A Fault Detection System Design for Uncertain Fuzzy Systems

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Abstract – This paper deals with a fault detection system design for uncertain nonlinear systems modelled as T-S fuzzy systems with the integral quadratic constraints. In order to generate a residual signal, we used a left coprime factorization of the T-S fuzzy system. From the filtered signal of the residual generator, the fault occurence can be detected effectively. A simulation study with nuclear steam generator level control system shows that the suggested method can be applied to detect the fault in actual applications.

Keywords: Fault detection, fuzzy system, coprime factorization, integral quadratic constraint.

1 Introduction

Recently, fault detection of control systems has been an active research area due to the growing complexity of modern automatic control systems. A fault can be defined as an unexpected change in a system, such as a component malfunction, which tends to cause undesirable behaviour in the overall system performance. The purpose of the fault detection system design is to detect the occurrence of a fault as fast as possible.

A lot of works for the fault detection system design have been developed based on generation of the residual signal [1-5]. In order to design the residual generator, the use of robust parity equations [1], eigenstructure assignment [2], unknown input observers [3] and H_∞ optimal estimation approaches [4,5] have been intensively investigated. But these works are mainly focused on linear systems. Comparatively little work has been reported for the fault detection system design of nonlinear systems.

In the past few years, there has been growing interest in fuzzy control of nonlinear systems, and there have been many successful applications. Among them, a controller design method for nonlinear dynamic systems modelled as a T-S(Takagi-Sugeno) fuzzy model has been intensively addressed [6-8]. Unlike a single conventional model, this T-S fuzzy model usually consists of several linear models to describe the global behaviour of the nonlinear system. Typically the T-S fuzzy model is described by fuzzy IF-THEN rules.

In this paper, using the coprime factorization approach we develop a fault detection scheme for T-S fuzzy systems with the integral quadratic constraints(IQC's). The IQC's can be used conveniently to describe uncertain parameters, time delays, unmodeled dynamics, etc [9]. An application to fault detection of a nuclear steam generator control system is presented in order to demonstrate the efficacy of the proposed scheme. We think that the primary contribution of this work is that the fault detection system design is extended from linear systems toward nonlinear systems using T-S fuzzy approach.

2 Uncertain Fuzzy Systems

We consider the following fuzzy dynamic system with the IQC's.

Plant Rule i $(i=1,\cdots,r)$:

IF $\rho_1(t)$ is M_{i1} and $\cdots$ and $\rho_g(t)$ is M_{ig},

THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + \sum_{j=1}^{q} F_{ij} w_j(t) + B_i u(t) \\ z_j(t) &= H_{ij} x(t) + J_{ij} u(t) \\ y(t) &= C_i x(t) + \sum_{j=1}^{q} G_{ij} w_j(t) + D_i u(t) \\ w_j(t) &= \theta_j z_j(t), \end{aligned} \tag{1}$$

where r is the number of fuzzy rules. $\rho_k(t)$ and M_{ik} ($k=1,\cdots,g$) are the premise variables and the fuzzy set respectively. $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input, $y(t)\in R^p$ is the output variable and $w_j(t)\in R^{k_j}, z_j(t)\in R^{k_j}$ are variables related to uncertainties. $A_i, F_{ij}, \cdots, D_i$ are real matrices with compatible dimensions. θ_j is an uncertain operator described by the following IQC's.

$$\int_0^\infty w_j(t)^T w_j(t)dt \le \int_0^\infty z_j(t)^T z_j(t)dt, \quad j=1,\cdots,q. \tag{2}$$

Let μ_i, $i=1,\cdots,r$, be the normalized membership function defined as follows:

$$\mu_i = \frac{\prod_{j=1}^{g} M_{ij}(\rho_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{g} M_{ij}(\rho_j(t))}. \quad (3)$$

Then, for all i, we obtain

$$\mu_i \geq 0, \ \sum_{i=1}^{r} \mu_i = 1. \quad (4)$$

For simplicity, we define

$$\mu^T = \begin{bmatrix}\mu_1 & \cdots & \mu_r\end{bmatrix}, \ w(t) = \begin{bmatrix} w_1(t)^T & \cdots & w_q(t)^T\end{bmatrix}^T,$$
$$z(t) = \begin{bmatrix} z_1(t)^T & \cdots & z_q(t)^T \end{bmatrix}^T,$$
$$F_i = \begin{bmatrix} F_{i1} & \cdots & F_{iq}\end{bmatrix}, \ G_i = \begin{bmatrix} G_{i1} & \cdots & G_{iq}\end{bmatrix},$$
$$H_i = \begin{bmatrix} H_{i1}^T & \cdots & H_{iq}^T\end{bmatrix}^T, \ J_i = \begin{bmatrix} J_{i1}^T & \cdots & J_{iq}^T\end{bmatrix}^T.$$

With the notations defined above, we rewrite the uncertain fuzzy system (1) as follows :

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i (A_i x(t) + F_i w(t) + B_i u(t)) \\
&= A_\mu x(t) + F_\mu w(t) + B_\mu u(t), \\
z(t) &= \sum_{i=1}^{r} \mu_i (H_i x(t) + J_i u(t)) \\
&= H_\mu x(t) + J_\mu u(t), \quad (5)\\
y(t) &= \sum_{i=1}^{r} \mu_i (C_i x(t) + G_i w(t) + D_i u(t)) \\
&= C_\mu x(t) + G_\mu w(t) + D_\mu u(t), \\
w(t) &= diag(\theta_1, \cdots, \theta_q) z(t) = \Theta z(t).
\end{aligned}$$

In a packed matrix notation, we express the fuzzy system (5) as

$$G(\mu) = \left[\begin{array}{c|c|c} A_\mu & F_\mu & B_\mu \\ \hline H_\mu & 0 & J_\mu \\ \hline C_\mu & G_\mu & D_\mu \end{array}\right]. \quad (6)$$

We present a coprime factor model which will be used in the next section. Let L_μ be an output injection matrix such that $A_\mu + L_\mu C_\mu$ is quadratically stable for all permissible Θ and μ satisfying (2) and (4). Then $G(\mu) = \tilde{M}(\mu)^{-1}\tilde{N}(\mu)$ where $\tilde{N}(\mu), \tilde{M}(\mu)$ are quadratically stable for all permissible Θ and are given by

$$\begin{bmatrix}\tilde{M}(\mu) & \tilde{N}(\mu)\end{bmatrix} = \left[\begin{array}{c|c|cc} A_\mu + L_\mu C_\mu & F_\mu + L_\mu G_\mu & L_\mu & B_\mu + L_\mu D_\mu \\ \hline H_\mu & 0 & 0 & J_\mu \\ \hline C_\mu & G_\mu & I & D_\mu \end{array}\right]. \quad (7)$$

3 Fault Detection System

In this section, we discuss a fault detection system design method for the fuzzy system (5). We construct a residual generator using the left coprime factors. Let $\tilde{M}_0(\mu)$ and $\tilde{N}_0(\mu)$ be the nominal system of $\tilde{M}(\mu)$ and $\tilde{N}(\mu)$. Thus ,

$$\begin{bmatrix}\tilde{M}_0(\mu) & \tilde{N}_0(\mu)\end{bmatrix} = \left[\begin{array}{c|cc} A_\mu + L_\mu C_\mu & L_\mu & B_\mu + L_\mu D_\mu \\ \hline C_\mu & I & D_\mu \end{array}\right]. \quad (8)$$

Let $u_f(t)$ and $y_f(t)$ be the actuator fault and the measurement sensor fault signals respectively. Then we construct a residual signal as follows :

$$e(t) = Q(\mu)\left(\tilde{M}_0(\mu) y_m(t) - \tilde{N}_0(\mu) u(t)\right), \quad (9)$$

where $Q(\mu)$ is a fuzzy filter which will be used as a design parameter and $y_m(t) = y(t) + y_f(t)$ is an output sensor measurement. Note that

$$0 = \tilde{M}(\mu) y(t) - \tilde{N}(\mu)(u(t) + u_f(t)). \quad (10)$$

Using (10) the residual signal $e(t)$ in (9) can be expressed as

$$e(t) = Q(\mu)\left(e_d(t) + e_f(t)\right), \quad (11)$$

where

$$\begin{aligned}
e_d(t) &= \left(\tilde{N}(\mu) - \tilde{N}_0(\mu)\right)u(t) - \left(\tilde{M}(\mu) - \tilde{M}_0(\mu)\right)(y(t) + y_f(t)), \\
e_f(t) &= \tilde{N}(\mu) u_f(t) + \tilde{M}(\mu) y_f(t).
\end{aligned}$$

In (11), $e_d(t)$ and $e_f(t)$ correspond to signals due to the model uncertainties and the fault signals respectively. Even if no fault occurs, the residual signal $e(t)$ is not zero due to the system model uncertainties.

A state space realization of the coprime factor uncertainty can be expressed as

$$\begin{bmatrix}\tilde{M}(\mu) - \tilde{M}_0(\mu) & \tilde{N}(\mu) - \tilde{N}_0(\mu)\end{bmatrix} = \left[\begin{array}{cc|c|cc} A_\mu + L_\mu C_\mu & 0 & F_\mu + L_\mu G_\mu & L_\mu & B_\mu \\ 0 & A_\mu + L_\mu C_\mu & 0 & L_\mu & B_\mu \\ \hline H_\mu & 0 & 0 & 0 & J_\mu \\ \hline C_\mu & -C_\mu & G_\mu & 0 & 0 \end{array}\right]. \quad (12)$$

We define

$$\begin{bmatrix} P_{11}(\mu) & P_{12}(\mu) \\ P_{21}(\mu) & P_{22}(\mu) \end{bmatrix} = \left[\begin{array}{cc|ccc} A_\mu + L_\mu C_\mu & 0 & F_\mu + L_\mu G_\mu & L_\mu & B_\mu \\ 0 & A_\mu + L_\mu C_\mu & 0 & L_\mu & B_\mu \\ \hline H_\mu & 0 & 0 & 0 & J_\mu \\ C_\mu & -C_\mu & G_\mu & 0 & 0 \end{array}\right]. \quad (13)$$

With the definition of (13), the coprime factor uncertainty (12) also can be expressed as

$$\begin{aligned} &\left[\tilde{M}(\mu) - \tilde{M}_0(\mu) \quad \tilde{N}(\mu) - \tilde{N}_0(\mu)\right] \\ &= P_{21}(\mu)\Theta(I - P_{11}(\mu)\Theta)^{-1} P_{12}(\mu). \end{aligned} \quad (14)$$

When no fault occurs the residual signal $e(t)$ becomes

$$e(t) = Q(\mu)P_{21}(\mu)w_e(t), \quad (15)$$

where

$$w_e(t) = \Theta(I - P_{11}(\mu)\Theta)^{-1} P_{12}(\mu) \begin{bmatrix} -y(t) \\ u(t) \end{bmatrix}. \quad (16)$$

In (15), $Q(\mu)$ is chosen to minimize the effect of disturbances or model uncertainties in the frequency range (w_1, w_2) that $w_e(t)$ manifests itself. For the above purpose, we design $Q(\mu)$ such that $\|W_r(s) - Q(\mu)P_{21}(\mu)\|_\infty < \gamma$ where $W_r(s)$ a stable transfer function for frequecy shaping.

The faults can thus be detected using a simple thresholding logic:

$$J = \|e(t)\| \begin{cases} \le J_{th} & \text{normal} \\ > J_{th} & \text{faulty} \end{cases} \quad (17)$$

where

$$J_{th} = \left(\max_{w \in (w_1, w_2)} \bar{\sigma}(W_r(jw)) + \gamma \right) \|\Delta\|_\infty \|z_e(t)\|,$$

$$\Delta = \Theta(I - P_{11}(\mu)\Theta)^{-1}, \; z_e(t) = P_{12}(\mu) \begin{bmatrix} -y(t) \\ u(t) \end{bmatrix}.$$

Note that Δ is a system with feedback connection of Θ and $P_{11}(\mu)$. Hence, a state space realization of Δ can be expressed as

$$\Delta = \left[\begin{array}{cc|c|c} A_\mu + L_\mu C_\mu & 0 & F_\mu + L_\mu G_\mu & 0 \\ 0 & A_\mu + L_\mu C_\mu & 0 & 0 \\ \hline H_\mu & 0 & 0 & I \\ \hline 0 & 0 & I & 0 \end{array}\right]. \quad (18)$$

By using the well developed LMI(Linear Matrix Inequality) tools and S-procedure, we can compute $\|\Delta\|_\infty$.

4 S/G Fault Detection System

The dynamics of a nuclear steam generator is described in terms of the feedwater flowrate, the steam flowrate and the steam generator water level. Irving[10] derived the following fourth order Laplace transfer function model based on the step response of the steam generator water level for step change of the feedwater flowrate and the steam flowrate:

$$\begin{aligned} y(s) &= \frac{g_1}{s}(u(s) - d(s)) - \frac{g_2}{1 + \tau_2 s}(u(s) - d(s)) \\ &+ \frac{g_3 s}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-1} s + s^2} u(s), \end{aligned} \quad (19)$$

where

τ_1 and τ_2 damping time constants;

T period of the mechanical oscillation;

g_1 magnitude of the mass capacity effect;

g_2 magnitude of the swell or shrink due to the feedwater or steam flowrates;

g_3 magnitude of the mechanical oscillation.

This plant has a single input(feedwater flowrate $u(s)$), a single output(water level $y(s)$) and a measurable known disturbance(steam flowrate $d(s)$). The parameter values of a steam generator at several power levels are given in Table 1, and the parameters are very different according to the power levels. The third term of the right hand side in (19) is extremely small in affecting the water level response [11].

Table 1: Parameters of a steam generator linear model.

Power (%)	τ_1	τ_2	g_2	g_3	g_1	T
30	43.4	4.5	1.83	0.310	0.058	7.7
50	34.8	3.6	1.05	0.215	0.058	4.2
100	28.6	3.4	0.47	0.105	0.058	1.7

By neglecting the third term in (19), we have the following reduced order state space model :

$$\dot{x}(t)=\begin{bmatrix}0 & 0\\ 0 & -\tau_2^{-1}(1+\beta_1\theta_1)\end{bmatrix}x(t)$$
$$+\begin{bmatrix}1\\ g_2\tau_2^{-1}(1+\beta_2\theta_2)\end{bmatrix}u(t)-\begin{bmatrix}1\\ g_2\tau_2^{-1}(1+\beta_2\theta_2)\end{bmatrix}d(t) \quad (20)$$
$$y(t)=\begin{bmatrix}g_1 & -1\end{bmatrix}x(t),$$

where θ_1 and θ_2 are introduced to describe possible parameter uncertainties and satisfy $|\theta_1|\le 1, |\theta_2|\le 1$. Thus, we assume that τ_2^{-1} is within β_1 % of the nominal value given in Table 1. We treat the third term as an additive uncertain dynamics $\delta_a(s)$ such that $\delta_a(s)=W_a(s)\theta_3$, $\|\theta_3\|_\infty \le 1$ where $W_a(s)$ is a frequency weight. For simulation, let $\beta_1=\beta_2=0.1$ and $W_a(s)=\dfrac{0.62s}{s^2+0.46s+0.126}$. Then we have the following T-S fuzzy model:

Plant Rule i $(i=1,\cdots,3)$:
IF the power level is M_{i1},
THEN

$$\dot{x}(t)=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & a_i & 0 & 0\\ 0 & 0 & -0.0461 & -0.1265\\ 0 & 0 & 1 & 0\end{bmatrix}x(t)+\begin{bmatrix}0&0&0\\1&1&0\\0&0&0\\0&0&0\end{bmatrix}w(t)+\begin{bmatrix}1 & -1\\ b_i & -b_i\\ 1 & 0\\ 0 & 0\end{bmatrix}u_a(t)$$
$$z(t)=\begin{bmatrix}0 & h_i & 0 & 0\\ 0&0&0&0\\ 0&0&0.62&0\end{bmatrix}x(t)+\begin{bmatrix}0&0\\ j_i & -j_i\\ 0&0\end{bmatrix}u_a(t)$$
$$y(t)=\begin{bmatrix}0.058 & -1 & 0 & 0\end{bmatrix}x(t)+\begin{bmatrix}0&0&1\end{bmatrix}w(t) \quad (21)$$
$$w(t)=diag(\theta_1,\theta_2,\theta_3)z(t)$$

where the membership function M_{i1} $(i=1,\cdots,3)$ are depicted in Fig. 1 and

$$u_a(t)=\begin{bmatrix}u(t)\\ d(t)\end{bmatrix},\ a_1=-0.222,\ a_2=-0.278,\ a_3=-0.294,$$
$$b_1=0.4067,\ b_2=0.2917,\ b_3=0.1382,\ h_1=0.0222,$$
$$h_2=0.0278,\ h_3=0.0294,$$
$$j_1=0.0407,\ j_2=0.0292,\ j_3=0.0138.$$

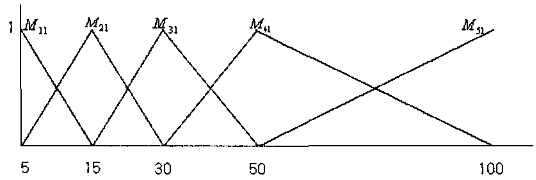

Fig. 1. Membership function

We obtain the left coprime factors in (7) using L_i $(i=1,\cdots,3)$ as follows :

$$L_1=\begin{bmatrix}-217\\ -7.84\\ 0\\ 0\end{bmatrix}, L_2=\begin{bmatrix}-226\\ -8.33\\ 0\\ 0\end{bmatrix}, L_3=\begin{bmatrix}-229\\ -8.47\\ 0\\ 0\end{bmatrix}.$$

We discuss a design method such that $\|\boldsymbol{W}_r(s)-\boldsymbol{Q}(\mu)\boldsymbol{P}_{21}(\mu)\|_\infty<\gamma$. Unfortunately, the above mentioned method does not give a good result in the steam generator level system since $\boldsymbol{P}_{21}(\mu)$ has the blocking zero at the origin for each rule. Let $\bar{\boldsymbol{P}}_{21}(\mu)=\frac{1}{s}\boldsymbol{P}_{21}(\mu)$. We obtain $\boldsymbol{Q}(\mu)=\bar{\boldsymbol{Q}}(\mu)\frac{1}{s}$ where $\bar{\boldsymbol{Q}}(\mu)$ is found such that $\|\boldsymbol{W}_r(s)-\bar{\boldsymbol{Q}}(\mu)\bar{\boldsymbol{P}}_{21}(\mu)\|_\infty<\gamma$ is satisfied. The maximum singular value plot of $\boldsymbol{W}_r(s)$ is shown in Fig. 2.

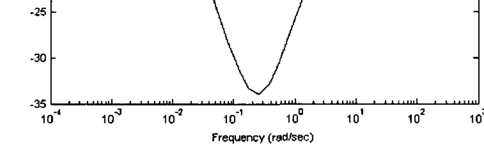

Fig. 2. The maximum singular value

A simulation of the steam generator control system has been done with the following two cases of scenario: case1: 100% power level, a 5% step disturbance on $d(t)$ at 500sec, and a drift (5%/min ramp) on the water level sensor at 1000sec.

case2: 30% power level, a 5% step disturbance on $d(t)$ at 500sec, and a -50% error on the water level sensor at 1000sec.

In order to compute J in (17), we use $J = \sqrt{\int_{t-T}^{T} e(t)^2 dt}$ for a finite time period T. The filtered residual signal is depicted in Fig. 2. The solid line shows the simulation result of case 2, and the dotted line shows the simulation result of case 1. In both cases, the effects of the step disturbance $d(t)$ on the filtered residual signal are very small while the effects of the sensor faults are remarkable.

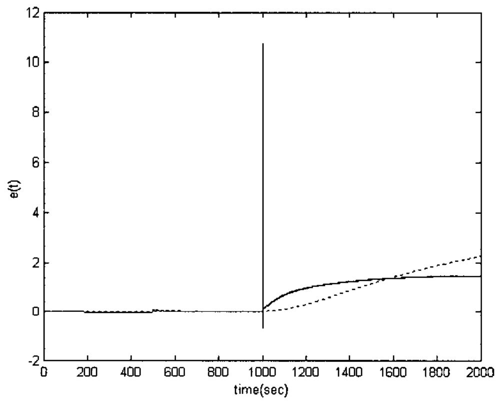

Fig. 3. The filtered residual signal

Fig. 3 shows the unfiltered residual signals(i.e. $Q(\mu) = I$) of case 1. In Fig. 3, we can observe that the effect of the disturbance $d(t)$ at 500sec is big and the effect of the drift on the water level sensor is very small.

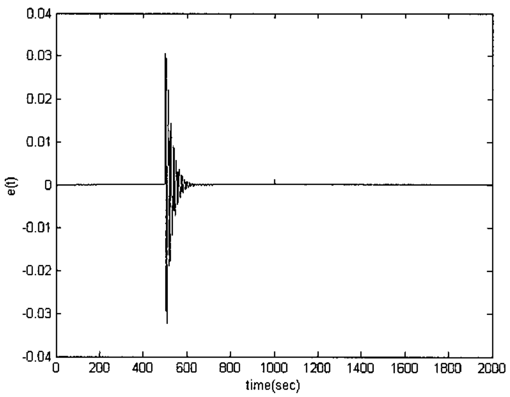

Fig. 4. The unfiltered residual signal

5 Conclusion

In this paper, we have described a fault detection scheme for T-S fuzzy systems with the IQC's. We design the residual generator from the left coprime factor model and the fuzzy filter minimizing the effect of disturbances. The norm of the residual signal is compared with a threshold level to determine whether or not a fault occurs. The developed scheme has been successfully implemented to detect fault occurrence of the nuclear steam generator water level control system.

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