Geoid of Western Mongolia from airborne gravity data 2004

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Abstract

This paper summarizes a preliminary geoid computation for western Mongolia, utilizing the airborne data collected fall 2004, as part of the NGA-DNSC-ALAGaC-MonMap cooperative airborne gravity project. A gravimetric geoid has been computed using the airborne gravity data, SRTM terrain models and GRACE/EGM global fields. The gravimetric geoid has subsequently been fitted to GPS-leveling data across Western Mongolia, as well as for a special Ulaanbaatar city geoid model.

Introduction

The Mongolian airborne gravity project will in 2004-5 cover all of Mongolia. The primary goal is to determine mean gravity anomaly data for global earth models (EGM06), as well as provide information for precise geoid models of Mongolia. This note outlines initial geoid computations from the 2004 data covering Western Mongolia. The airborne survey was done using an Air Greenland Twin-Otter aircraft, flying at varying altitudes to clear the topography, cf. Figure 1. The accuracy of the airborne gravity data is estimated to be approx. 2 mgal, cf. separate note on processing results.

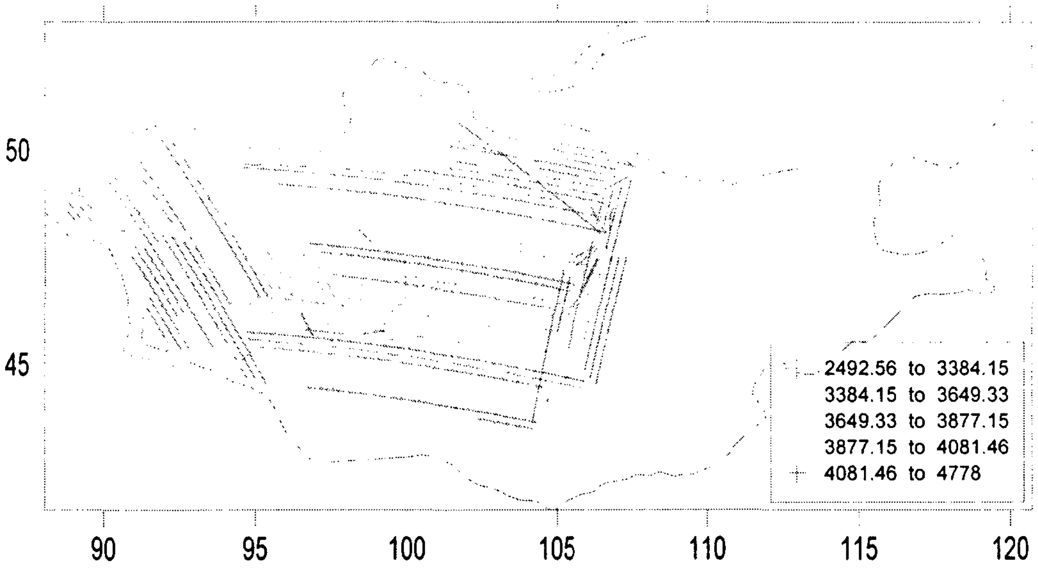

Fig. 1. Airborne gravity data of western Mongolia 2004. Colors show flight elevation.

The primary aim of the new Mongolian geoid model is to be able to compute orthometric heights H in the national height system by

$$H = h^{GPS} - N \qquad (1)$$

where h^{GPS} is the GPS height above the ellipsoid (e.g. from RTK-GPS) and N the geoid. In the above equation it is important to realize that H refers to the Baltic Sea datum, h^{GPS} refers to the geocentric MONREF system (ITRF/WGS84). The geoid is computed in the same system, based on accurate long-wavelength gravity information from the GRACE mission.

The gravimetric geoid height N is in principle determined by Stokes' equation of physical geodesy, which gives the expression of the geoid height N as an integral of gravity anomalies around the earth (σ)

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} \Delta g \, S(\psi) d\sigma \qquad (2)$$

where Δg is the gravity anomaly, R earth radius, γ normal gravity, and S a complicated function of spherical distance ψ (Heiskanen and Moritz, 1967). The actual computation is done in the frequency domain by spherical FFT methods.

In practice global models of the geopotential from analysis of satellite data and global mean gravity anomalies are used, e.g. for the current global model EGM96 (Lemoine et al., 1996)

$$N_{EGM96} = \frac{GM}{R\gamma}\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\sin\phi) \qquad (3)$$

Here the spherical harmonic coefficients C_{nm} and S_{nm}, for EGM96 complete to degree and order 360 define the long-wavelength gravity field (degree 360 corresponds to a resolution of 55 km). Background and online information on EGM96 can be found at www.nga.mil. Currently a new global model EGM06 is being prepared in cooperation with the International Association of Geodesy; this new spherical harmonic model will have a maximal degree of 2160 (5' resolution). The Mongolian airborne gravity project is part of the necessary data gathering activities to support EGM06.

For the Mongolian geoid the new GRACE satellite data model GGM02S has been used, linearly merged with EGM96 in the spherical harmonic band 90-100.

A third data source for the geoid determination is digital terrain models (DEM's), which provide details of the gravity field variations in mountainous areas (the mass of the mountains can change the geoid by several 10's of cm locally). The handling of digital terrain models is done by analytical prism integration assuming known rock density (Forsberg, 1984). The new 30" satellite data SRTM was used for this purpose cf. Fig. 2.

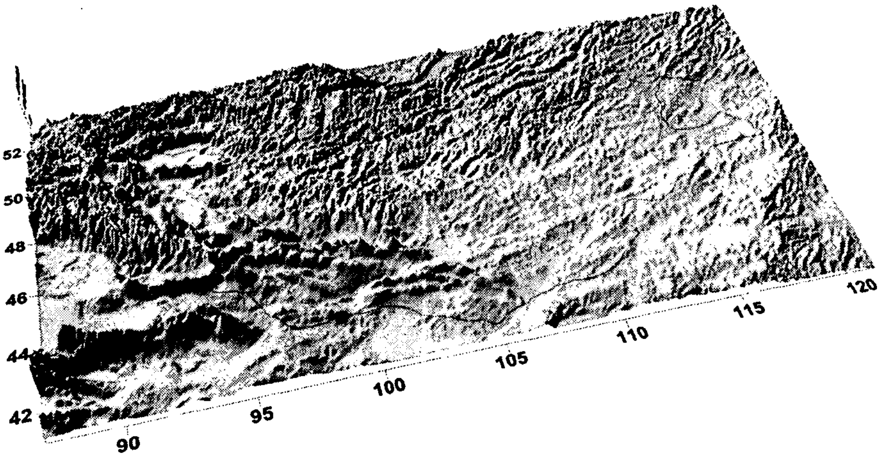

Fig. 2. SRTM digital terrain model of Mongolia

The RTM method was used to only take into account the shorter wavelengths of the topography (Forsberg, 1984). The used reference topography is shown in Fig. 3. It has a resolution of approx. 80 km, and was generated from the detailed DEM by filtering. By the used method the computed geoid is technically a quasi-geoid, but due to the errors and fitting is can be viewed as a classical geoid as well.

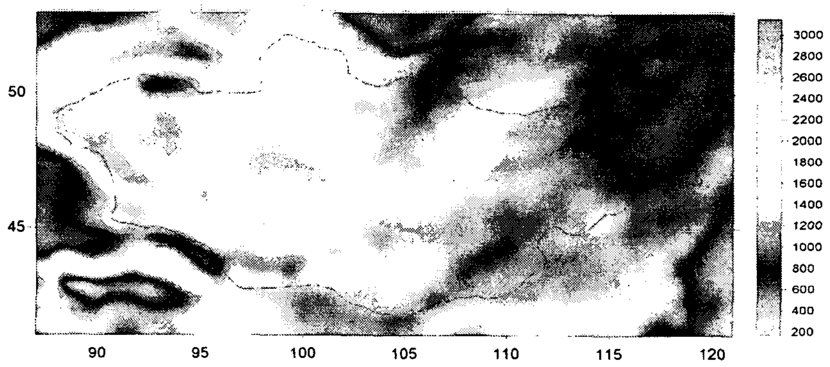

Fig.3. Smoothed reference DEM of Mongolia for RTM terrain reduction

With the data from spherical harmonic models, local or airborne gravity, and DEM's, the (gravimetric) geoid is constructed by remove-restore techniques as a sum

$$N = N_{EGM} + N_{gravity} + N_{DEM} \tag{4}$$

However, to be consistent with GPS and local leveling systems, a correction between the *global* and *local* vertical datums must be made:

$$N^{GPS} = N + \varepsilon \tag{5}$$

where ε is a *GPS-corrector surface* taking into account datum differences and possible errors in GPS, leveling and the gravimetric geoid N.

In practice ε is determined by *fitting* the gravimetric geoid at points with coincident GPS and leveling; at these points ε can be directly determined by

$$\varepsilon = N_{GPS} - N = h_{GPS} - H_{levelling} - N \tag{6}$$

and ε then interpolated to other points by e.g. least-squares collocation; in practice, however, the N^{GPS} will not be a classical geoid (an equipotential surface) because any error in H or h will be "inherited" in the final GPS geoid N^{GPS}. This is presently the major source of error in the Western Mongolian geoid.

Gravimetric geoid computation of Western Mongolia

The spherical harmonic reference field effects on gravity and geoid were computed in grids. Subsequently the terrain effects were computed using prism integration for the airborne gravity data. The computed terrain effects were filtered along-track with a filter corresponding to the airborne gravity processing. Table 1 shows the effect of the data reductions (airborne data were thinned to 20 sec, and an atmospheric correction applied prior to data reductions).

Table 1. Reduction statistics of airborne gravity data (unit: mgal)

Data	Mean	Std.dev.	Min	Max
Airborne free-air anomalies	-16.79	30.92	-106.25	176.33

- GRACE/EGM ref. field	0.71	23.89	-120.65	188.39
- GRACE/EGM and terrain	0.77	16.07	-83.35	87.98

From Table 1 it is seen that a good reduction in the standard deviation is found, and that the reduced airborne data have little or no bias relative to GRACE. This is a good indication of the excellent quality of the airborne data.

The reduced airborne gravity data were merged with a limited set of ALAGaC surface gravity points in Ulaanbaatar city, and gridded using rapid least-squares collocation. No formal downward continuation was done, since only very limited surface data was used and the present computations were preliminary, awaiting the completion of the 2005 survey (formal collocation downward continuation was done, however, for all airborne data to a constant level of 3500 m for purposes of checking the x-over statisctics).

The gridded, reduced gravity anomalies at the basic 3' x 3' grid are shown in Fig. 4, and the computed geoid effects in Fig 5. The FFT process involve FFT transforms of 480 x 1200 zero-padded data points, using 3 reference parallels. The magnitude of the computed residual geoid effects illustrates the major improvements in the gravity field determination of western Mongolia due to the airborne survey.

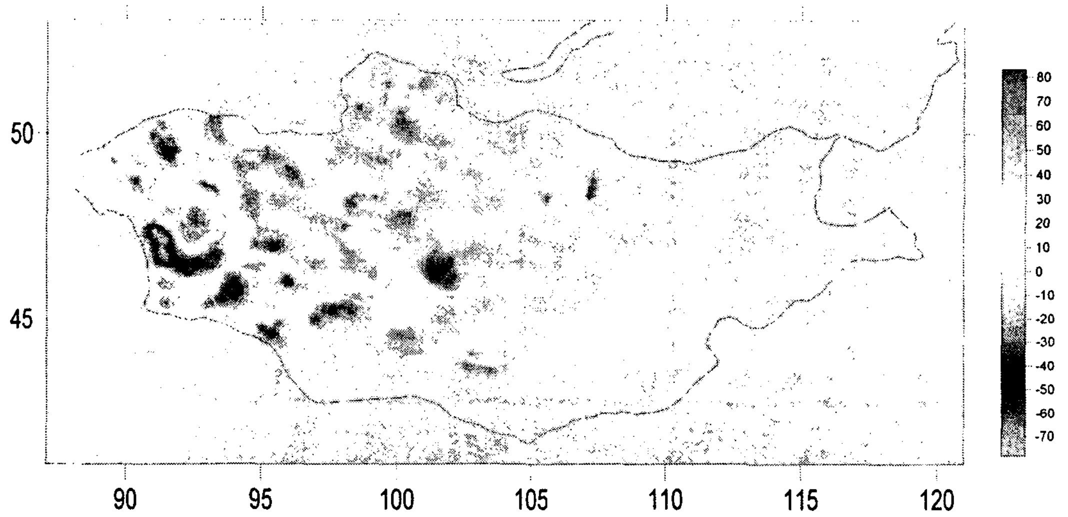

Fig. 4. Residual terrain-reduced gravity anomalies from the airborne data. Unit mgal.

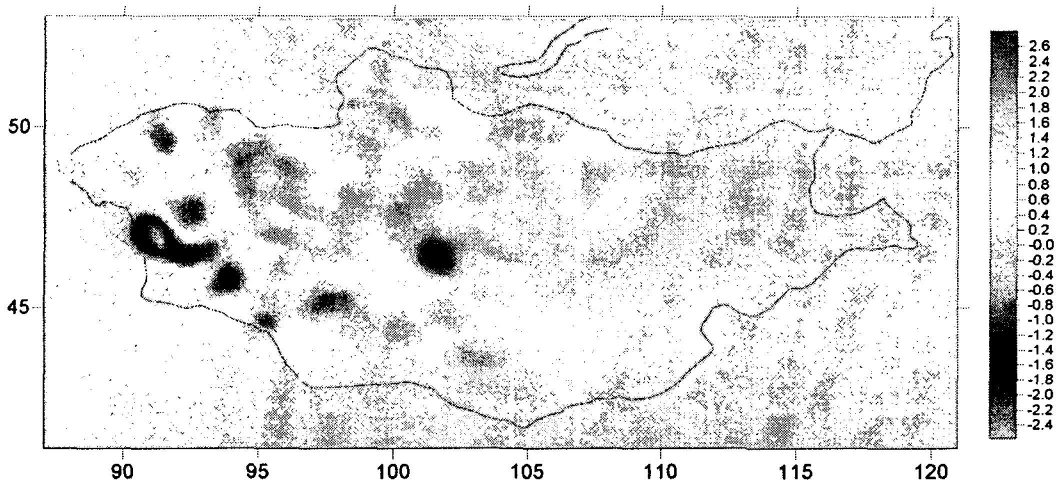

Fig. 5. Geoid effects computed by FFT. Unit meter.

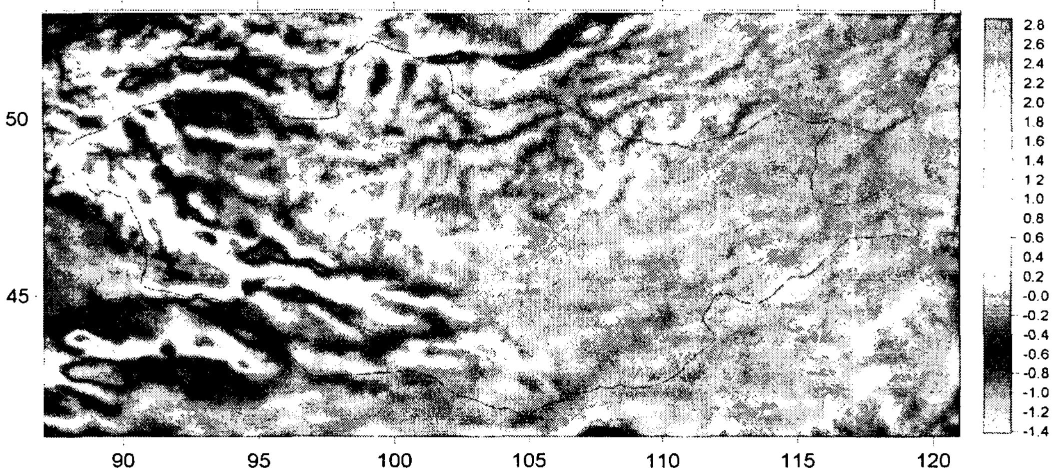

Fig. 6. Geoid RTM restore effects due to terrain. Unit meter.

Fig 6 shows the terrain RTM geoid restore effect, and Fig. 7 shows the final gravimetric geoid after addition of the reference field. Table 2 shows the statistics of the restore effects. The geoid terrain effects were computed by Fourier methods as well.

Table 2. Statistics of geoid restore effects (unit: meter)

Data	Mean	Std.dev.	Min	Max
Reduced geoid from FFT	0	0.26	-2.33	2.80
Terrain RTM geoid effects	-0.01	0.30	-1.42	2.87
Final geoid	-35.97	16.95	-68.84	4.64

The gravimetric geoid was computed using modified Wong-Gore Stokes' kernel functions, balancing the role of the airborne data and the global reference field. The kernel used in the final geoid solution was transitioning from zero to one in the spherical harmonic band 40-50. This means that local data have full "power" above harmonic degree 50, and the GRACE model full power below 40. The "optimal" kernel modification was found by comparison to GPS-levelling data, confirming results from other regions that the accuracy of the GGM02S in the higher bands (50-90) is not as good as expected.

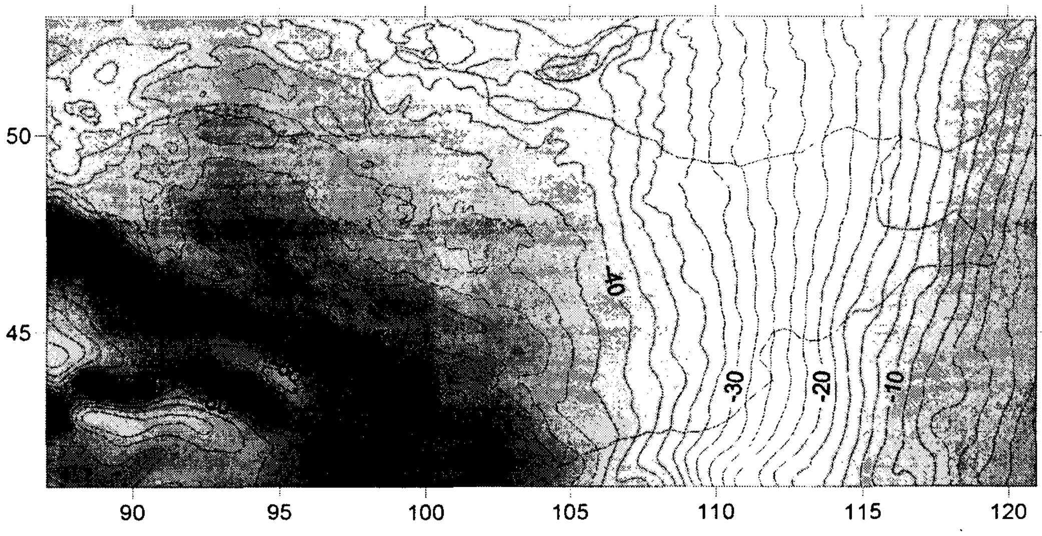

Fig. 7. Final gravimetric geoid for Western Mongolia. Contour interval 2 m.

Fit to GPS-levelling data

The computed gravimetric geoid was subsequently fitted to the GPS control by least squares collocation. Two data sets were available:

1) A national set of GPS on leveling benchmarks, of which a subset of 14 points were selected to coincide with the region of the airborne survey.

2) A set of GPS leveling points in Ulaanbaatar city (15 points).

A separate fit of the geoid was done for Western Mongolia, and for UB city. After some experiments, a correlation length of 120 km and apriori standard deviation of 5 cm was used for the Western Mongolia fit, and 15 km / 1 cm for the much more dense GPS net in UB. The fitting process was done by least squares collocation. The GPS fit signals are shown in Fig. 8. It should be stressed that the errors in N^{GPS} coming from the leveling, crustal movements or GPS antenna height errors will directly affect the fitted GPS-geoids. The fitted geoid heights should therefore always be treated with some care, as errors propagate into the regions around such outlier points.

Table 3 below shows the fit of the different geoid models to the GPS-derived geoid heights (in the Baltic Sea level datum). It is seen that the vertical datum in Mongolia seems to be off by more than 1 m from the global vertical datum implied by the gravimetric geoid computation.

Table 3. Fit of geoids to GPS-levelling data(unit: meter)

Data / geoid model	Mean	Std.dev.	Min	Max
GPS-geoid data (14 pts.)	-44.00	3.37	-51.51	-36.76
GPS minus GGM02EGM ref.field.	1.04	0.56	0.41	2.61
GPS minus gravimetric geoid	1.11	0.16	0.79	1.44
GPS minus fitted geoid (Western Mongolia)	*-0.01*	*0.03*	*-0.06*	*0.05*
GPS minus fitted geoid (UB, 15 pts)	*0.00*	*0.02*	*-0.03*	*0.03*

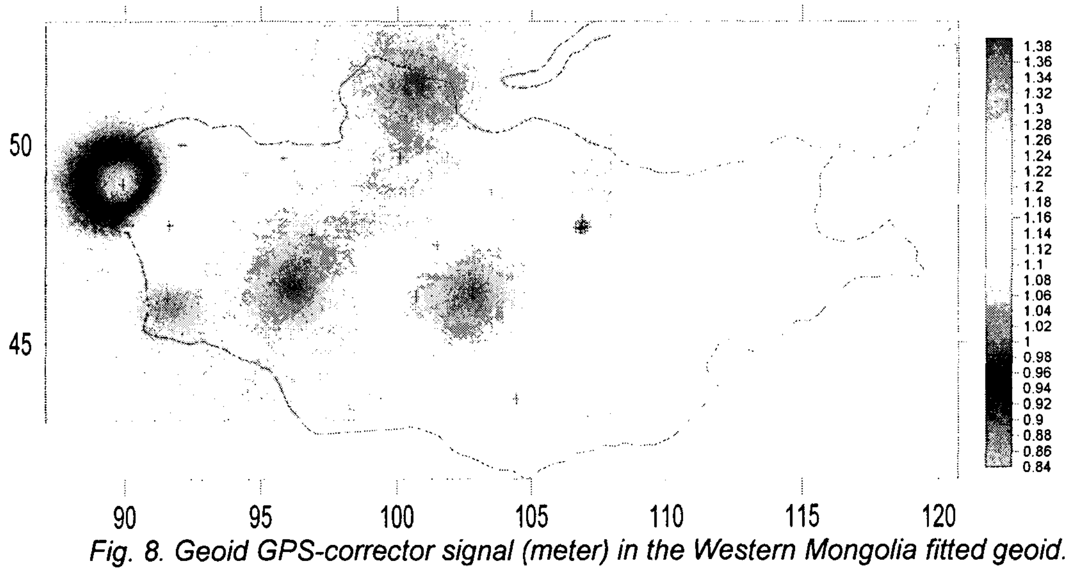

Fig. 8. Geoid GPS-corrector signal (meter) in the Western Mongolia fitted geoid.

Conclusions

A set of three new geoid models have been computed:

- A gravimetric geoid based on the airborne gravity data, GRACE and SRTM satellite data, EGM96, and a few surface gravity points. This geoid is in a global vertical reference system.

- A geoid fitted to GPS stations across Western Mongolia. The accuracy of this geoid is estimated to be 5-10 cm. When using this geoid heights are obtained in the Baltic Sea vertical datum.

- A special fitted version of the geoid in Ulaanbaatar city. This geoid should be accurate to 2-3 cm within the central parts of UB, with larger errors in the surrounding mountains.

In 2005 additional airborne gravity data will be collected, and all available surface data will at that point be used for an improved "final" GPS geoid for all of Mongolia. This would represent a major step forward in the national geodetic infrastructure of Mongolia.

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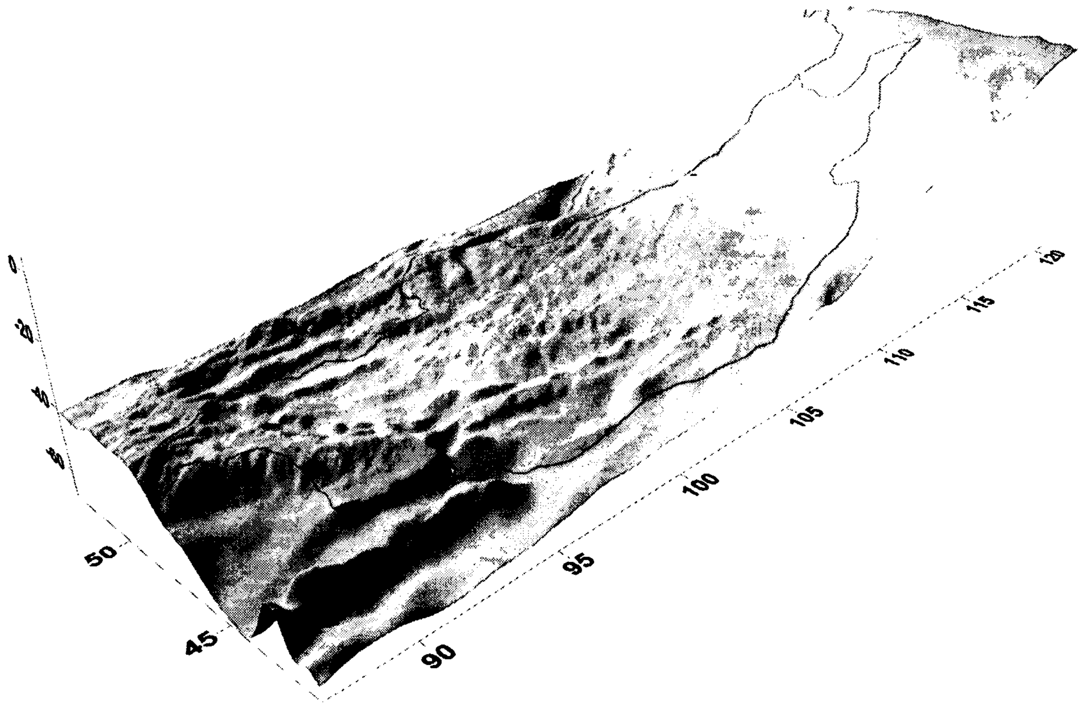

Mongolian geoid