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Techniques of Internally Generating Waves on A Curve and Specifying Partial Reflection Conditions

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Abstract

The techniques of internally generating waves on a curve in a rectangular grid system are developed using the line source method. Numerical experiments are conducted using the extended mild-slope equations of Suh et al. (1997). For five different types of wave generation layout, numerical experiments are conducted in the cases of the propagation of waves on a flat bottom, and the refraction and shoaling of waves on a plane slope. The fifth type of wave generation, which consists of two parallel lines connected to a semicircle, shows the best solutions especially when the grid size is small enough.

Keywords: Internal Generation of Waves, Wave Generation Curve, Numerical Experiment, Wave Refraction and Shoaling.

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1. INTRODUCTION

Waves reflected from the model domain should pass through the offshore boundary without any numerical distortions. Otherwise, the waves re-reflected at the offshore boundary may influence the numerical solution. To avoid the re-reflection problem, internal wave generation techniques have been used and sponge layers are placed at the offshore boundary. The techniques are categorized as the line source method and the source function method. In the line source method, the values of water surface elevation or velocity potential are added with desired energy to the corresponding values that are computed by the model equations.

Until now, the technique of internal wave generation has been developed only on a straight line. If waves in arbitrary direction are generated on a horizontally two-dimensional domain, we need at least two wave generation lines, i.e., one parallel to the x-axis and the other parallel to the y-axis. However, the generated wave energy would be more or less than the target one near the point which meets two wave generation lines orthogonally. Such a problem may be solved by connecting two orthogonal lines with an arc with large radius. In

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the present study, we develop a technique of internally generating waves on a curve. In section 2, a theory is presented for internally generating waves on the arc. In sections 3, numerical experiments are conducted to simulate the propagation of waves on a flat bottom, and the refraction and shoaling of waves on a plane slope. Finally, a brief summary is made.

2. THEORY OF INTERNAL GENERATION OF WAVES ON A CURVE

In the line source method, we add the values η^* of water surface elevation with desired energy to the corresponding values $\eta^{\it model}$ that are computed by the model equations as

$$\eta^{n+1} = \eta^{model} + \eta^* \tag{1}$$

where η is the water surface elevation and the superscript n+1 means the stage of time for predicting the values. The values η^* added at the wave generation line is given by (Lee and Suh, 1998):

$$\eta^* = 2\eta^I \frac{C_e \Delta t}{\Delta x} \cos \theta \tag{2}$$

where η^{I} is the water surface elevation of incident waves, and $\theta (= \cos^{-1}(k_{x}/k))$ is the angle of incident waves from the x-axis. The wave generation line is assumed to be parallel to the x-axis. When waves are generated on an arc instead of the line, the added values η^* should be different from those given by Eq. (2). For waves in the direction of a positive x-directional component, there may be four cases of generating waves on the arc. Fig. 1 shows four cases of the points a, b, cand d on the wave generation arc. For the cases of ray 1, 2, and 4, the values η^* added at the generation curve would be (see Fig. 2) Fig. 1. wave generation points



on an arc added at the wave

$$\eta^{*} = 2\eta^{I} \frac{C_{e}\Delta t}{\Delta y} \frac{\sin\left(\alpha + \theta\right)}{\cos\alpha} \text{ (point a)}, \quad \eta^{*} = 2\eta^{I} \frac{C_{e}\Delta t}{\Delta x} \frac{\sin\left(\alpha - \theta\right)}{\cos\alpha} \text{ (point b)} \quad (3)$$
$$\eta^{*} = 2\eta^{*} \frac{C_{e}\Delta t}{\Delta x} \frac{\cos\left(\alpha + \theta\right)}{\cos\alpha} \text{ (point c)}, \quad \eta^{*} = 2\eta^{I} \frac{C_{e}\Delta t}{\Delta y} \frac{\sin\left(\alpha - \theta\right)}{\cos\alpha} \text{ (point d)}$$

where α is the angle of the line normal to the wave generation curve from either the y-axis (for points a and d) or the x-axis (for points b and c). For the cases of ray 3 and 5, the direction of incident waves is outside the curve. Thus, waves are not generated in these cases.

3. NUMERICAL EXPERIMENTS

Numerical experiments are conducted using the time-dependent extended mild-slope equations of Suh et al. (1997) given by:



Fig. 2. Incident wave rays at a point of the wave generation curve. (a) point a, (b) point b.

$$\frac{\partial \eta}{\partial t} = -\nabla \cdot \left(\frac{CC_g}{g}\nabla \tilde{\phi}\right) + \frac{\omega^2 - k^2 CC_g}{g}\tilde{\phi} + \frac{\omega^2}{g}\left\{R_1(\nabla h)^2 + R_2\nabla^2 h\right\}\tilde{\phi}$$

$$\frac{\partial \tilde{\phi}}{\partial t} = g\eta - \omega D_s\tilde{\phi}$$
(5)

where $\tilde{\phi}$ is the velocity potential at mean water level, C and C_g are the phase velocity and group velocity, respectively, and D_s is the damping coefficient at the sponge layer. Eqs. (4) and (5) are discretized by a fourth-order Adams-Moulton predictor-corrector method in time and by a three-point symmetric formula in space.

Tests are made for five different layouts of wave generation lines and arcs on a flat bottom. Figs. 3(a)-(c) show the 1st to 3rd types with two orthogonal lines. Fig. 3(d) shows the 4th type with two orthogonal lines connected to a quarter-circle. Fig. 3(e) shows the 5th type with two parallel lines connected to a semicircle. The depth is shallow ($kh = 0.1\pi$). The amplitudes of water surface elevations are measured in a test area of $6L \times 6L$.

Figs. 4(a)-(b) show the mean and maximum values of absolute error versus incident wave directions in the test area. The absolute error is defined as the absolute value of the difference between the computed and target wave amplitudes. The grid size is $\Delta x = \Delta y = L/10$. The comparison of all the averaged errors yields that the 4th and 5th types of wave generation give the smallest errors particularly around $\theta = 36^{\circ}$. The maximum errors for the five types of wave generation have almost the same trend as the averaged errors. For the case of smaller grid size with $L/\Delta x = 20$, the 5th type of wave generation yields smaller errors than the 4th type in any direction of incident waves. For the 5th type, the averaged absolute error is below 0.02 at the range of wave directions between $\theta = 3^{\circ}$ and $\theta = 54^{\circ}$, which is wide enough for generating multi-directional waves.

Tests are also made for waves propagating obliquely on a plane slope to verify the generation of waves over varying water depths. Waves propagating obliquely on a plane slope would experience the refraction as well as the shoaling.

The 3rd to 5th types of wave generation are used. Fig. 5 shows the computational domain for testing waves propagating obliquely on a plane slope using the 5th type of wave



generation. The wave period is 10 s. The water depths on the up-wave and down-wave

Fig. 3. five types of internal wave generation. (a) 1st type, (b) 2nd type, (c) 3rd type, (d) 4th type (radius of arc = 5L), (e) 5th type (radius of arc = 5L).



Fig. 4. absolute error versus incident wave direction $(L/\Delta x = 10)$; dash-dotted line: 1st type, dash-dot-dotted line: 2nd type, dash-dot-dotted line: 3rd type, dashed line: 4th type, solid line: 5th type. (a) mean value of absolute error, (b) maximum value of absolute error.

sides of the slope are 156 m ($kh = 2\pi$) and 2 m ($kh = 0.09\pi$). The incident wave direction is $\theta = 30^{\circ}$ in deep water. Wave amplitudes are measured along six transects.

Fig. 6 shows the comparison of the measured wave amplitudes against the exact solution on the transects. The 5th type of wave generation yields the most accurate solution at all the sections and the 3rd type of wave generation yields the worst solution.





Fig. 5. Computational domain for testing waves propagating obliquely on a plane slope using the 5th type of wave generation.

Fig. 6. Comparison of wave heights on transects;dash-dotted line: 3rd type, dashed line: 4th type, solid line: 5th type, solid line with O: exact solution. (a) section 3, (b) section 6.

4. CONCLUSIONS

The techniques of internally generating waves on a curve in a rectangular grid system are developed using the line source method. The extended mild-slope equations of Suh et al. (1997) are used for numerical experiment. For five different types of wave generation layout, numerical experiments are conducted in the cases of the propagation of waves on a flat bottom, and the refraction and shoaling of waves on a plane slope. The 5th type of wave generation shows the most accurate solution especially when the grid size is small enough.

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