Implicit Restarted Arnoldi Method를 적용한 전력시스템 미소신호안정도 모델링 방법 연구

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A Study of Power System Modeling of Small-Signal Stability Using Implicit Restarted Arnoldi Method

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Abstract - This paper describes implicit restated arnoldi method algorithm and its application to small size power systems in order to observe the salient features of IRAM algorithm. Two area system with 36 state variables and England 39-bus system with 150 state variables have been tested using IRAM, and the eigenvalue results of IRAM are compared with those of the results obtained from QR method.

1. Introduction

Conventional QR [1,2] method for small signal stability analysis are not applicable to very large-scale power systems because of limitation of memory capacity, computing time, and computation accuracy. In order to evaluate the small signal stability of power systems, it is usually required to calculate only a specific set of eigenvalues with certain features of interest, for example, local mechanical modes, inter-area modes, etc. Therefore, significant effort has been expended to develop new methods with such basic properties as sparcity based techniques, finding a few specific set of eigenvalues, and mathematical robustness with good convergence characteristics and numerical stability [3,4].

Recently another new effective approach which can be appliable to very large eigenvalue analysis has been suggested in the society of applied mathematics. This approach is called implicitly restarted Arnoldi method (IRAM) [5]. IRAM is a technique for combining the implicitly shifted QR mechanism with a k-step Arnoldi factorization to obtain a truncated form of the implicitly shifted QR-iteration. The numerical difficulties and storage problems normally associated with Arnoldi process is avoided. The algorithm is capable of computing a few (k) eigenvalues with user specified features such as largest real part or largest magnitude. Implicit Restarting provides a means to extract interesting information from very large Krylov subspaces while avoiding the storage and numerical difficulties associated with the standard approach.

This paper describes IRAM algorithm and its application to small size power systems in order to observe the features of IRAM. Two power systems, in which one has 36 state variables and the other has 150 state variables, have been tested by ARPACK [6] program which has IRAM algorithm, and the results are compared with QR method.

2. IRAM Algorithm and its Applications

2.1 Implicit Restarted Arnoldi Method

The IRAM determines the restart vector implicitly using the QR iteration with shifts. The restart occurs after every m steps and we assume that $m > j$ where j is the number of sought-after eigenvalues. The choice of the Arnoldi length parameter m depends on the problem dimension n, the effects of orthogonality loss, and system storage constraints. After m steps we have the Arnoldi factorization

$$AQ_C = Q_C H_C + r_C e_m^T$$

The subscript "c" stands for "current". The QR iteration with p shifts is then applied to H_C. Here $p = m - j$ and we have $H_+ = V^T H_C V$ because $V_i^T H^{(i)} V_i = H^{(i+1)}$. The orthogonal matrix $V = V_1 \cdots V_p$, with V_i the orthogonal matrix associated with the shift μ_i, has two crucial properties:

(1) $[V]_{mi} = 0$ for i=1:j-1. This is because each V_i is upper Hessenberg and so $V \in \mathbb{R}^{m \times m}$ has lower bandwidth $p = m - j$.

(2) $Ve_1 = \alpha (H_C - \mu_p I)(H_C - \mu_{p-1} I) \cdots (H_C - \mu_1 I) e_1$ where α is a scalar.

We obtain the following transformation:

$$AQ_+ = Q_+ H_+ + r_C e_m^T V$$

where $Q_+ = Q_C V$. In view of property (1),

$$AQ_+(:,1:j) = Q_+(:,1:j) H_+(1:j,1:j) + v_{mj} r_C e_m^T$$

is a length-j Arnoldi factorization. Back to the basic Arnold j iteration at step $j+1$ and performing p steps, we can have a new length-m Arnoldi factorization.

2.2 Shift and Invert Spectral Transformation

The shift and invert spectral transformation with IRAM enhance convergence to a desired portion of the spectrum. If $(x, \lambda)$is an eigenpair for A and $\sigma \neq \lambda$ then

$$(A - \sigma)^{-1} x = xv, \text{ where } v = 1/(\lambda - \sigma)$$

These transformed eigenvalues of largest magnitude are precisely the eigenvalues that are easy to compute with a Krylov method. Once they are found, it is easy to be transformed back to the original problem.

$$\lambda_j = \sigma + \frac{1}{v_j}$$

In addition, the complex shift-invert method needs two times of storage requirements compared to real shift-invert method.

2.3 Modeling of linearized power system

The linearized model of all machines and its control device can be expressed in the following form:

$$px_g = A_g x_g + B_g v_{gg} + B_c v_c \quad (1)$$

$$i_{gg} = C_g x_g - D_g v_{gg} \quad (2)$$

$$y_c = C_c x_g + D_c v_{gg} + D_c v_c \quad (3)$$

where $x_g = [x^t_{g1}, ..., x^t_{gn}]^t$, $v_{gg} = [v^t_{gg1}, ..., v^t_{ggn}]^t$, $v_c = [v^t_{c1}, ..., v^t_{cn}]^t$, $i_{gg} = [i^t_{gg1}, ..., i^t_{ggn}]^t$. y_c are output variables. $A_g \sim D_c$ are block diagonal matrices composed of the corresponding device matrices.

The interconnecting transmission network is represented by the node equations:

$$\begin{bmatrix} i_{ng} \\ i_{nl} \end{bmatrix} = \begin{bmatrix} Y_{gg} & Y_{gl} \\ Y_{lg} & Y_{ll} \end{bmatrix} \begin{bmatrix} v_{ng} \\ v_{nl} \end{bmatrix} \quad (4)$$

$$i_{nl} = J_l v_{nl} \quad (5)$$

where,

J_l : nonlinear load bus' linearized coefficient

i_{ng}, v_{ng}: voltage and current of generator bus ,

$$i_{ng} = [i^t_{ng1}, ..., i^t_{ngn}]^t, \ v_{ng} = [v^t_{ng1}, ..., v^t_{ngn}]^t$$

i_{nl}, v_{nl}: voltage and current of load bus

$$i_{nl} = [i^t_{nl1}, ..., i^t_{nln}]^t, \ v_{nl} = [v^t_{nl1}, ..., v^t_{nln}]^t$$

$Y_{gg} \sim Y_{ll}$: admittance matrix of network

Equating equation (4) associated with the admittance of load bus and generator bus, and equation (5), we obtain

$$i_{ng} = [Y_{gg} - Y_{gl}(Y_{ll} - J_l)^{-1} Y_{lg}] v_{ng} \quad (6)$$

Network equations are written in a synchronouly rotating R-I reference frame. For synchronous machines, Park's equations are expressed in a local d-q coordinates fixed on the generator rotor. It is necessary to transform the network input variable such as terminal voltage into the local d-q coordinate fixed on the generator rotor. The following transformation matrices are used for changing reference frame:

$$i_{dq} = i_{gg} = T_1 i_{ng} + T_2 \delta \quad (7)$$

$$v_{dq} = T_1 v_{ng} + T_3 \delta \quad (8)$$

$$v_{RI} = v_{ng} = T_4 v_{\theta V} = T_4 v_{gg} \quad (9)$$

Using transformation matrices above, we can obtain the complete system state matrix[12]:

$$px_g = A x_g + B_c v_c \quad (10)$$

$$y_c = C x_g + D_c v_c \quad (11)$$

where,

$$A = A_g + B_g (T_1 Y_g T_4 + D_g)^{-1} (C_g - T_2) \quad (12)$$

$$C = C_c + D_c (T_1 Y_g T_4 + D_g)^{-1} (C_g - T_2) \quad (13)$$

$$Y_g = [Y_{gg} - Y_{gl}(Y_{ll} - J_l)^{-1} Y_{lg}] \quad (14)$$

$$T_2 = T_2 T_5 \quad (15)$$

2.4 Case Study

In this section, two systems, 11-bus system and 39-bus system, are tested with ARPACK program [6] in which the IRAM algorithm was implemented with Fortran 77 partly using BLAS and LAPACK. In order to apply IRAM with single precision to the power system for small-signal stability, the PSS tuning program, PWRSTAB [12], and ARPACK program are integrated into one program. In addition, the eigenvalues from IRAM in ARPACK are compared with the eigenvalues obtained from QR method in PWRSTAB program.

2.4.1 Two-Area system

11-bus system as shown in Fig. 1, which has 4 machines equipped with static exciters, is 36 order system and has one unstable mode which is caused by high response exciter systems. Two eigenvalues are calculated by ARPACK (slightly modified to consider complex matrices) with every shift point from (0.0, j15.0) to (0.0, j1.0) with decrement step as -1.0. Table 1 shows the comparison of two results between QR method and IRAM. They give almost identical results.

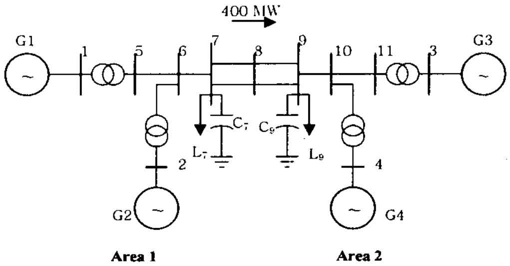

Fig. 1 Two-Area System:4 Machine and 11-Bus System

Table 1 Comparison of the eigenvalue results of QR method and IRAM

No.	*QR*		*IRAM*	
	Real	***Imag***	***Real***	***Imag***
1	-18.659	±16.458	-18.659	±16.458
2	-19.171	±10.152	-19.171	±10.152
3	-0.466	±7.332	-0.466	±7.332
4	-0.665	±7.162	-0.665	±7.162
5	0.049	±3.867	0.049	±3.867

Table 2 and 3 are, as an example, the single IRAM results and run statistics with shift (0,j7.0), respectively. ARPACK can obtain the eigenvalue of interest by setting *Which* character parameter. There are 6 modes to be selected for the concerned eigenvalues. For example, if smallest imaginary parts are concerned, 'SI' is set as *Which* ='SI'. The fastest way to find the most unstable mode or rightmost eigenvalues in ARPACK is to set *Which*='LR', which possibly tries to find the eigenvalues of largest real part. *OP* in Table 3 is the symbol of matrices to be manipulated to calculate the eigenvalue.

Table 2 ARPACK eigenvalue results with shift (0, j7.0)

Ritz values (Real, Imag) and relative residuals			
	Col 1	Col 2	Col 3
Row 1:	-4.66479E-01	7.33208E+00	1.00031E+00
Row 2:	-6.65237E-01	7.16232E+00	1.40694E+00

Table 3 ARPACK run statistics with shift (0, j7.0)

Size of the matrix is	36
The number of Ritz values requested is	2
The number of Arnoldi vectors generated (NCV) is	10
What portion of the spectrum: SI (smallest Imag. part)	
The number of converged Ritz values is	2
The number of IRAM update iterations taken is	1
The number of *OP**x is	10
The convergence criterion is	5.9604645E-08

2.4.2 England 39-bus system

England 39-bus system has 150 state variables including 10-machine models, 9-exciter models and 9-governor models. Table 4 shows the identical eigenvalue results from two methods as previous one. This indicates that IRAM provide reliable results regardless of system size. In this paper, the speed of calculation of IRAM against a large-scale power system could not be investigated because of not using large scale eigenvalue program exploiting sparcity technique and not studying large-scale power system, but will be studied in the next research phase.

Table 5 and 6 are the results of single IRAM run and run statistics with shift (0, j7.0), respectively. IRAM did not missed the eigenvalues of oscillation even if two pairs of eigenvalues are closely located. For example, mode 1 and mode 2 shown Table 4 are closely clustered, but IRAM calculates two pairs of eigenvalues.

Table 4 Comparison of the eigenvalue results of QR method and IRAM (150 order system)

No.	*QR*		*IRAM*	
	Real	*Imag*	*Real*	*Imag*
1	-0.283	±7.715	-0.283	±7.715
2	-0.145	±7.612	-0.145	±7.612
3	-0.091	±7.105	-0.091	±7.105
4	-0.196	±6.261	-0.196	±6.261
5	-0.127	±3.987	-0.127	±3.987
6	-0.069	±1.338	-0.069	±1.338

Table 5 ARPACK eigenvalue results with shift (0, j7.0)

Ritz values (Real, Imag) and relative residuals	Col 1	Col 2	Col 3
Row 1:	-9.13388E-02	7.10457E+00	9.99996E-01
Row 2:	-1.44625E-01	7.61160E+00	1.49239E+00

Table 6 ARPACK run statistics with shift (0, j7.0)

Size of the matrix is	150
The number of Ritz values requested is	2
The number of Arnoldi vectors generated (NCV) is	10
What portion of the spectrum: SI	
The number of converged Ritz values is	2
The number of Implicit Arnoldi update iterations taken is	2
The number of OP*x is	17
The convergence criterion is	5.9604645E-08

3. Conclusions

This paper describes implicit restated arnoldi method algorithm and its application to small size power systems in order to observe the salient features of IRAM algorithm. ARPACK program is used to apply IRAM with shift and invert spectral transformation to power system for small-signal stability. Two area 11-bus system with 36 state variables and England 39-bus system with 150 state variables have been tested using IRAM, and the eigenvalue results are compared with those of results obtained from QR method. They shows the identical eigenvalue results in both systems. Therefore, the research results of this paper indicate that IRAM provides reliable calculation results of the concerned eigenvalues regardless of system size.

In the on-going research phase, an efficient sparcity-based eigenvalue algorithm applicable to very large power system will be developed using IRAM algorithm.

4. References

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