from the bottleneck to the lead time of initial order. The purpose of rope is to decide the time of initial order that the logistics can back up bottleneck via non-bottleneck activities [11].

According to PERT, the beginning of bottleneck expected time or the completion time of bottleneck predecessor activity can be figured out. The length of rope is as same as the duration of bottleneck predecessor activity which is using the concept of pull system for concurrent production. The rope scheduling is the milestone for all bottleneck predecessor activities after the commencement of work. The completion time of all the predecessor activities must not exceed it so that the bottleneck can start as expected as in the buffer management.

3.2 Simulation of Scheduling in DBR Model

As shown in the right half of Fig. 2 is the logic operation procedure of computer simulation. In the process of setting up the scheduling model, the choosing and combining of the schedule components, the parameter needed, and the PERT scheduling need to be considered. As shown in table 1, DBR scheduling components in the schedule model consist of drum, buffer and rope. For example, when the schedule model contains 2 scheduling components (E=2), there are three possible types, such as Drum-Buffer schedule, Drum-Rope schedule and Buffer-Rope schedule. Totally, there are therefore eight schedule models.

Table 1. DBR Scheduling Components for MCS

No.	Schedule Components	Drum	Buffer	Rope
1	Traditional schedule			
2	Drum schedule	■		
3	Buffer schedule		■	
4	Rope schedule			■
5	Drum-Buffer schedule	■	■	
6	Drum-Rope schedule	■		■
7	Buffer-Rope schedule		■	■
8	Drum-Buffer-Rope schedule	■	■	■

Remark: Shaded sections indicate which DBR scheduling components are utilized in the simulation

4. CASE STUDY

4.1 Case background

The actual construction project of steel-structure building in Taichung of Taiwan is used to test the viability of DBR scheduling in the construction project through. Table 2 shows the durations of all the activities.

The work includes steel structure manufacturing in the shop (activities 1~12), structure work on site (activities

13~24) and the steel structure assembly (activities 25~27). There are three phases in total, 27 activities among three work phases. All activities relate to each other by FS (Finish to Start). For example, the successor activity of activity item 7 “assembling” is activity item 8 “check on”.

Table 2. Activity Duration Information (* critical path)

activity item	optimistic	the most likely	pessimistic
Shop work			
1.drawing	20	25	30
2.assessment & accept	7	10	15
3.layout	4	6	10
4.cutting	3	7	10
5.drilling	3	5	7
6.straighten	2	3	5
7.Assembling	5	7	10
8.check on	2	3	5
9.welding	10	13	18
10.examination & adjustment	3	5	7
11.Painting	5	7	10
12.transportation	3	5	7
Basement work			
13.B1 earthwork *	5	7	10
14.B1 horizontal support *	3	5	7
15.B2 earthwork *	5	7	10
16.B2 horizontal support *	3	5	7
17.B3 earthwork *	7	10	15
18.B3 horizontal support *	3	5	7
19.B4 earthwork *	7	10	15
20.B4 horizontal support *	4	6	10
21.B4 structural construction *	14	24	38
22.B3 structural construction *	14	24	38
23.B2 structural construction *	14	24	38
24.B1 structural construction *	15	25	39
Site work			
25.Erection	16	20	25
26.Adjustment	2	3	5
27.Welding	7	10	15

4.2 Case analysis

Based on the left half of DBR Schedule model on Fig. 2, the case in this study is analyzed as follows:

Step1. Defining basic project network:

Basic project network is defined to be utilized by PERT calculations. Work out the activities duration’s information based on equation 1 to 5.

Step 2. Verifying bottlenecks:

The critical path in the project is on the activities 13 to 27. The standard deviation (SD) for activities 20 to 24 is 4 days respectively, which activities are all candidates to become bottlenecks. Considering the statistical influence of assembly node on the completion date, the bottleneck is activity 24.

Step 3. Bottleneck Scheduling:

This step limits the time uncertainty of bottlenecks. In other words, the uncertainty of bottleneck duration can be minimized reasonably. The engineering resources must fully support bottleneck to stop the pessimistic time from delaying the successor activities or the project completion date. Therefore, the assumed distribution of activity time is {15,25}. With the complete support from project resources, the bottleneck will not be delayed.

Step 4. Identifying buffer:

1. Constraint Buffer: CB is acquired by equation 6 as 27.5 days. Equation 9 is used to calculate the size of buffer for the bottleneck predecessor activities 13 to 23 and equation 12 is used to bring out the pessimistic time of activities 13 to 23 without the buffer.

2. Assembly Buffer: AB is acquired by equation 7 as 16.42 days. Equation10 is used to figure out the size of the buffer for the constraint buffer predecessor activities 13 to 23. Equation 13 is used to calculate the pessimistic time of activities 1 to 12 without the buffer. The pessimistic time of the non-critical path activities 1 to 12 is 124 days. In fact, four and half months before the first stage of steel work (activity 25), the construction company signed the contract with the steel supplier with the minimum buffer time of 13 days, therefore the AB is not necessary in this case.

3. Shipping Buffer: According to equation 8 the SB is 6.67 days. Equation 11 is used to figure out the buffer size of SB predecessor activity 24, and equation 14 is used to calculate the pessimistic time of activity 24 without the buffer.

Step 5. Managing Buffer:

Based on the progress the project manager can adjust the mobilization of resources in time and minimize the project completion date uncertainty.

Step6. Controlling rope:

According to above mention of 4.1, the rope in this case is the expected time of bottleneck predecessor activity 23, which is 151.09 days based on equation 15.

4.3 Analysis of the simulation result

Table 3 and Fig. 3 show the result of DBR schedule model to be simulated in the PERT for 1000 times. The mean completion time using the traditional schedule (No. 1) is 172.35 days. It is not the longest duration, but the uncertainty reaches 10.35 days (SD 10.35 days, min. 136.7 days, max. 212.4 days). Obviously, there is still plenty of room for PERT scheduling to be improved. As shown in Fig. 3, the performances of single component schedules are comparing with the traditional schedule. In the drum schedule (No. 2), the uncertainty of bottleneck activity is reduced. Although the completion time is reduced by 2.36% (4.07 days), the uncertainty is only reduced by 1.12 days (SD.9.23 days, Min.135.78 days, Max.198.09 days). Due to the additional buffer time in the buffer schedule (No. 3), the project completion time rises by 6.31% (10.88 days) when the uncertainty is 5.09 days (SD.5.26 days, Min.168.01 days, Max.198.97 days). In the rope schedule (No. 4), completion time is down 1.89 days (SD.8.46 days, Min.155.40 days, Max.206.21 days).

The two components schedules which are Drum-Buffer schedule (No. 5), Drum-Rope schedule (No. 6), and Buffer-Rope schedule (No.7) are comparing with the traditional schedule, the durations are slightly different, but the duration uncertainty reduces drastically. Especially, the Buffer-Rope schedule (No. 7) has the best result of 7.49 days (SD.2.86 days, Min.161.77 days, Max.182.07 days).

Drum-Buffer-Rope schedule (No.8) reduces the uncertainty of project duration by 7.88 days (SD.2.47 days, Min.161.45 days, Max.176.47 days). When compare with the other seven schedule models, the project duration of 168.76 days is not larger than others.

Table 3. Results of simulation

	Mean	Standard Deviation	Range Minimum	Range Maximum	Difference in standard deviation compared to PERT
Simulation 1	172.35	10.35	136.7	212.4	Not applied
Simulation 2	168.28	9.23	135.78	198.09	1.12
Simulation 3	183.23	5.26	168.01	198.97	5.09
Simulation 4	175.22	8.46	155.4	206.21	1.89
Simulation 5	178.99	3.45	167.39	188.69	6.90
Simulation 6	170.21	6.96	154.87	199.04	3.39
Simulation 7	169.25	2.86	161.77	182.07	7.49
Simulation 8	168.76	2.47	161.45	176.47	7.88

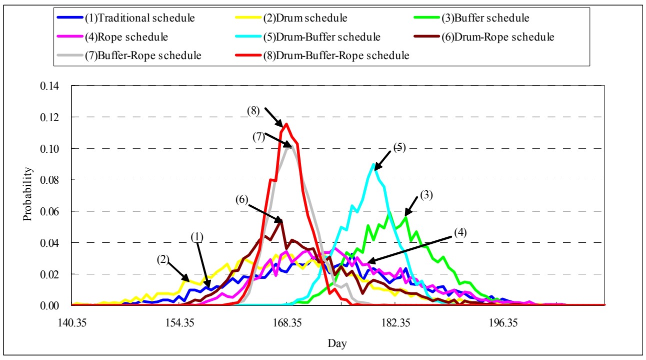

Figure 3. Schedule Components vs. Traditional Schedule

5. CONCLUSIONS

According to the study, each of DBR scheduling component responses to the pertaining defect of project schedule and reduces the uncertainty of project duration. Eventually the result shows that the project uncertainty of DBR scheduling (SD.2.47days) is lower than that of the traditional PERT scheduling (SD.10.35days). It is helpful to the future policy decisions of labor, cost, and finance planning.

This study shows:

(1) The application of TOC in the construction scheduling is available.

(2) A comprehensive project schedule model was developed through adjusting the activity and inter-activities uncertainty.

The succeeding related study may consider:

(1) The application in different cases, such as civil work engineering, harbor & river engineering etc. to demonstrate the value of the schedule model in different types of engineering.

(2) The relationship of different activities combination, such as Start to Start (SS), Start to Finish (SF), and Finish to Finish (FF) to build the DBR schedule model.

REFERENCES

[1] Goldratt, E., and Fox. R., *The Race*, North River Press, 1986.

[2] Spencer, M. S., *Using the Goal in the MRP System*, Production and Inventory ,1991.

[3] Cook, D. P., "A Simulation Comparison of Traditional, JIT and TOC Manufacturing System in a Flow Shop with Bottlenecks.", *Production and Inventory Management Journal*, Vol.1, pp.73-78, 1994.

[4] Blackstone, J.H., Gardiner, L.R. and Gardiner, S.C., "A Framework for the systemic Control of Organizations", *International Journal of Production Research*, Vol.35, No.3, pp597-609, 1997.

[5] Demmy, W. S., and Demmy B. S., "Drum-Buffer-Rope Scheduling and Pictures for the Yearbook", *Production and Inventory Management Journal*, Third Quarter, pp.45-47, 1994.

[6] Leu S. S. and Yang C. H., "GA-based Multicriteria Optimal Model for Construction Scheduling.", *Journal of Construction Engineering and Management*, Vol. 125, pp420-427, 1999.

[7] Daud Nasir, Brenda McCabe, and Loesie Hartono, "Evaulating Risk in Construction-Schedule Model(ERIC-S): Construction Schedule Risk Model.", *Journal of Construction Engineering and Management*, Vol.129, pp518-527, 2003.

[8] Fry, T. D., Blackstone, J. H. and Cox, J. F., "An analysis and discussion of the optimized production technology software and its use.", *Production and Operations Management Journal*, Vol.1, pp229-242, 1992.

[9] Chua, D. and Dhen, L., *Constraint modeling and buffer management with integrated production scheduler.*, Proc., 9th Int. Group for Lean Construction Conf., Singapore, 2001.

[10] Schragenheim, E., and Ronen, B., "Buffer Management: A Diagnostic Tool for Production Control", *Production and Inventory Management Journal*, Second Quarter, pp.74-79,1991.

[11] Gardiner, S. C., Blackstone, J. H., and Gardiner L. R., "Drum-Buffer-Rope and Buffer Management: Impact on Production Management Study and Practice", *International Journal of Operation and Production Management*, Vol.13, No6, pp68-78, 1993.