

## 건물-지반 시스템에 관한 진동대실험 (1) : 반무한지반위의 구조물

### Shaking table test on soil-structure interaction system (1) : Superstructure with foundation on half-space soil

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#### ABSTRACT

This paper presents the shaking table testing method, only using building specimen as an experimental part, taking into account the dynamic soil-structure interaction based on the substructure method. The Parmelee's soil stiffness is used as an assumed soil model in here. The proposed methodologies are summarized as:

- ① Acceleration feedback method is the one that the shaking table is driven by the motion, corresponding to the acceleration at foundation of the total SSI system. This is found by observing the fed-back accelerations of superstructure and using the interaction force based on the acceleration formulation.
- ② Velocity feedback method is the one that the shaking table is driven by the motion, corresponding to the velocity at foundation of the total SSI system. This is found by observing the fed-back accelerations of superstructure and using the interaction force based on the velocity formulation.

The applicability of the proposed methodologies to the shaking table test is investigated and experimentally verified in this paper.

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#### 1. INTRODUCTION

It is important to evaluate the seismic response of dynamic soil-structure interaction system, and is required to appropriately establish the earthquake load based on the evaluation of seismic response reflecting the underground characteristics on it, in aseismic design of structures. The conventional shaking table testing method of SSI system has revealed the limitations in its experimental performing[Iguchi et al, 1993]; ① Too small building specimen comparing to a soil one to satisfy the similarity between them ② Boundary treatment of soil specimen ③ Material selection for soil specimen and ④ High expenses for carrying out the test.

For these reasons, in case of the vibration control experiment considering the non-linearities in the superstructure of SSI system, the following things are the challenging subjects in earthquake engineering;

- ① Investigation on the shaking table test considering the dynamic soil-structure interaction, without any

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physical soil specimen. ② Driving the shaking table with the interaction force, which is produced by the feedback of the responses of experimental specimen, for reflecting the effect of soil on the superstructure. Authors have implemented the studies on the simulation of shaking table test considering the dynamic soil-structure interaction (SSI) effects [Lee et al, 2002a, b]. This paper presents the shaking table testing method of SSI system, only using the building specimen as an experimental part, and its experimental verification is also addressed.

## 2. OUTLINE OF EXPERIMENTAL SYSTEM

Fig. 1 shows the experimental devices implemented in Earthquake Disaster Reduction Laboratory of Disaster Control Research Center in Tohoku University. The motion of the building model mounted on the shaking table is measured by the accelerometers attached at each floor and converted into digital signals by A/D board installed in the control computer. The shaking table is driven by the control signal converted into the analog signal by D/A board installed in control computer.

The electro-magnetic type shaking table in the figure, with the maximum excitation force of 313.6(N), the weight of 470(N) and the maximum moving distance of  $\pm 15$ (cm), can uniaxially generate horizontal or vertical vibrations by changing the exciting direction. This shaking table acts as a vibration generator when the table is fixed at the top of the shaking table and also can be used as an Active Mass Driver when the shaking table is fixed at the top of some building model. The size of mounted table is 30(cm) $\times$ 30(cm).

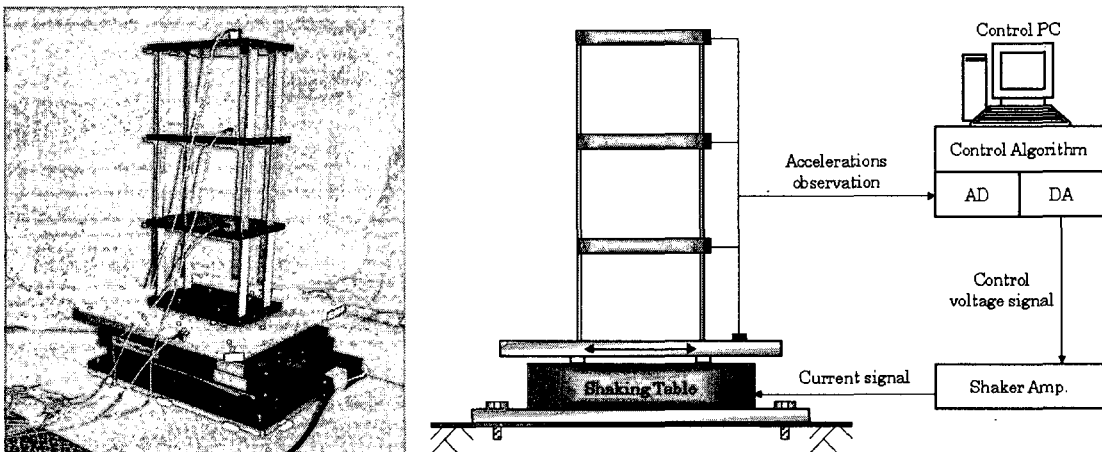


Figure 1. Experimental set-up

### 2.1 Controller of shaking table

An ideal shaking table used in structural vibration test is one that moves as its reference signal. Usual shaking table, however, has its own dynamic characteristic; the observed response at the shaking table is different from its reference signal in both amplitude and phase. Accordingly, we need specific controller of

shaking table in order to drive it as an intended motion and promote its command tracking performance to reference signal. In this study, the controller was designed based on the experimentally obtained transfer function of shaking table. To obtain the transfer function, Banded-white noise input signal(0~20Hz) was sent to, and the output acceleration at the shaking table was observed. The final transfer function of shaking table was obtained from dividing the cross spectrum between the input and output by the auto spectrum of the input itself, as shown in Fig 2. In the figure, the dark dotted line is the transfer function of shaking table without building model, the light dotted line denotes that with building model,  $\bar{G}(\omega)$ , and the solid line expresses its fitted transfer function with building model,  $G(\omega)$ .

Based on the fitted transfer function,  $G(s)$ , the two degree-of-freedom controller[John, 1992], which has 2 inputs of reference and the fed-back acceleration and 1 output of control signal, was used for compensating the dynamic characteristic and controlling the motion of shaking table, as shown in Fig. 3. In the figure, the target model,  $F(s)$ , having the type of low-pass Butterworth filter with the cut-off frequency of 20 Hz, is selected to satisfy the condition that the filter,  $F(s)/G(s)$ , has to be stable. The feedback controller,  $K(s)$ , is also designed based on the mixed sensitivity problem considering the frequency weighting on the complimentary sensitivity and the sensitivity function in  $H_\infty$  control.

The following Fig. 4 shows the compensated results by the above 2 D.O.F. controller for El Centro earthquake wave as a reference signal. Fig. 4 compares a reference with the fed-back acceleration at the shaking table in Fig. 3.

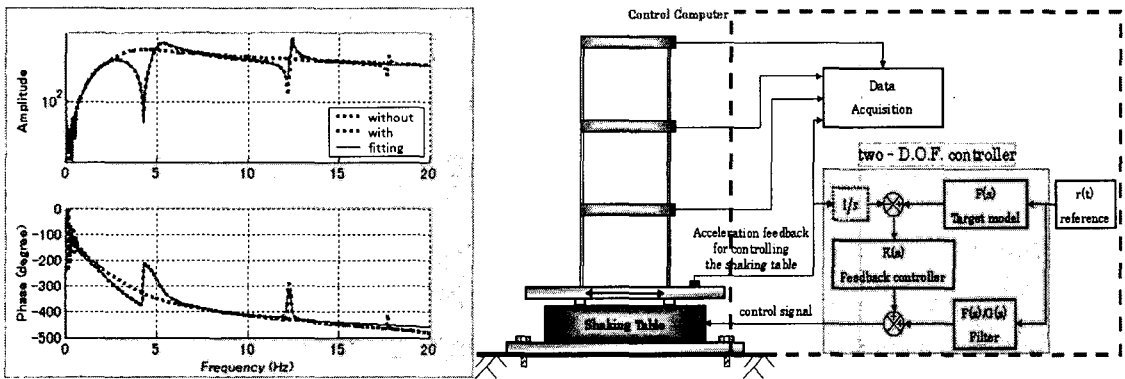


Figure 2. Transfer function of shaking table      Figure 3. Two degree-of-freedom controller

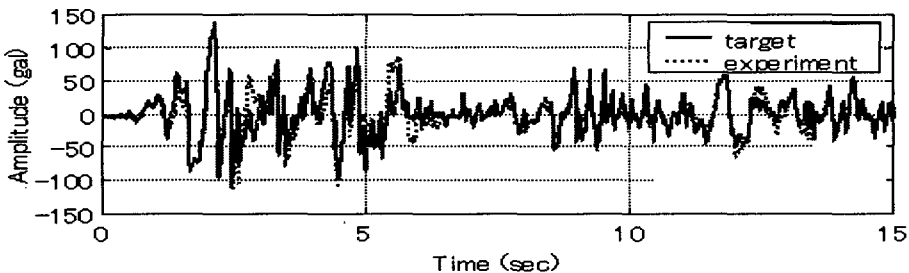


Figure 4. Compensated El Centro earthquake wave

## 2.2 Parametric identification of building specimen

The building model shown in Figs. 1 and 3, with the equally distributed floor weight of 13N, is identified on the basis of the acceleration data observed at each story and shaking table in Fig. 3. The damping and stiffness coefficients are identified in such a way that the sum of the squares of errors between the absolute accelerations of building model observed from the experiment such as Fig. 3 and those calculated based on Eq. (1).

$$[M]\{\ddot{y}_i(t)\} + [C]\{\dot{y}_i(t)\} + [K]\{y_i(t)\} = -[M]\{1\}\ddot{Y}_b(t) \quad (1)$$

where,  $\{\ddot{y}_i(t)\}$  is the analytical relative accelerations at each story  $i$  of building model to the experimentally observed absolute acceleration at the shaking table,  $\{1\}$  is a column vector whose components are one, and  $[M]$ ,  $[C]$ ,  $[K]$  are the mass, damping and stiffness matrix of building model, respectively, which are expressed as;

$$[M] = \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}, [C] = \begin{bmatrix} c_3 & -c_3 & 0 \\ -c_3 & c_3 + c_2 & -c_2 \\ 0 & -c_2 & c_2 + c_1 \end{bmatrix}, [K] = \begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_1 \end{bmatrix} \quad (2)$$

where,  $m_i$ ,  $c_i$  and  $k_i$  are the mass, damping and stiffness coefficients of building model at each story  $i$ , respectively.

Table 1. Identification of building specimen

Story	1st	2nd	3rd	Mode	1st	2nd	3rd
$k_i$ (N/cm)	43.1	45.1	54.9	Frequency (Hz)	4.1	12.1	17.3
$c_i$ (N·s/cm)	1.7	0.002	0.3	Damping (%)	0.292	0.634	0.296

## 3. ANALYTICAL SOIL-STRUCTURE INTERACTION SYSTEM

The following Fig. 5 shows the superstructure, which is identified in the previous chapter, with the assumed circular foundation having the weight of 7.74N and the radius of 15cm, rested on the assumed half-space soil with Poisson's ratio of 0.3, the shear velocity of 416.7cm/s and the specific weight of 10.1kN/m<sup>3</sup>. Parmelee's soil stiffness[Parmelee, 1970], which has the form of the constant value not depending on frequencies and is frequently adopted as the simple soil stiffness of half-space for engineering use, is used for soil's one such as the following Eq. (3).

$$k_b^g = \frac{6.77}{1.79 - \nu} \rho V_s^2 r = 121.9 \text{ N/cm}, \quad c_b^g = \frac{6.21}{2.54 - \nu} \rho V_s r^2 = 2.7 \text{ N·s/cm} \quad (3)$$

where, the mass density of soil  $\rho = \gamma/g$  and the gravity acceleration  $g = 9.805 \text{ m/s}^2$ .

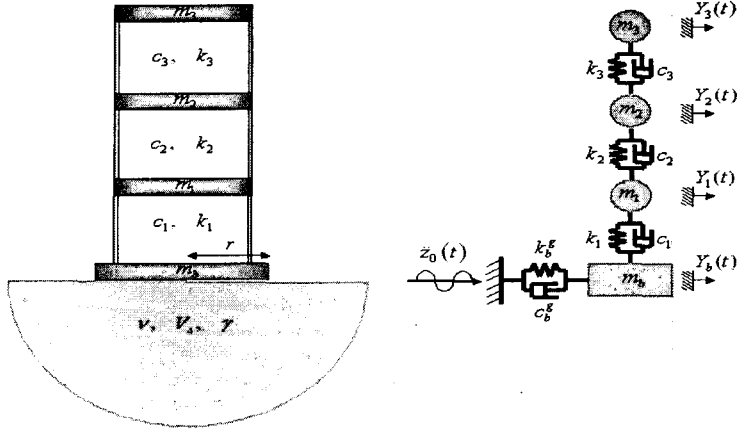


Figure 5. Superstructure with foundation on half-space

The corresponding differential equation is expressed as:

$$\begin{bmatrix} [M_{ss}] & [M_{sb}] \\ [M_{bs}] & [M_{bb}] \end{bmatrix} \begin{Bmatrix} \ddot{Y}_s(t) \\ \ddot{Y}_b(t) \end{Bmatrix} + \begin{bmatrix} [C_{ss}] & [C_{sb}] \\ [C_{bs}] & [C_{bb}] \end{bmatrix} \begin{Bmatrix} \dot{Y}_s(t) \\ \dot{Y}_b(t) \end{Bmatrix} + \begin{bmatrix} [K_{ss}] & [K_{sb}] \\ [K_{bs}] & [K_{bb}] \end{bmatrix} \begin{Bmatrix} Y_s(t) \\ Y_b(t) \end{Bmatrix} = - \begin{Bmatrix} \{0\} \\ R_b(t) \end{Bmatrix} \quad (4)$$

where,  $[M_{ss}] = [M]$ ,  $[C_{ss}] = [C]$  and  $[K_{ss}] = [K]$  in Eq. (2), and

$$\begin{aligned} [M_{sb}] &= [M_{bs}]^T = [0 \ 0 \ 0]^T, & [C_{sb}] &= [C_{bs}]^T = [0 \ 0 \ -c_1]^T, & [C_{bb}] &= c_1 \\ [K_{sb}] &= [K_{bs}]^T = [0 \ 0 \ -k_1]^T, & [K_{bb}] &= k_1 \end{aligned} \quad (5)$$

### 3.1 Acceleration feedback

In case of acceleration feedback, the mass of foundation  $[M_{bb}]$  in Eq. (4) is expressed as:

$$[M_{bb}] = 0 \quad (6)$$

This means that the foundation is not included in the superstructure but in the soil-foundation system for the numerical calculation of Eq. (4); instead of foundation's being included in the superstructure, it is considered for the calculation of the interaction force. Therefore, the interaction force in Eq. (4) expressed as:

$$R_b(t) = m_b \{ \ddot{Y}_b(t) - \ddot{Y}_b^g(t) \} + c_b^g \{ \dot{Y}_b(t) - \dot{Y}_b^g(t) \} + k_b^g \{ Y_b(t) - Y_b^g(t) \} \quad (7)$$

where,  $Y_b^g(t)$  is the effective foundation input motion.

The superstructure under the interaction force excitation expressed in Eq. (4) also satisfy the dynamic equilibrium expressed as the following Eq. (8).

$$\sum_{i=1}^3 m_i \ddot{Y}_i(t) = -R_b(t) \quad (8)$$

### 3.2 Velocity feedback

Differently from the case of the acceleration feedback, in case of velocity feedback using the acceleration data of structural responses, the mass of foundation  $[M_{bb}]$  in Eq. (4) is given by,

$$[M_{bb}] = m_b \quad (9)$$

This implies that the foundation is not included in the soil-foundation system but in the superstructure for the numerical calculation of Eq. (4); since the foundation is contained in the superstructure, the interaction force is calculated from the mass-less foundation on half-space. Therefore, the interaction force in Eq. (4) expressed as:

$$R_b(t) = c_b^g \{ \dot{Y}_b(t) - \dot{Y}_b^g(t) \} + k_b^g \{ Y_b(t) - Y_b^g(t) \} \quad (10)$$

The superstructure under the interaction force excitation expressed in Eq. (10) also meets the dynamic equilibrium expressed as the following Eq. (11).

$$m_b \ddot{Y}_b(t) + \sum_{i=1}^3 m_i \ddot{Y}_i(t) = -R_b(t) \quad (11)$$

## 4. EXPERIMENTAL VERIFICATION OF NUMERICAL SSI SYSTEM

The controller design to experimentally realize the motion of SSI system expressed in Eq. (4) and its experimental verification are investigated in here.

### 4.1 Acceleration feedback

To obtain the soil stiffness, a foundation-soil system is separated from the total soil-structure interaction system shown in Fig. 5, and then a unit displacement excitation is enforced on the mass of foundation as shown in the following Fig. 6 (a).

The resulting reaction force in the frequency domain gives:

$${}_a S_b^g(\omega) = -\omega^2 m_b + i\omega c_b + k_b \quad (12)$$

Meanwhile, The resulting response at foundation for the incident input acceleration also becomes the effective foundation input acceleration, as shown in the following Fig. 6 (b).

This is expressed as:

$$\ddot{Y}_b^g(\omega) = \frac{i\omega c_b + k_b}{{}_a S_b^g(\omega)} \cdot \ddot{Z}_0(\omega) = \frac{i\omega c_b + k_b}{-\omega^2 m_b + i\omega c_b + k_b} \cdot \ddot{Z}_0(\omega) \quad (13)$$

where,  $\ddot{Z}_0(\omega)$  is the incident input acceleration.

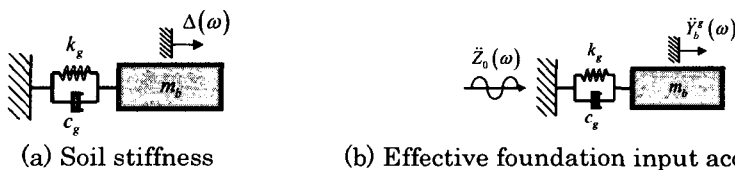


Figure 6. Soil stiffness and effective foundation input acceleration in case of acceleration feedback

The interaction force based on the acceleration formulation is given by[Motosaka et al, 1990]:

$$R_b(\omega) = \frac{{}_a S_b^g(\omega)}{-\omega^2} \cdot [\dot{Y}_b(\omega) - \dot{Y}_b^g(\omega)] \quad (14)$$

Therefore, the acceleration at the shaking table, which is required to give the building specimen the interaction force expressed as Eq. (14), is derived from substituting Eqs. (12) and (13) into Eq. (14) and rearranging it.

$$\ddot{Y}_b(\omega) = \frac{-\omega^2}{-\omega^2 m_b + i\omega c_b + k_b} \cdot R_b(\omega) + \frac{i\omega c_b + k_b}{-\omega^2 m_b + i\omega c_b + k_b} \cdot \ddot{Z}_0(\omega) \quad (15)$$

As known from Eq. (8), the interaction force in Eq. (15) is observed from the absolute acceleration at the building specimen.

The following Fig. 7 illustrates the experimental set-up and its signal flow in the control computer. Laplacian variable  $s$  in the figure equals to  $i\omega$  in Eq. (15). The part surrounded in dotted line specifies the realization in control computer of the filters in the above Eq. (15). Two types of closed-loop systems exist in this figure. One is for soundly driving the shaking table, through the two degree-of-freedom controller explained in before, to comply with the reference signal. Another is for giving the shaking table the motion of Eq. (15); the interaction force is observed from the absolute accelerations of building specimen, and then the shaking table moves through the above filters.

The following Figs. 8 and 9 show the experimental results obtained from converting the filters and 2 D.O.F. controller in Fig. 7 into their digital version and reflecting them on software for control use. Fig. 8 compares the results observed from the shaking table test in Fig. 7(solid line) with those calculated from the numerical analysis in Eq. (4)(dotted line). Fig. 9 expresses the experimental results between the foundation-fixed system shown in Fig. 3 and the SSI system shown in Fig. 7.

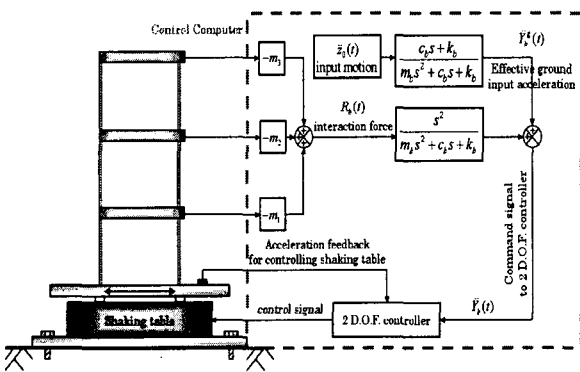


Figure 7. Signal flows in control computer in case of acceleration feedback

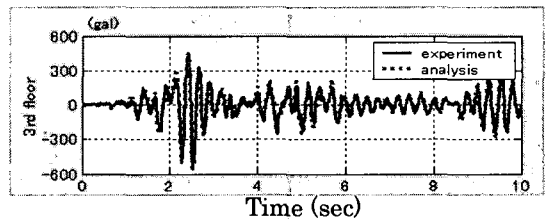


Figure 8. Comparison of results between the numerical analysis and the experiment of SSI system (Acceleration feedback)

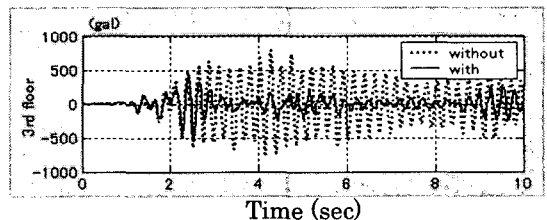


Figure 9. Comparison of experimental results between the foundation-fixed and SSI system (Acceleration feedback)

## 4.2 Velocity feedback

As for obtaining the soil stiffness in case of velocity feedback, the same procedure as the case of acceleration feedback is applied for the mass-less foundation-soil system, as shown in Fig. 10 (a).

The obtained soil stiffness is expressed as:

$${}_v S_b^g(\omega) = i\omega c_b + k_b \quad (16)$$

The corresponding velocity at the node of foundation for the incident input velocity also becomes the effective foundation input velocity, as like the following Fig. 10 (b).

The result becomes the same as incident input velocity:

$$\dot{Y}_b^g(\omega) = \frac{i\omega c_b + k_b}{{}_v S_b^g(\omega)} \cdot \dot{Z}_0(\omega) = \dot{Z}_0(\omega) \quad (17)$$

where,  $\dot{Z}_0(\omega)$  is the incident input velocity.

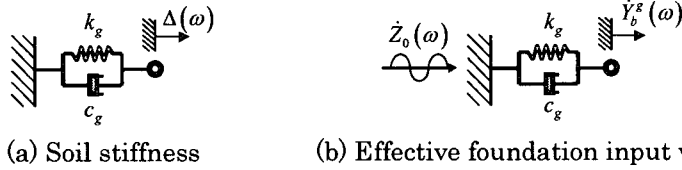


Figure 10. Soil stiffness and effective foundation input velocity in case of velocity feedback

The interaction force based on the velocity formulation is given by[Motosaka et al, 1990]:

$$R_b(\omega) = \frac{{}_v S_b^g(\omega)}{i\omega} \cdot [\dot{Y}_b(\omega) - \dot{Y}_b^g(\omega)] \quad (18)$$

Finally, the absolute velocity, which is needed to cause soil-structure interaction effect to the building specimen by the shaking table, is obtained by substituting Eqs. (16) and (17) into Eq. (18) and rearranging it.

$$\dot{Y}_b(\omega) = \frac{i\omega}{{}_v S_b^g(\omega)} \cdot R_b(\omega) + \dot{Z}_0(\omega) \quad (19)$$

As known from Eq. (11), the interaction force in the above Eq. (19) can be observed from measuring the absolute accelerations from all floors of building specimen and shaking table.

The following Fig. 11 shows the experimental set-up and its signal flow in control computer for the experiment of SSI system with foundation on half-space in case of velocity feedback. This expresses the reflection of the above Eq. (19) on the control computer. The acceleration observation at the shaking table is added in the figure due to foundation's inclusion in the calculation of interaction force, comparing to Fig. 7 in case of acceleration feedback. The digital integrator is also inserted for performing integration of acceleration data, since the given reference signal is the velocity at the shaking table, and the 2 degree-of-freedom controller explained in before has to drive the shaking table according to that.

Fig. 12 compares the results observed from the test in Fig. 11(solid line) with those calculated from the numerical analysis in Eq. (4)(dotted line). Fig. 13 expresses the experimental results between the



foundation-fixed system shown in Fig. 3 and the SSI system shown in Fig. 11.

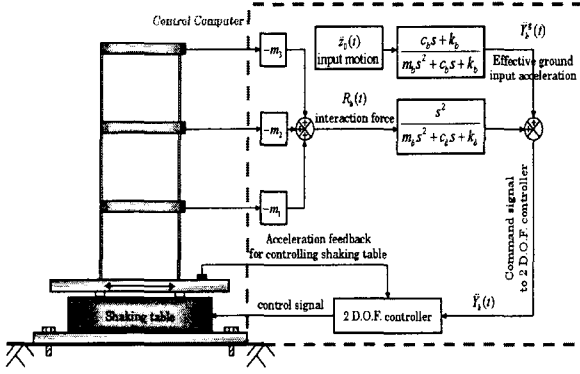


Figure 11. Signal flows in control computer in case of velocity feedback

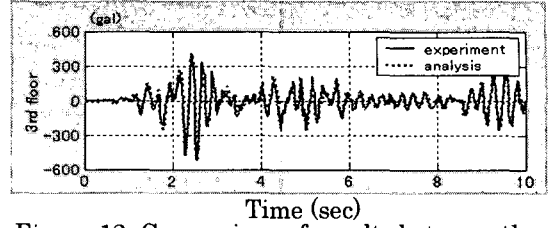


Figure 12. Comparison of results between the numerical analysis and the experiment of SSI system (velocity feedback)

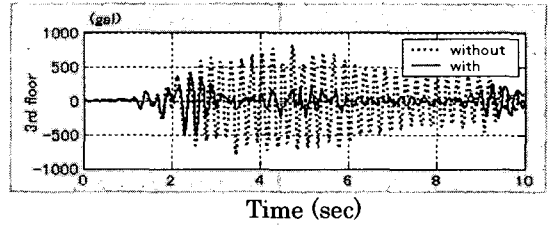


Figure 13. Comparison of experimental results between the foundation-fixed and SSI system (velocity feedback)

### 4.3 Identification of experimental soil stiffness

#### (1) Acceleration feedback

Substituting Eq. (13) into Eq. (15) and rearranging it with  $i\omega c_b + k_b$  leads to,

$$i\omega c_b + k_b = -\omega^2 \left[ \frac{R_b(\omega)}{\ddot{Y}_b(\omega) - \ddot{Y}_b^g(\omega)} - m_b \right] \quad (20)$$

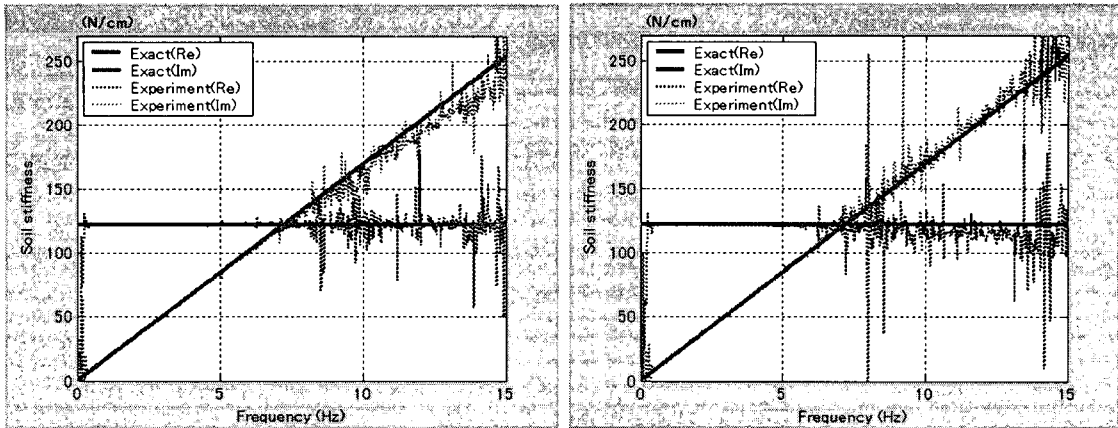
The interaction force,  $R_b(t)$ , the absolute acceleration at the shaking table,  $\ddot{Y}_b(t)$ , and the effective ground input acceleration,  $\ddot{Y}_b^g(t)$ , can be experimentally measured in the control computer, as shown in Fig. 7. Finally, the experimentally obtained soil stiffness such as the following Fig. 14(a) is calculated from taking the Fourier transform into them and from the relation of the above Eq. (20). The 'Exact' in Fig. 14(a) denotes the analytical soil stiffness used in the numerical analysis, assumed as Eq. (3).

#### (2) Velocity feedback

Substituting Eq. (17) into Eq. (19) and rearranging it with  $i\omega c_b + k_b$  leads to,

$$i\omega c_b + k_b = \frac{i\omega \cdot R_b(\omega)}{\dot{Y}_b(\omega) - \dot{Y}_b^g(\omega)} \quad (21)$$

The interaction force,  $R_b(t)$ , the velocity at the shaking table,  $\dot{Y}_b(t)$ , and the effective ground input velocity  $\dot{Y}_b^g(t)$ , can be experimentally measured in the control computer, as shown in Fig. 11. The experimentally obtained soil stiffness in case of velocity feedback is shown in the following Fig. 14(b).



(a) Acceleration feedback

(b) Velocity feedback

Figure 14. Comparison between the analytical and the experimental soil stiffness

## 5. CONCLUSIONS

The shaking table testing methods of SSI system having constant soil stiffness was proposed, and their experimental verification was performed in this paper. The experimental results in the time domain and the identified soil stiffness with experimental data showed that the proposed methodologies can be applied to the shaking table test of SSI system with good accuracies.

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