RESEARCH ON LOAD-BEARING PROPERTY AND DESIGN OF CABLE DOMES

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ABSTRACT : The cable dome, proposed by Geiger after developing Fuller's idea of tensegrity and improved by Levy, is a new type of large span space structures. In this paper, formulations of the initial forces distribution in members of two main systems of cable dome, which are Geiger dome and Levy dome, are presented. By analyzing the static performance of Levy dome and the variation of internal forces in members of the structure, four groups of design parameters in cable dome structure are represented in terms of: (1) the numbers of rings and the spaces between the rings; (2) the slopes of ridge cables; (3) the lengths of struts; (4) the initial force in one member of the structure.

Keywords: *Cable dome structure; Distribution of initial forces by prestressing; Load-carrying capacity; Levy dome*

1. INTRODUCTION

Cable dome structure, which is developed in recent 20 years, is a new type of large span space structures. As a representative self-equilibrium system composed of continuous prestressed cables and individual compressed struts, cable dome can be divided into two types according to the shape of the geometry grids: (1)Geiger dome, which has wedge shape of grids; (2)Levy dome, which has triangulated shape of grids. The structural efficiency of cable dome is extremely high and the tensile strength of cables can be utilized sufficiently. Furthermore, by small increase of the self-weight and cost of unit area of the structure, cable dome can reach a large span, that is, this kind of structure is particularly fit for large span roof designing[1].

2. THE DEVELOPMENT AND CHARACTERISTICS OF CABLE DOME STRUCTURE

The concept of 'Tensegrity' was first conceived by R.B. Fuller in 1950s, which reflected his idea of 'nature relies on continuous tension to embrace islanded compression elements'[2]. In 1980s, D.H. Geiger, an American designer, had developed this concept and designed an innovative structure 'cable dome', which is put into practice in the circular roof structures of Gymnastic and Fencing Arenas for the Seoul Olympic Games in 1986 and other large span structures, such as 'Sun Coast Dome' (constructed in Florida, U.S.) and 'Red Bird Arena' (constructed in Illinois, U.S.). In 1990s, two engineers of Weidlinger Associates: M.P. Levy and T. F. Jing, had further improved the layout of cable dome and built the Georgia Dome in quasi-elliptical shape for the Atlanta Olympic Games in 1996. Figure.1 shows the structural layout of Georgia Dome. The system, which is improved by M.P. Levy and named as 'Levy system' or 'Levy dome', has triangulated shape of grids. Compared with Geiger system which has wedge shape of grids, Levy system has higher rigidity, so the stability of structure has been improved.

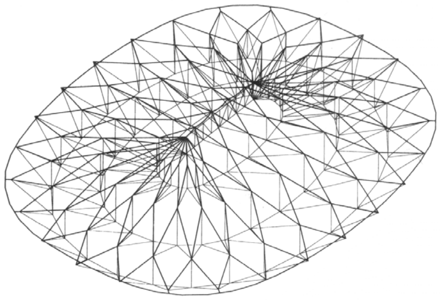


Figure.1. The structural layout of Georgia Dome

The main characteristics of cable domes are as follows[1].

1. The members of the structure are in state of all-tensioned;
2. There is a close relation between the working capacity and shape of structure;
3. The rigidity of structure is provided by prestress;
4. The structure is a self-supported, self-equilibrum and non-conservative system;
5. The process of getting shape is also the process of constructing.

3. ANALYSIS OF PRESTRESSES IN CABLE DOME STRUCTURE

The cables of cable domes have little compressive strength, so some of the ridge cables will be slack when they bear loads without prestresses. Having been prestressed appropriately, all the cables can keep tight and get a certain geometric shape and enough rigidity while the loads are put on the structure. To obtain a high rigidity, high level of prestresses is necessary. The cable domes.

The analysis of prestresses aims to find out the relation between geometric shape and initial forces by prestressing in members of cable domes in initial equilibrium state, then confirm the corresponding distribution of initial forces by prestressing in members of cable domes for a certain geometric shape of structure. Cable dome has high symmetry property so the distribution of initial forces may also have similar characteristics. Accordingly, it is only a few variables needed to express the geometric parameters of cable domes so that some formulations can be provided, by which the distribution of initial forces in structure can be determined.

For a cable dome which has a certain geometric shape, the equilibrium equations of cables and struts which connect the joint k can be provided. All of the equations must be tenable in three directions of the integral coordinate system:

$$f_i^k = \sum_{j=1}^{M_k} \frac{x_i^k - x_i^{k'}}{L_j} s_j \qquad (i = 1, 2, 3; k = 1, 2, \cdots, K) \tag{1}$$

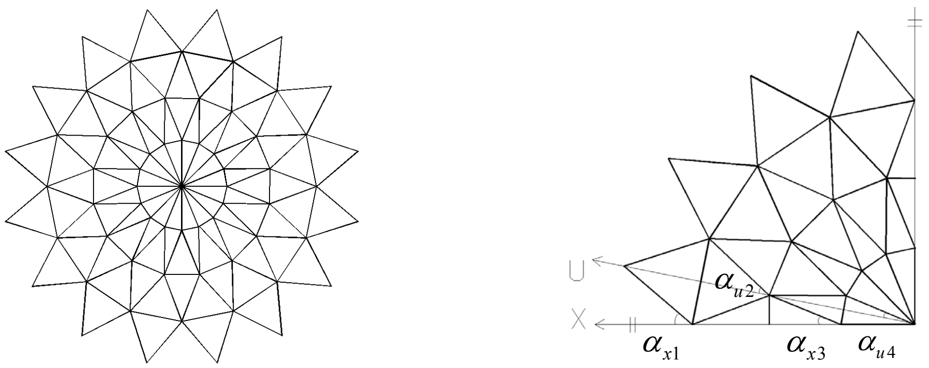
Where:

$k^{'}, k$	Joints on two ends of cable/strut j
K	Number of joints in cables and struts
M_k	Number of elements which connect joint k
L_j	Length of cable/struts j in initial state
s_j	Internal force in cable/strut j in initial state
x_i^k	Coordinate value of joint k in the i-direction
f_i^k	Force on joint k in initial state in the i-direction

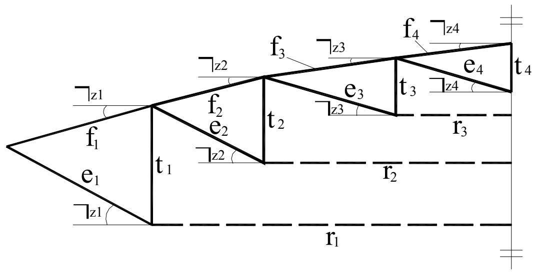
Eq. (1) can be written in a matrix form as Eq.(2).

$$[A]_{N\times M}\{S\}_M = \{F\}_N \tag{2}$$

Where N is the number of degrees of freedom of unlimited joints of cables and struts; $[A]_{N\times M}$ indicates the coefficient matrix; $\{S\}_M = [s_1, s_2, \cdots, s_M]^T$ are the vectors of distribution of initial forces in cables and struts, the number of cables and struts is M ; $\{F\}_N = [f_1, f_2, \cdots, f_N]^T$ are the vectors of loads.



(a) Plan view (b) Parameters in plan view



(c) Parameters in section view

Figure.2. Levy dome with central strut

For Levy dome with central strut, the distribution of initial forces in cables and struts can be expressed as

In struts:

$$\begin{cases} t_i = 2e_i \sin\alpha_{zi} & (i = 1,2,\dots,w) \\ t_{w+1} = n\cdot e_{w+1}\sin\alpha_{z,w+1} \end{cases} \tag{2}$$

In hoop cables:

$$\begin{cases} r_i = e_i\dfrac{\cos\alpha_{xi}\cos\alpha_{zi}}{\sin(\pi/n)} & (i = w, w-2,\dots) \\ r_i = e_i\dfrac{\cos\alpha_{ui}\cos\alpha_{zi}}{\sin(\pi/n)} & (i = w-1, w-3,\dots) \end{cases} \tag{3}$$

In diagonal cables:

$$e_i = f_i\frac{\sin\beta_{zi}}{\sin\alpha_{zi}} \qquad (i = 1,2,\dots,w) \tag{4}$$

In ridge cables:

$$\begin{cases} f_{w+1} = e_{w+1}\dfrac{\sin\alpha_{z,w+1}}{\sin\beta_{z,w+1}} \\ f_w = \dfrac{f_{w+1}\cos\beta_{z,w+1} + e_{w+1}\cos\alpha_{z,w+1}}{2\cos\beta_{zw}\cos\alpha_{xw}} \\ f_i = \dfrac{f_{i+1}\cos\beta_{z,i+1} + e_{i+1}\cos\alpha_{z,i+1}}{\cos\beta_{zi}\cos\alpha_{ui}}\cos(\alpha_{x,i+1} - \alpha_{u,w+1}) \\ f_i = \dfrac{f_{i+1}\cos\beta_{z,i+1} + e_{i+1}\cos\alpha_{z,i+1}}{\cos\beta_{zi}\cos\alpha_{xi}}\cos(\alpha_{x,i+1} - \alpha_{u,w+1}) \end{cases} \tag{15}$$

n	number of sides in the polygon
w	number of rings (in Figure.2, w=3)
i	a series of number of rings which are shown in Figure.2 ,used for identification of members
r_i	axial force in the hoop cable at the i th ring
t_i	axial force in the strut at the i th ring
t_{w+1}	axial force in the $(w+1)$ th strut (central strut)
e_i	axial force in the diagonal cable at the i th ring
e_{w+1}	axial force in the diagonal cable at the $(w+1)$ th ring
f_i	axial force in the ridge cable at the i th ring
f_{w+1}	axial force in the ridge cable at the $(w+1)$ th ring
α_{xi}	inner angle between the diagonal cable at the i th ring and x -axis
α_{ui}	inner angle between the diagonal cable at the i th ring and u -axis
α_{zi}	inner angle between the diagonal cable at the i th ring and the horizontal plane
β_{xi}	inner angle between the ridge cable at the i th ring and x -axis
β_{ui}	inner angle between the ridge cable at the i th ring and u -axis
β_{zi}	inner angle between the ridge cable at the i th ring and the horizontal plane

Most of the parameters in the formulations above are shown in Figure.2, the meanings of other symbols are as follows.

To check up the formulations above, we should set geometric parameters of an analytical model. The values of spaces between neighboring rings and lengths of struts are listed in Table.1, $\tan^{-1}(\beta_{z1})=0.24$, $\tan^{-1}(\beta_{z2})=0.20$, $\tan^{-1}(\beta_{z3})=0.16$, $\tan^{-1}(\beta_{z4})=0.12$.

Applying the formulations (12)～(15), we can get the distribution of initial forces in the Levy system dome, which is close to the result obtained by the Finite Element Analysis(FEA) software package, the results are shown in Table.2. The formulations are proved to be accurate.

Table.1 Parameters of the analytical model

Spaces between neighboring hoops	Lengths of struts(from central strut to the outmost strut)
15m、15m、15m、15m	9m、11m、13m、15m

Table.2 Comparison of the computing results of initial forces by prestressing in members of the structures (kN)

	Number	Results obtained by applying formulations (F1)	Results obtained by software computing (F2)	|F1-F2|/F2（%）		Number	Results obtained by applying formulations (F1)	Results obtained by software computing (F2)	|F1-F2|/F2（%）
Diagonal cable	1	2838.37	2839.08	0.03%	Ridge cable	1	6159.11	6162.56	0.06%
	2	1446.75	1446.63	0.01%		2	3707.21	3709.36	0.06%
	3	811.304	811.166	0.02%		3	2463.23	2464.7	0.06%
	4	1000	1000	0		4	3631.96	3633.83	0.05%
Strut	1	-2874.74	-2876.85	0.07%	Hoop cable	1	9580.95	9584.5649	0.04%
	2	-1454.09	-1454.84	0.05%		2	5446.4326	5448.2759	0.03%
	3	-778.332	-778.325	0.001%		3	3381.1849	3382.5944	0.04%
	4	-6923.7	-6926.11	0.03%					

The initial force by prestressing in diagonal cable4 is 1000kN.

All the formulations above are based on a circular shape of structure which is symmetric. There is only one independent variable of initial forces by prestressing in members of the structure.

4. ANALYSIS OF LOAD-BEARING PROPERTY OF LEVY DOME

4.1. Analytical model

Though there are some literatures about the behavior of cable domes[3][4][5], only a few of them refer to the behavior of Levy dome[6], so we analyze a Levy dome, the layout of the cable dome structure is shown in Figure.4, the symbol of the members of structure are shown in Figure.3. The rise of dome is 1/10 of the diameter. The slopes of ridge cables are set as follows: $\tan^{-1}(\beta_{z1}) = 0.24$, $\tan^{-1}(\beta_{z2}) = 0.20$, $\tan^{-1}(\beta_{z3}) = 0.16$, $\tan^{-1}(\beta_{z4}) = 0.12$. The lengths of struts from the outmost one to the central one are: $l_1 = 15m$, $l_2 = 13m$, $l_3 = 11m$, $l_4 = 9m$. The prestresses in diagonal cables, struts, ridge cables, hoop cables are respectively $240Mpa$, $-200Mpa$ (Compressed)、$400Mpa$、$200Mpa$. The initial force of inner diagonal cables is $500kN$. The cross-sections of the members are determined by the distribution of prestresses and initial force in themselves.

The modulus of elasticity are:

$E = 1.8 \times 10^8 kN/m^2$ (steel cable),

$E = 2.1 \times 10^8 kN/m^2$ (steel strut).

Vertical live load: $p_{\perp} = 0.3kN/m^2$, that converting to joints load are:

Load on one joint at 1th ring: $78kN$,

Load on one joint at 2th ring: $52kN$

Load on one joint at 3th ring: $26kN$

Load on one joint at 4th ring: $69kN$

Two loading cases taken into account are:

Load case 1: dead load + live load on all parts of the structure;

Load case 2: dead load + live load on half of the structure

4.2. Analysis of the results

Under load-case 1, the vertical displacement of joints and tensions in members of structure are shown in Figure.4.

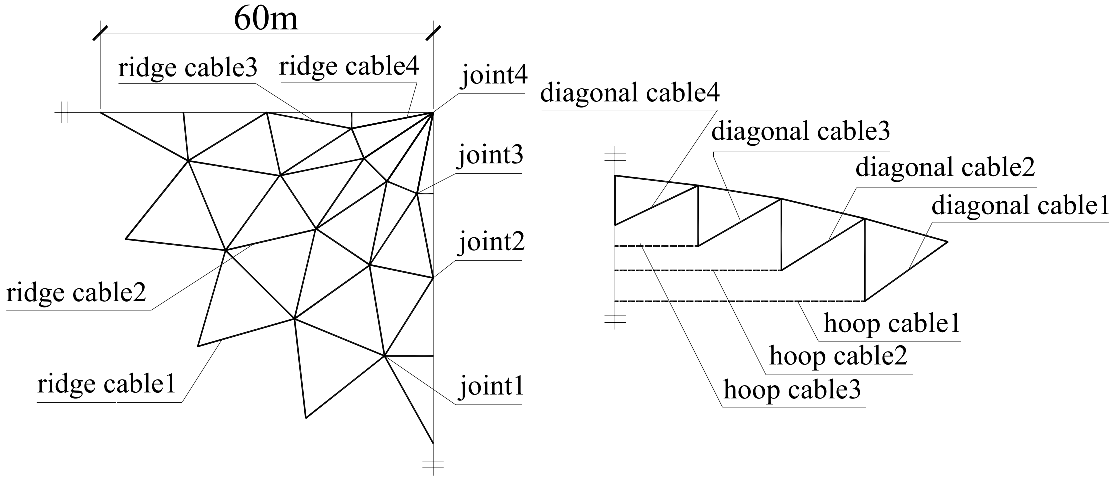
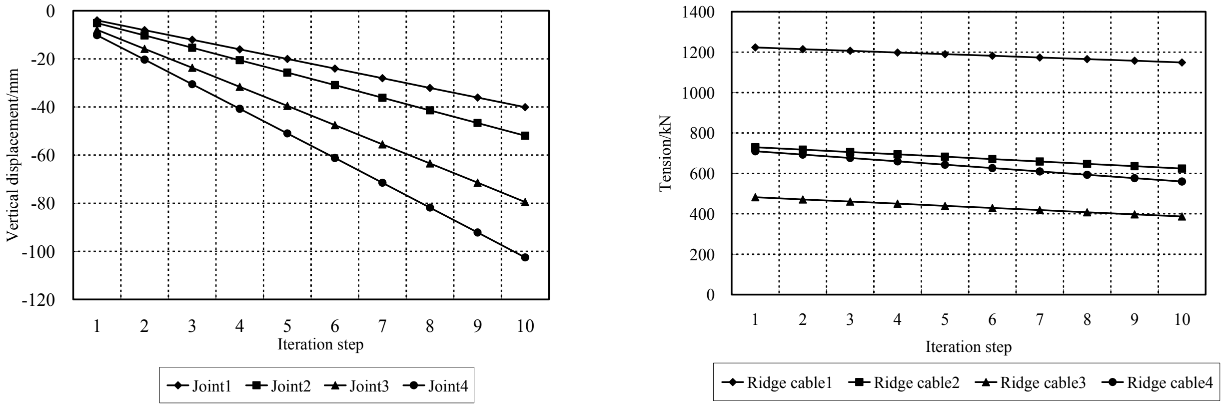
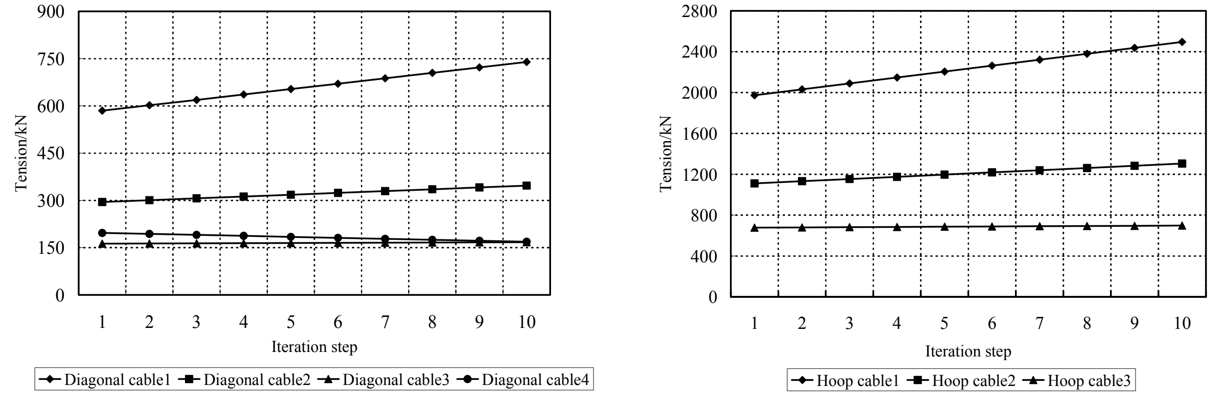


Figure.3 Symbols in analytical model



(a) Variation of joint vertical displacement under load-case 1

(b) Variation of rigid cable tension under load-case 1



(c) Variation of diagonal cable tension under load-case 1

(d) Variation of loop cable tension under load-case 1

Figure.4 Variations of displacement and tensions of Levy dome under load-case 1

By analyzing results shown in Figure.4, we can conclude that:

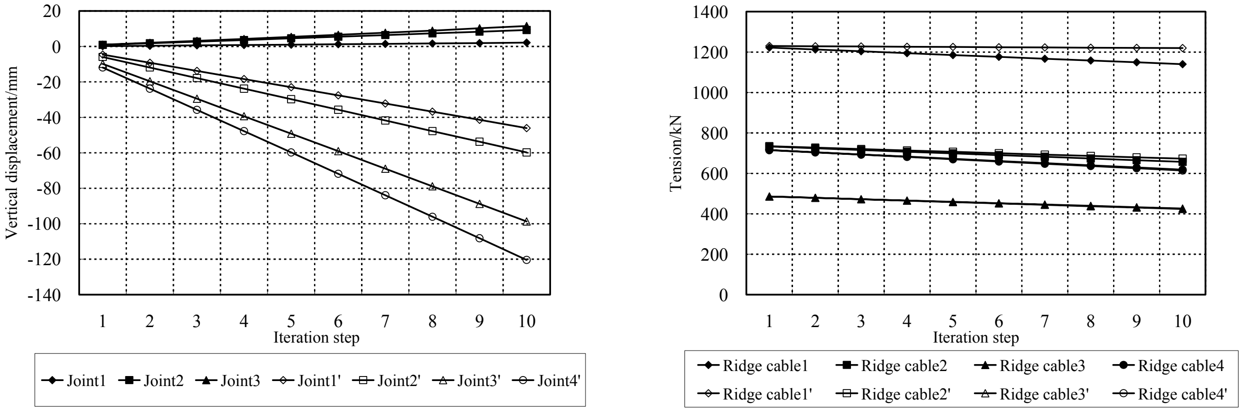
The vertical displacement in joints, the tensions in diagonal cables and ridge cables, except for diagonal cable 4, are growing along with the increase of load. The tensions in outmost cables are growing faster. At the same time, the tensions in ridge cables decrease. For the accumulation of the vertical displacements from the outmost ring to the central ring, the displacement in the center of the structure is bigger than others. When the load attains a certain value, the tension in one of the ridge cables, usually in the central part of the dome, will decrease to zero at one time and the structure becomes slack so that it can not work properly. So we define the value of load at this time as the limit of load-carrying capacity.

If the level of prestresses is low, the tension in ridge cable 1 will fall along with the increase of load at the beginning, then turn rising. The cause will be analyzed later.

Under load-case 2, the vertical displacements of joints and tensions in members of structure are shown in Figure.5.The cables or joints □’, such as ridge cable4’ or joint2’, figure the cables or joints in the half of the structure which don’t bear load.

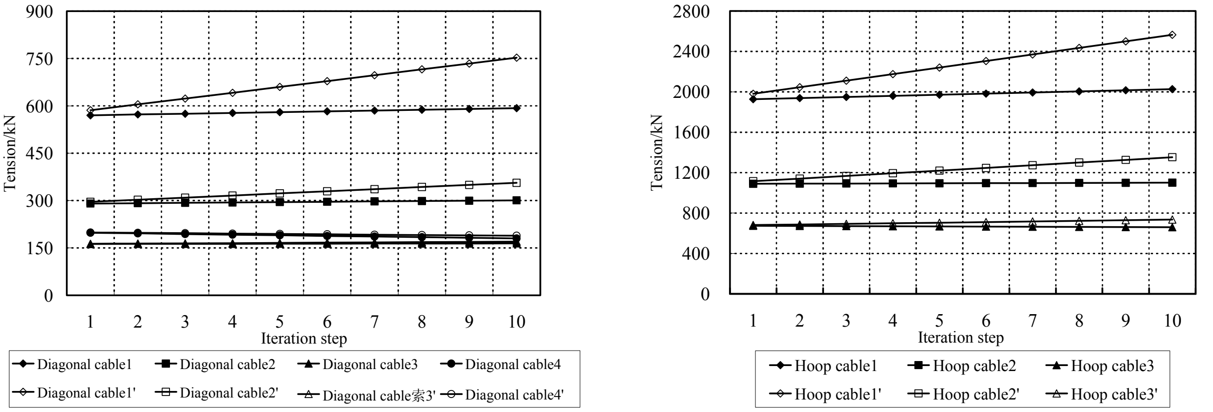
By analyzing results shown in Figure.5, we can conclude that:

1. In the members in the half of the structure which bear the load, the variation tendency of the vertical displacements of joints, the tensions in ridge cables, diagonal cables and hoop cables are similar with the variation tendency of vertical displacements of joints and tensions of those members in the structure under load-case 1.
2. In the members in the half of the structure which don’t bear the load, compared with the load-bearing characteristics of the members which bear the load, the directions of vertical displacements are opposite,



(a) Variation of joint vertical displacement under load-case 2

(b) Variation of rigid cable tension under load-case 2



(c) Variation of diagonal cable tension under load-case 2

(d) Variation of loop cable tension under load-case 2

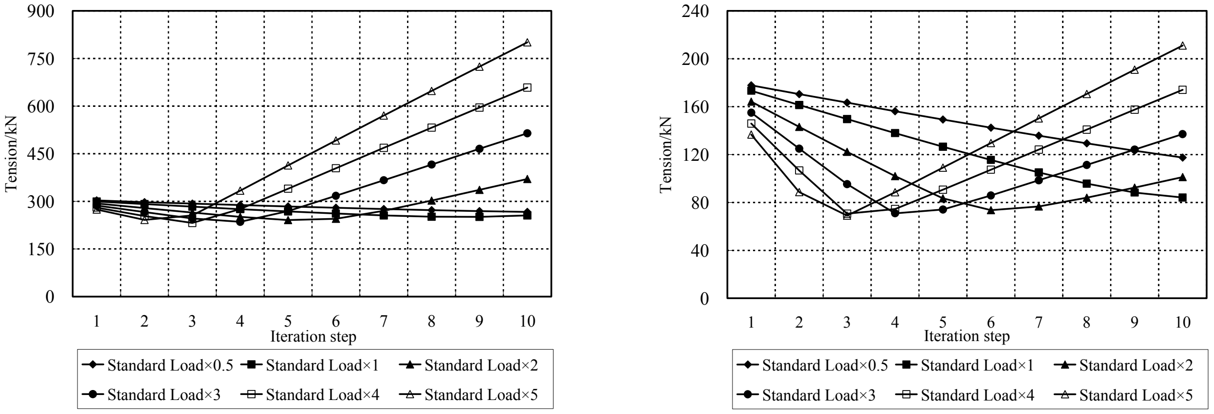
Figure.5 Variations of displacement and tensions of Levy dome under load-case 2

that is, they are upward. However, the variation tendency of tensions in members in either half of the structure, no matter they bear the load or not, are similar. Only in the cables in the central part of the dome, the tendency has a little difference.

As we have already discussed, when the load increase to a certain value, one of the ridge cables will be slack and the structure can not work properly. So we name the vertical load taken on joints as 'the standard load', which are: $78kN$ in joint1、 $52kN$ in joint2、 $26kN$ in joint3、 $69kN$ in joint4 . We set the load on the joints as several times 'the standard load'. The tensions in ridge cables are shown in Figure.9, the numbers of ridge cables are shown in Figure.6. When the load comes to be nearly 6 times the 'standard load', the results can not be convergent. At this time, we could consider that the value of internal force in ridge cable4 approaches to zero and the values of loads are the limit of load-carrying capacity.

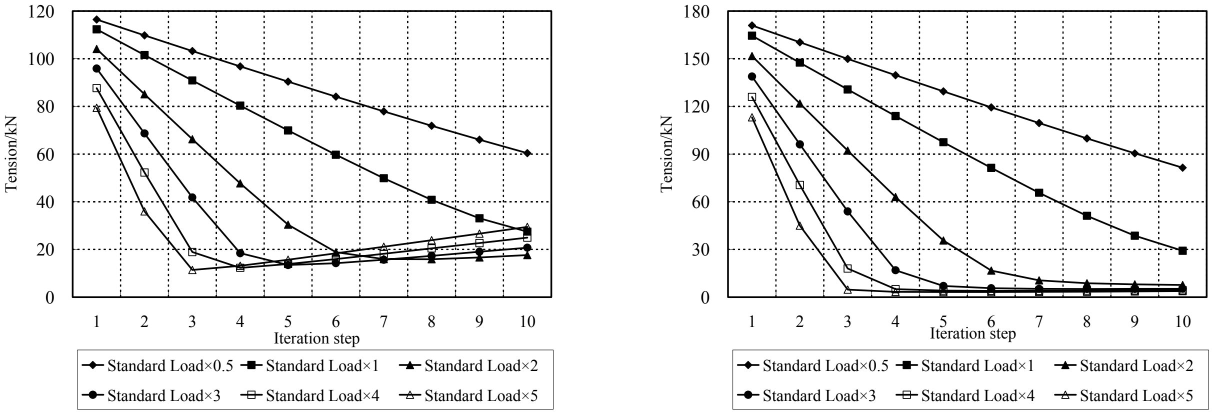
By analyzing results shown in Figure.6, we can conclude that:

The value of tensions in ridge cable4 will approach to zero fast along with the increase of load. At that time, ridge cable4 can not restrict the central strut, the structure is not stable.



(a) Variation of rigid cable 1 tension under different loads

(b) Variation of rigid cable 2 tension under different loads



(c) Variation of rigid cable 3 tension under different loads

(d) Variation of rigid cable 4 tension under different loads

Figure.6 Variation of rigid cable tension under different loads

Except for ridge cable4, the tensions in other ridge cables don’t fall all the time. When the load comes to be a certain value, the tensions will turn rising. The cause is: by analyzing the relation of force on the upper joint of central strut (shown in Figure.7), we can conclude that the internal force in ridge cable 4 is determined by its vertical component of forces. When there is no external load, the vertical component of forces in ridge cable 4 is equal to the vertical components of forces in diagonal cable 4. Along with the increase of load, the upper joint of central strut has downward displacement, which makes the geometric deformation of ridge cable 4, so the force in this cable is decreasing, ultimately approaching to zero. But the responses of outer ridge cables are not the same. Searching the force in members connected with the upper joint of strut 1(shown in Figure.8), there is an equilibrium of horizontal component of forces in members: the horizontal component of forces in ridge cable 1 is equal to the sum of the horizontal component of forces in diagonal cable 2 and ridge cable 2. Along with the increase of load, in the horizontal direction, the decrease of force in ridge cable 2 is faster than the increase of force in diagonal cable 2 at the beginning, the sum of them is decreasing, so the force in ridge cable 1 is decreasing simultaneously. When the load attains a certain value, in the horizontal direction, the increase of force in diagonal cable 2 is faster than the decrease of force in ridge cable 2, thus the force in ridge cable 1 begin to increase.

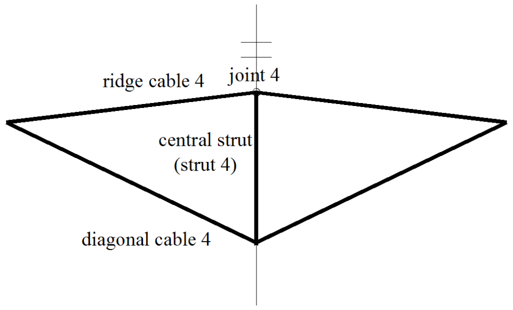


Figure.7 Members of structure around upper joint of central strut

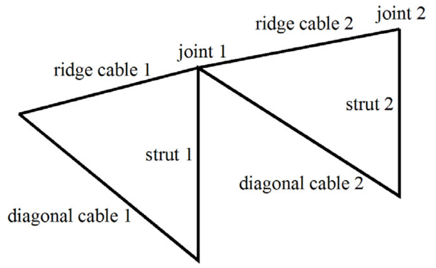


Figure.8 Members of structure around upper joint of strut 1

Table 3. 6 groups of design parameters of the layouts

Number of layout	Spaces of neighboring rings	Lengths of struts
1	15m、15m、15m、15m	9m、11m、13m、15m
2	15m、15m、15m、15m	7m、9m、11m、13m
3	12m、18m、18m、12m	7m、9m、11m、13m
4	10m、20m、20m、10m	7m、9m、11m、13m

5 .DESIGN SUGGESTIONS OF CABLE DOME

It is well known that to determine the shape of a cable dome structure, it is not only involving the factor of architecture, but also the consideration from structural view. Generally，there are four groups of independent parameters should be set to determine the shape of a cable dome structure:

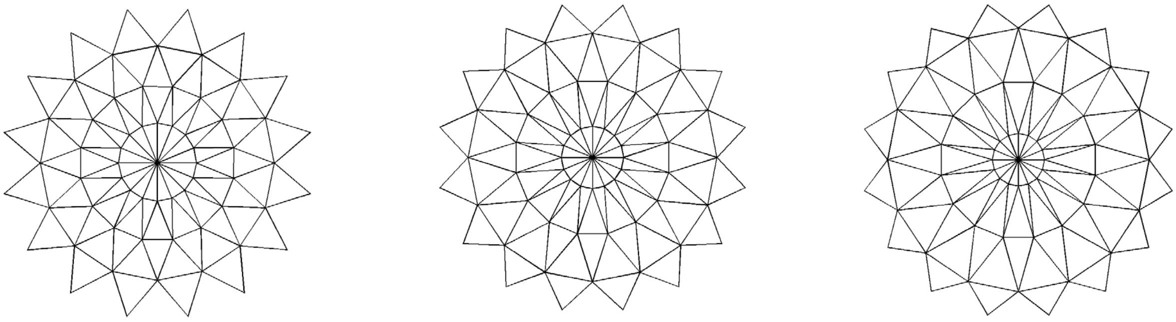
the numbers of rings and the spaces between the rings
the slopes of ridge cables
the lengths of struts
the initial force in one members of the structure

The process of determining the design parameters is also the process of optimizing the shape of the structure. If the values of the parameters above are chosen properly, the cable dome structure will have a good-looking shape. The parameters are also the deciding factors of the distribution of initial forces in members of the structure. So both the factors of obtaining a good-looking shape and keeping a proper level of prestresses should be included into consideration simultaneously. The first two groups of the parameters can be determined by factor of architectural view, then the designer can determine the last two groups of parameters by structural consideration.

The number of rings should be determined by the scale of the structure. If the diameter of the structure is less than 100m, two rings will be enough, if it is more than 100m, three rings will be needed.

The value of load on the outmost members is bigger, so the lengths of the outmost struts should be longer than others in order to strengthen the vertical component of forces in outmost diagonal cables (such as diagonal cable 1). Long struts will disturb the view of some audience seating in rearward and need to enlarge the cross-sections to fulfill the demand of stability. Considering the restrictions above, the space between ring 1 and ring 2 can be set smaller than the spaces between others. The space between other rings can be equal.

Generally speaking, if the rise in the center of the dome is smaller, the level of forces in ridge cables is higher, the dome would attain a higher rigidity and



(a) Layout 1 and 2 (b) Layout 3 (c) Layout 4

Figure.9. Plan view of six layouts

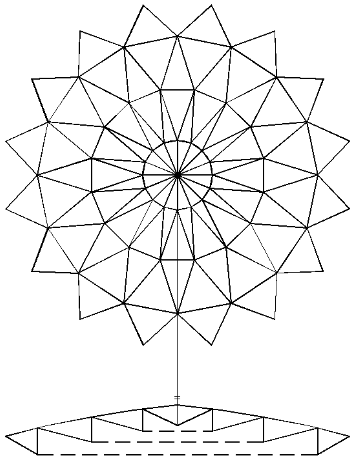


Figure.10. Plan and section views of the preferable layout

load-bearing capacity. But if the rise of the dome is too small, the dome will be easy to collapse. To get a suitable shape of the dome which has a gently curve, the slopes of ridge cables need to diminish gently from the outmost part to the central part.

Considering the shape of cable dome, the lengths of struts can be set in an arithmetical progression. For the design of cable dome which has a diameter of more than 100m, two groups of lengths are suggested. The listing sequence is from the central strut to the outmost strut:

Group 1: 9m, 11m, 13m, 15m;

Group 2: 7m, 9m, 11m, 13m.

The initial force in one member of cable domes is determined by the analysis of the load-bearing capacity.

Collecting the suggestions given above, several testing layouts of a cable dome structure which have different design parameters are listed in Table.3.

The plan views of the layouts are shown in Figure.9. We can see that the plan views of the first three layouts are symmetrical, and the struts in layout 2 and 3 are shorter. Considering the collapsing possibility of the cable dome structure, layout 3(shown in Figure.10) is better.

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