# Batch Size Distribution in Input Flow to Queues with Finite Buffer

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***Abstract*** *- Queueing models are good models for fragments of communication systems and networks, so their investigation is interesting for theory and applications. Theses queues may play an important role for the validation of different decomposition algorithms designed for investigating more general queueing networks. So, in this paper we illustrate that the batch size distribution affects the loss probability, which is the main performance measure of a finite buffer queues.*

*Key words: Batch Markovian Arrival Process, Finite Buffer, Loss Probability, and Queueing Models*

## 1. Introduction

Queueing models with a finite buffer are useful in design of many computers and telecommunication systems. So, they got a lot of attention of researchers. Extensive work in this direction was done by research group leaded by P.P. Bocharov. A very general model of the *BMAP/SM/1/N* type was investigated in [3]. Here the *BMAP* stands for the *Batch Markovian Arrival Process*, for description and more details see [1], [5], *SM* service process, see e.g.[6], assumes that the successive service times can be dependent.

In [3], the *BMAP/SM/1/N* model was considered in detail. The numerically stable algorithms for calculating the embedded and arbitrary time stationary distributions of the queue length are presented and the loss probability is calculated. In paper [4], this research is supplemented by consideration of another customer admission disciplines.

These disciplines define customer's admission in situation when the buffer is not full at a batch arrival epoch, but the whole batch cannot be put into the buffer. In [3], the partial admission disciplines were considered. This discipline suggests that a part of a batch corresponding to the available buffer space is accepted into the system while the rest leaves the system forever. In [4], the complete admission and complete rejection disciplines are dealt with. The former one suggests that the whole batch is admitted to enter the system. The latter one assumes that the whole batch is rejected.

In the paper [7], the *BMAP/SM/1/N* models with partial admission and complete rejection disciplines are considered. Additionally, vacations and setup times are taken into an account. Based on their numerical experience, the authors by [7] assert that only the mean batch size, but not the batch size distribution affects essentially performance measures of the system. So they conclude that the contribution of the accurate batch size distribution is trivial to the system's performance. Our own experience contradicts to this assertion. The aim of this paper is to illustrate that the batch size distribution affect the loss probability, which is the main performance measure of a finite buffer queue.

## 2. Mathematical Model

We consider a single queue with a finite buffer of capacity N. The customers arrive in the *BMAP*. The *BMAP* is described in the following way. Continuous time Markov chain with $v_t, t \ge 0$ a finite space $\{0,1,\cdots,W\}$, which is called as the directing process of the BMAP, is defined as follows. The sojourn time of the chain in state has exponential distribution with parameter $\lambda_v$. After this time expires, the chain makes a transition into some state $v'$ and with probability $p_k(v,v')$ this transition is accompanied by generation of a batch consisting of $k$ customers, $k \ge 0$. It is assumed that $p_0(v,v)=0$ and $\sum_{k=0}^{\infty}\sum_{v'=0}^{W} p_k(v,v')=1$ for any $v=\overline{0,W}$. Lucantoni [5] offered to keep information about the transitions of the process $v_t, t \ge 0$ in matrices $D_k, k \ge 0$ having entries $(D_k)_{v,v'}$ defined as follows:

$$(D_0)_{v,v} = -\lambda_v ,$$

$$(D_0)_{v,v'} = -\lambda_v \; p_0(v,v'), v' \ne v ,$$

$$(D_k)_{v,v'} = -\lambda_v \; p_k(v,v'), k \ge 1, \; v, v' = \overline{0,W} .$$

In turn, information about the matrices $D_k, k \ge 0$ can be kept by means of the matrix generating function $D(z)=\sum_{k=0}^{\infty} D_k z^k, |z| \le 1$. The matrix $D(1)$ is the generator of the Markov chain $v_t, t \ge 0$. Row vector $\vec{\theta}$ of stationary distribution of the chain $v_t, t \ge 0$ is calculated as the unique solution to the system $\vec{\theta}D(1)=\vec{0}$, $\theta\vec{e}=1$. Here $\vec{e}$ is the column vector consisting of one's, $\vec{0}$ is the row vector consisting of zeroes. Average intensity $\lambda$ (fundamental rate) of the *BMAP* is calculated as $\lambda = \vec{\theta}D'(1)\vec{e}$, average intensity $\lambda_g$ of groups is calculated by $\lambda_g = \vec{\theta}(-D_0)\vec{e}$. The *BMAP* is very popular in literature because it takes into account possible correlation of successive inter arrival times what is important from the point view of modeling the modern telecommunication networks. Following to [7], we assume that the service process is recurrent. Service time is characterized by the distribution function $B(t)$ with the finite mean $b_1 = \int_0^{\infty} t dB(t)$.

## 3. Analysis of the Model

Calculation of the stationary distribution of a number of customers in the system $i_t, t \ge 0, i_t = \overline{0, N+1}$, can be implemented in several ways. Probably, the simpler way is to apply the techniques of imbedded Markov chains. Consider behavior of the queueing model at the service completion epochs $t_k, k \ge 1$. Two-dimensional process $\{i_{t_k+0}, v_{t_k}\}$, $k \ge 1$, $i_{t_k+0} = \overline{0,N}$, $v_{t_k} = \overline{0,W}$ is the Markov chain.

Dennote $\pi(i,v) = \lim_{k\to\infty} P\{i_{t_k}=i, v_{t_k}=v\}$, $i=\overline{0,N}$, $v=\overline{0,W}$ and $\pi_i = [\pi(i,0), \cdots, \pi(i,W)]$.

In [3], [4], stable algorithms for calculation of vectors $\pi_i, i \ge 0$ are elaborated. Having these vectors been computed, the arbitrary time distribution of the process $\{i_t, v_t\}, t \geq 0$, $i_t = \overline{0, N+1}$, $v_t = \overline{0, W}$ is calculated using the Markov renewal process theory [2].

Loss probability $P_{loss}$ of arbitrary customer is calculated by

$$P_{loss} = 1 - (\lambda \tau)^{-1}$$

where $\lambda$ is the fundamental rate of the *BMAP* and $\tau$ is the average inter departure time.

*For different admission disciplines, the value of* $\tau$ is calculated as follows:

$\tau = b_1 + \pi_0 (-D_0)^{-1} \vec{e}$ for partial admission and complete admission disciplines,

$\tau = b_1 + \pi_0 (-1) \left( D_0 + \sum_{k=N+2}^{\infty} D_k \right) \vec{e}$ for complete rejection discipline.

## 4. Impact of the Batch Size Distribution

One of the main conclusions of the paper [7] is that, under the fixed average batch size, say $K$, loss probability is almost insensitive with respect to the distribution of a batch size.

Incorrectness of this conclusion is intuitively clear in case when the average batch size distribution $K$ is close to the buffer capacity $N$. So, to illustrate the effect of the batch size distribution we do not consider this obvious case and admit the same assumption as was done in [7], namely, we suppose that $N = 10K$, so the values $K$ and $N$ are quite different. Aiming to be more close to the input data of the paper [7], we also assume that the arrival process is the *BIPP*- Batch Interrupted Poisson Process. This process having fundamental rate $\lambda = 0.452817$, correlation coefficient $c_{cor} = 0$, and squared variation coefficient $c_{\text{var}} = 5$ is constructed based on the *IPP*- Interrupted Poisson Process defined by the following matrices:

$$D_0 = \begin{bmatrix} -1.01884 & 0.113204 \\ 0.113204 & -0.113204 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.905634 & 0 \\ 0 & 0 \end{bmatrix}$$

Concrete mechanisms for constructing different flows having the same mean batch size value but different batch size distribution will be explained below. We assume that service time is constant, i.e., it has degenerate distribution. To illustrate the influence of the traffic intensity $\rho$ (load of the system), we take three different values $T$ of the service time. These values of $T$ are taken as 1.10420, 1.76672, 2.65008; the corresponding traffic intensities $\rho$ are equal to 0.5, 0.8, and 1.2.

To show that the batch size distribution influence the value of the loss probability, we consider five different *BIPP*s having the same mean batch size $K$, but different batch size distribution.

- the *BIPP*, corresponding to the curve, which has legend $\{K\}$ on the following Figures, has a constant batch size $K$;
- the *BIPP*, corresponding to the curve, which has legend $\{K-1, K+1\}$, has a constant batch size $K-1$ or $K+1$ with probability 0.5.
- the *BIPP*, corresponding to the curve, which has legend $\{1, 2K-1\}$, has a constant batch size 1 or $2K-1$ with probability 0.5.

- the *BIPP*, corresponding to the curve, which has legend *geometric*, has a restricted geometric distribution. Thus, probability that the batch size is equal to $k$ is equal to $\gamma^{k-1}(1-\gamma)/(1-\gamma^{M})$, $k=\overline{1,M}$, where $M$ is the maximal batch size and $\gamma$ is some parameter.

Figures 1-2 illustrate dependence of the loss probability $P_{loss}$ on the mean batch size $K$ for different values of the load and the *BIPP*s presented above. One should see that the batch size distribution essentially influences the value of the loss probability. Note, that we included the curve corresponding to the *BIPP*, which has legend $\{1,3K-1\}$, only on one figure between Figures 1 and 2 and present the curves corresponding to this *BIPP* separately on Figure 5 because loss probability in this case is much higher than in other cases and the difference between these is not shown properly. Mention, that if we allow having much essential difference in a batch size, e.g., $\{1,4K-1\}$, $\{1,5K-1\}$, etc, the difference in loss probability essentially increases. Situations when the difference in a batch size is essential are not rare in practical situations. For example, it takes place when some customers correspond to short network control message while the others are long messages sent by users. Thus numerical examples confirm the conclusion that account of a batch size distribution is essential in analysis.

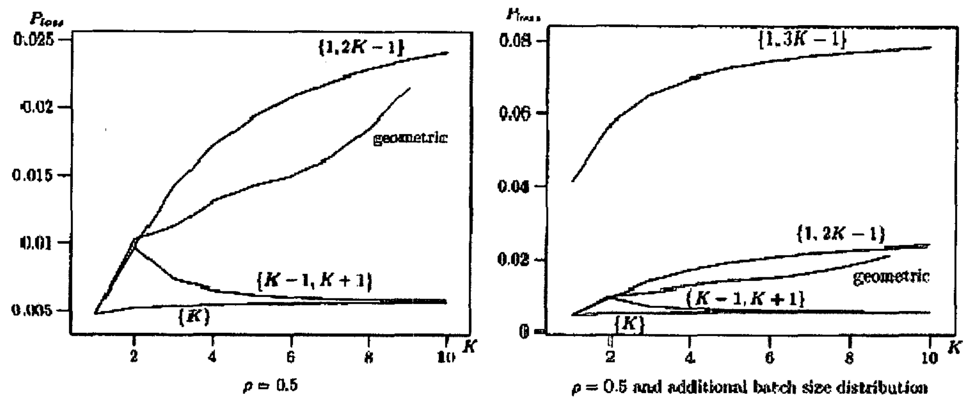


Figure 1. Loss probability $P_{loss}$ depending on the mean batch size $K$ for different distribution of customers in a batch for the constant system load

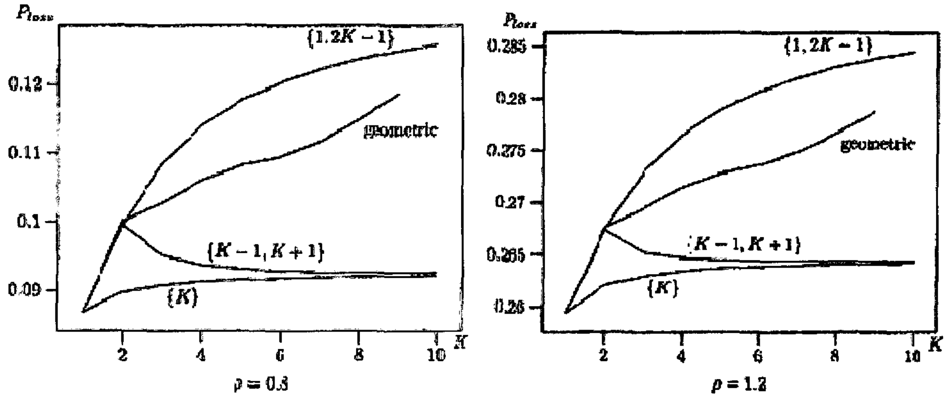


Figure 2. Loss probability $P_{loss}$ depending on the mean batch size $K$ for different distribution of customers in a batch for the constant system load

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