SCHEDULING REPETITIVE PROJECTS WITH STOCHASTIC RESOURCE CONSTRAINTS

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ABSTRACT: Scheduling repetitive projects under limitations on the amounts of available resources (labor and equipment) has been an active subject because of its practical relevance. Traditionally, the limitation is specified as a deterministic (fixed) number, such as 1000 labor-hours. The limitation, however, is often exposed to uncertainty and variability, especially when the project is lengthy. This paper presents a stochastic optimization model to treat the situations where the limitations of resources are expressed as probability functions in lieu of deterministic numbers. The proposed model transfers each deterministic resource constraint into a corresponding stochastic one and then solves the problem by the use of a chance-constrained programming technique. The solution is validated by comparison with simulation results to show that it can satisfy the resource constraints with a probability beyond the desired confidence level.

Key words : Scheduling, Chance-constrained Programming, Stochastic Optimization, Risk Analysis, Project Management

1. INTRODUCTION

Pipelines, multistory buildings, highways, and mass housing are good examples that exhibit repetitive characteristics where crews perform similar or identical tasks from one working location to another. These projects are therefore called "repetitive projects". Since repetitive projects represent a large portion of the construction industry, they need an effective scheduling method to ensure the project can be completed in the most efficient manner.

Resources employed in repetitive projects, such as labor and equipment, are often subject to limited availability. This leads to a practical concern of how to distribute the limited resources among various activities so that the project can be completed within a minimal period of time. This class of problem, so-called "limited resource allocation" or "resource-constrained scheduling", has been an active subject for decades because of its practical relevance.

This paper aims to schedule repetitive projects when the limitations of resources are estimated as probability functions. The problem is formulated as a chance-constrained program, which is converted to a deterministic equivalent and solved by means of common linear programming techniques.

2. REVIEW OF EXISTING APPROACHES

Traditional scheduling techniques, such as the Critical Path Method (CPM) and bar chart, have been shown less than satisfactory in addressing the limited resource allocation problem. Therefore three groups of alternatives were proposed. Operational research techniques used to solve the mathematical model included linear programming [11], mixed integer programming [10], branch-and-bound [3], and dynamic programming [5]. The second group specified a variety of heuristics (a set of rules to prioritize activities in the assignment of resources) to provide a near-optimal solution in practical time. Examples of this group dated back to the works of Kelley [8] and Wiest [13] while some recent efforts can be found in [4]. The third group capitalized on the development of evolutionary computation techniques, such as genetic algorithms [9], and local search techniques, such as simulated annealing [7] and tabu search [6,12].

Previous methods typically assumed the limits of resources can be expressed as deterministic numbers. Yet this assumption may be inappropriate when the supply of resources may in reality contain a great deal of uncertainty when extending several months, or even years, into the future. For instance, the situation of labour shortage leads to a sharing of crews between multiple projects. As a result, the actual manpower allocated to the project may differ from what was originally planned.

3. OPTIMIZATION MODEL

Within repetitive projects, scheduling is usually done by considering crews "flowing through" the whole project, similar to manufacturing assembly lines. The flow model of repetitive projects leads to the use of production lines whose slopes represent individual production rates. To accelerate the project, a well-grounded treatment is to "balance" the production lines (i.e., to make all the lines have the same slope) as decreed in the line-of-balance procedure.

Figure 1 illustrates a balanced situation where three consecutive activities A, B, and C are repetitively performed from unit 1 to unit u. Given the duration of the first unit (calculated by CPM), the rate of construction is then the result of dividing the remaining number of units (u-1) by the difference between project deadline and the duration of the first unit. Once the rate of construction has been calculated, the start times of activities in different units and the number of each required crew can be computed.

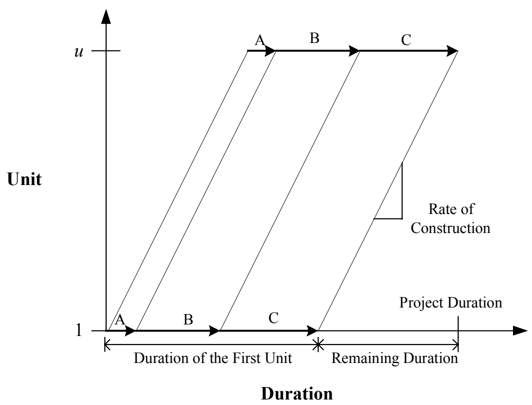

Figure 1. Line of balance

The objective function of the optimization model is to minimize the project duration, which can be computed as follows

$$D = D_1 + (M-1)/Q \tag{1}$$

where D is the project duration; M is the number of progress units; and D_1 is the duration of the first unit. As M being a constant, to minimize the project duration is equivalent to maximizing the system production rate, Q (measured in unit/day), which can be expressed as

$$\text{Maximize } \; Q = (M-1)/(D-D_1) \tag{2}$$

The constraints comprise the following sets of equations. The first set describes that the system production rate is governed by the slowest activity.

$$Q \le Q_i \quad \forall \; i \tag{3}$$

where Q_i denotes the production rate of activity i (measured in unit/day).

The second set defines coefficients of resource consumption for individual activities (RC_i):

$$RC_i = D_i Q_i \quad \forall \; i \tag{4}$$

Since the unit duration of activity i (D_i, measured in day/unit) is a known constant, the coefficient of resource consumption is directly proportional to the production rate of activity i (Q_i, measured in unit/day). The mechanism is that an increase in the production rate of an activity would require more resources working at different units simultaneously.

In the third set, we derive the daily working hours of resource j allocated to activity i (RD_{ij}, measured in resource-hour/day) by dividing the working hours of resource j required by activity i (R_{ij}, measured in resource-hour/unit) by the unit duration of activity i (D_i, measured in day/unit). The equation is

$$RD_{ij} = R_{ij} / D_i \quad \forall \; i, j \tag{5}$$

The fourth set ensures the feasibility of resources. That is, the daily working hours of each resource (sum of the allocated amounts for different activities, i=1, 2, …, n) should be less or equal to the available amount.

$$\sum_{i=1}^{n} (RD_{ij} \times RC_i) \le T_j \quad \forall \; j \tag{6}$$

where T_j denotes the availability of resource j per day (measured in resource-hour/day). Substitute the left-hand-side (LHS) of Eq. (6) by Eqs. (4) and (5), the constraint can be rewritten as

$$\sum_{i=1}^{n} R_{ij} Q_i \le T_j \quad \forall \; j \tag{7}$$

The last set of constraints defines the upper and lower bounds of the production rate for each activity. The former may be caused by technical constraints, such as limits on equipment capacity or crew productivity whereas the latter, being always positive, may be the result of economical consideration.

$$Q_i^L \le Q_i \le Q_i^U \quad \forall \; i \tag{8}$$

To deal with the uncertainty associated with the supply of resources, we modify Eq. (7) to a probabilistic constraint

$$\Pr\left[\sum_{i=1}^{n} R_{ij} Q_i \le T_j \right] \ge \alpha_j \quad \forall \; j \tag{9}$$

where Pr[.] is the probability of the event in [.]; T_j is now a random value with a distribution of uncertainty; α_j is a pre-specified confidence level. A solution set is said to be feasible if and only if the probability measure of the set is at least α_j In other words, the constraint may be violated at most (1-α_j) of time.

4. CHANCE-CONSTRAINED PROGRAMMING

The stochastic model can be solved by chance-constrained programming (CCP), which was first developed by Charnes and Cooper [1]. The basic technique of CCP is to convert the stochastic constraints to their respective deterministic equivalents. Here we focus on the case of Eq. (9).

Given the distribution density of T_j, the deterministic equivalent is to replace the right-hand-side (RHS) by T_j such that

$$\int_{\tilde{T}_j}^{+\infty} \varphi_j(T_j)dT_j = \alpha_j \quad \forall j \tag{10}$$

where $\varphi_j(T_j)$ is the distribution density function of T_j.

Suppose the cumulative distribution function (CDF) of T_j, denoted by Ψ_j, is continuous and strictly monotonic, T_j can be computed directly based on the inverse of the CDF

$$\tilde{T}_j = \Psi_j^{-1}(T_j, 1-\alpha_j) \quad \forall j \tag{11}$$

Thus the deterministic equivalent of Eq. (9) is

$$\sum_{i=1}^{n} R_{ij}Q_i \le \Psi_j^{-1}(T_j, 1-\alpha_j) \quad \forall j \tag{12}$$

Because Eq. (12) is a linear combination of the decision variables, the deterministic equivalent of the stochastic model is still a linear programming problem. So it can be solved in polynomial time by the use of very efficient linear programming techniques, such as simplex or interior point methods.

Our formulation in Eq. (12) is applicable for commonly-used types of distributions, such as normal, lognormal, beta, gamma, uniform, or triangular. When T_j is normally distributed, the RHS can be reduced to a combination of the estimated mean and standard deviation

$$\sum_{i=1}^{n} R_{ij}Q_i \le m_j + k(1-\alpha_j)\sigma_j \quad \forall j \tag{13}$$

where m represents the mean; $k(.)$ is the normal value corresponding to lower-tail probability of (.);σ_j is the st andard deviation.

For other types of distributions, the computation of the RHS of Eq. (13) is straightforward when the inverse CDF can be expressed as a closed form (e.g., uniform and triangular distributions). Otherwise, numerical algorithms are readily available to approximate the inverse value, such as lognormal and beta distributions [2].

5. APPLICATION

The proposed chance-constrained programming model is applied to a housing project. The project is to construct 100 units of housing, each of which consists of four consecutive activities: foundation, retaining wall, floor slab, and exterior wall. The necessary resources are carpenters, steelworkers, laborers, masons, and pumps. Table 1 lists the amount of resources required to complete each activity and unit duration. Table 2 lists the unit price for each resource

Table 1. Activity and resource information

Act.	Resource (resource-hour/unit)					Unit duration (day)
	Carpen.	Steel.	Lab.	Mason	Pump	
Found.	12	6	16	6	-	1.125
Retain. wall	150	36	114	15	10	4
Floor slab	160	40	156	36	24	7.75
Exter. wall	130	24	96	15	10	3.25

Table 2. Unit prices of resources

Resources	Unit Price ($/resource-hour)
Carpenter	11.67
Steelworker	13.33
Laborer	8.33
Mason	15
Pump	25

Suppose the supply of resources is estimated to be normally distributed with the respective means and standard deviations shown in Table 3.

Table 3. Resource supply condition

Resource	Supply condition		
	Mean	Coeff. of variation	Stand. deviation
Carpen.	112 hours	5%	5.6 hours
Steel.	57 hours	20%	11.4 hours
Lab.	87 hours	20%	17.4 hours
Mason	25 hours	5%	1.25 hours
Pump	13 hours	5%	0.65 hours

Here we use the coefficient of variation to estimate the level of uncertainty associated with the supply condition because it is a simple and direct measure of variability. If the required confidence level for every constraint is 90% (critical value is -1.285), the chance-constrained programming model has the following deterministic equivalent:

Maximize Q (14)

Subject to

$$Q \le Q_1; Q \le Q_2; \quad Q \le Q_3; Q \le Q_4 \tag{15)-(18}$$

Carpenter:

$$12 \times Q_1 + 150 \times Q_2 + 160 \times Q_3 + 130 \times Q_4 \le 112 + 5.6 \times (-1.285) \tag{19}$$

Steelworker:

$$6 \times Q_1 + 36 \times Q_2 + 40 \times Q_3 + 24 \times Q_4 <= 57 + 11.4 \times (-1.285); \tag{20}$$

Laborer:

$$16 \times Q_1 + 114 \times Q_2 + 156 \times Q_3 + 96 \times Q_4 <= 87 + 17.4 \times (-1.285); \tag{21}$$

Mason:

$$6 \times Q_1 + 15 \times Q_2 + 36 \times Q_3 + 15 \times Q_4 <= 25 + 1.25 \times (-1.285); \tag{22}$$

Pump:

$$0 \times Q_1 + 10 \times Q_2 + 24 \times Q_3 + 10 \times Q_4 <= 13 + 0.65 \times (-1.285); \tag{23}$$

$$Q_1 \ge 0; Q_2 \ge 0; \quad Q_3 \ge 0; Q_4 \ge 0 \tag{24)-(27}$$

The optimal solution set is that all the activities should be performed at the same pace: 0.169 units/day. This should not be surprising because it is reasonable to keep all the activities at the same pace if the lowest one would eventually govern the production rate of the entire project. Given that the first unit takes 16.125 days (sum of the unit durations), the project can be completed within 601.9 days according to Eq. (1).

The proposed model can treat a practical concern, when activities cannot be performed at the same pace due to technical constraints, directly by simply setting the bounds of the production rates for every activity as in Eq. (8). Suppose the fastest pace at which activities “foundation” and “retaining wall” can be performed is 0.18 units/day whereas the slowest pace at which activities “floor slab” and “exterior wall” can be performed is 0.2 units/day. The optimal production rates for 4 activities will be {0.11, 0.11, 0.2, 0.2}, which brings down the optimal production rate of the project to 0.11 and the project duration will be prolonged to 920.2 days.

The proposed model is also helpful in determining possible strategies to expedite the project. The analysis starts with indicating the most influential resource by checking which resource constraint has zero slack. In this example, the binding resource is laborer. Therefore the strategy is to hire more laborers (i.e., increase the mean) and/or to decrease the uncertainty associated with the supply of laborers (i.e., reduce the standard deviation).

If we double the mean and halve the standard deviation f or the supply of laborers, the optimal production rate becomes 0.213 units/day, which leads to a much shortest project duration: 480.9 days. Taking the latter alternative alone (halve the standard deviation without increasing the mean) can bring down the project duration to 516.1 days. The significant difference of 85.8 days (601.9-516.1) stresses the importance of uncertainty management while serving as a convincing motivation for the contractor to take action in ensuring the stability of labor supply (e.g., sign a contract with labor unions in advance). The analysis above demonstrates the capability of the proposed model in quantifying the influence of uncertain resource availability on the project duration.

The sensitivity analysis of the proposed model helps evaluate the impacts of different levels of uncertainty in the supply conditions of resources. If we model funding as a resource, Figure 2 compares the project durations caused by different levels of uncertainty of project financing, represented by the coefficients of variation of the funding. The comparison reflects the importance of maintaining a steady supply of funding. For instance, by reducing the coefficient of variation from 50% to 10%, the expected project duration can be shortened from 908.0 days to 442.8 days.

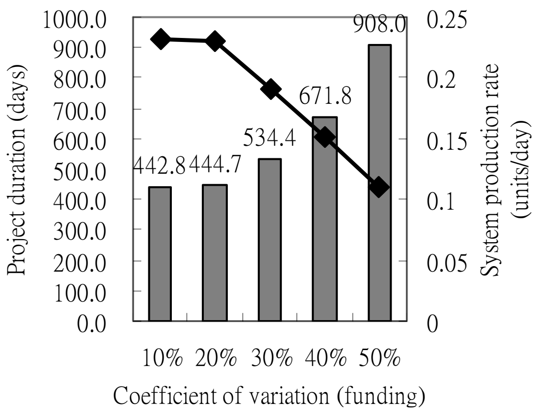

Figure 2.
Sensitivity analysis on the level of funding uncertainty

6. VALIDATION

A Monte Carlo simulation is performed to verify whether the solution set generated by the proposed model, when exposed to uncertainty, can actually be satisfied beyond the required confidence level. For each constraint, a random number is generated in accordance with the underlying distribution of the RHS. On the other hand, the optimal solution set is inserted into the LHS of the constraint and the outcome is compared to the random number to check if the constraint can indeed be fulfilled. Among 1000 simulation runs, all the constraints can be fully fulfilled (1000 times) except the laborer constraint, which is fulfilled 903 times out of 1000 (probability of 90.3%). This confirms that the solution generated by the proposed model can be fulfilled beyond the 90% confidence level.

To draw a comparison with the deterministic model, we compare the solution set generated by both models (stochastic and deterministic). In the deterministic model, the RHS of the constraints contains only the mean (or a single estimate) for the supply of resources. The optimal solution set is then {0.228, 0.228, 0.228, 0.228}, which is a lot more optimistic than the one considering uncertainty. By the same Monte Carlo simulation described above, when this solution set is exposed to uncertainty, the probability for each constraint to be fulfilled is merely {0.941, 0.998, 0.5, 1.0, 1.0, 0.996}. This means the entire system can be satisfied with only a probability of 46.8% (multiplication of all the probabilities if the supply conditions are mutually independent). In other words, it is more than half of the chance that the project will be in serious troubles due to neglecting the uncertainty in the supply conditions of resources.

7. CONCLUSION

In scheduling repetitive projects, the amounts of resources are not only subject to certain limits but also be exposed to uncertainty in today's dynamic and complex business environment. Such uncertainty requires caution and should be incorporated into the decision-making process. To achieve this goal, the present study develops a chance-constrained programming model, derives its deterministic equivalent, and solves the equivalent by classical linear programming techniques.

The proposed model is applied to a project comprising 100 units of housing. For the example project, the proposed model has been shown useful in (1) determining the acceleration strategy, (2) setting the technical bounds of individual activity production rates, and (3) evaluating the impact of different levels of uncertainty on the project duration. The solution set generated by the proposed model is verified by confirming that the constraints can be satisfied beyond the pre-specified confidence level. In contrast, the optimal production rates generated by a model without considering uncertainty would fail to reach the required confidence level and therefore may put the project in great jeopardy.

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