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A Note on Intuitionistic Fuzzy Ideals of Semigroup

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Abstract

We give the characterization of an intuitionistic fuzzy ideal[resp. intuitionistic fuzzy left ideal, an intuitionistic fuzzy right ideal and an intuitionistic fuzzy bi-ideal] generated by an intuitionistic fuzzy set in a semigroup without any condition. And we prove that every intuitionistic fuzzy ideal of a semigroup S is the union of a family of intuitionistic fuzzy principle ideals of S. Finally, we investigate the intuitionistic fuzzy ideal generated by an intuitionistic fuzzy set in S^1.

1 Introduction

In his pioneering paper[21], Zadeh introduced the notion of a fuzzy set in a set X as a mapping from X into the closed unit interval [0, 1]. Since then, some researchers[16,17,19,20] applied this notion to semigroup and group theory. In 1986, Atanassov[2] introduced the concept of intuitionistic fuzzy sets as the generalization of fuzzy sets. Recently Coker and his colleagues[6,7,8], Hur and his colleagues [13] , and Lee and Lee[18] introduced the concept of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets and investigated some of their properties. In 1989, Biswas[3] introduced the concept of intuitionistic fuzzy

subgroups and studied some of it's properties. In 2003, Banerjee and Basnet[2] investigated intuitionistic fuzzy subrings and intuitionistic fuzzy ideals using intuitionistic fuzzy sets. Also, Hur and his colleagues[1,9-11, 14, 15] applied the notion of intuitionistic fuzzy sets to algebra. Moreover, Hur and his colleagues[12] applied one to topological group. In this paper, we give the characterization of an intuitionistic fuzzy ideal[resp. intuitionistic fuzzy left ideal, an intuitionistic fuzzy right ideal and an intuitionistic fuzzy bi-ideal] generated by an intuitionistic fuzzy set in a semigroup without any condition. And we prove that every intuitionistic fuzzy ideal of a semigroup S is the union of a family of intuitionistic fuzzy principle ideals of S. Finally, we investigate the intuitionistic fuzzy ideal generated by an intuitionistic fuzzy set in S^1.

2. Preliminaries

We will list some concept and one result needed in the later sections. For sets X, Y and Z, $f=(f_1 f_2): X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mappings.

Throughout this paper, we will denote the unit interval [0,1] and I.

Definition 2.1[2,6]. Let X be a nonempty set. A complex mapping $A=(\mu_A,\nu_A): X \to I \times I$ is called an *intuitionistic fuzzy set*(in short, IFS) in X if $\mu_A(x)+\nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (navely $\mu_A(x)$) and the degree of non-membership(namely $\nu_A(x)$) of each $x \in X$ to A, respectively. In particular, $0_\sim$ and $1_\sim$ denote the *intuitionistic fuzzy empty set* and the *intuitionistic fuzzy whole set* in a set X defined by $0_\sim(x)=(0,1)$ and $1_\sim(x)=(1,0)$ for each $x \in X$, respectively.

We will denote the set of all **IFS**s in X as **IFS**(X).

Definition 2.2[2]. Let X be a nonempty sets and $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be an **IFS**s in X.. Then

(1) $A \subset B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A=B$ if and only if $A \subset B$ and $B \supset A$.

(3) $A^c=(\nu_A,\mu_A)$

(4) $A \cap B=(\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$

(5) $A \cup B=(\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$

(6) $[]A=(\mu_A, 1-\mu_A), < > A=(1-\nu_A, \nu_A)$

Definition 2.3[6]. Let $\{A_i\}_{i \in J}$ be an arbitrary family of **IFS**s in X, where $A_i=(\mu_{A_i},\nu_{A_i})$ for each $i \in J$. Then

(1) $\cap A_i = (\wedge \mu_{A_i}, \vee \nu_{A_i})$

(2) $\cup A_i = (\vee \mu_{A_i}, \wedge \nu_{A_i})$

Definition 2.4[18]. Let λ, $\mu \in I$ with $\lambda+\mu \leq 1$. An *intuitionistic fuzzy point*(in short, ***IFP***) $x_{(\lambda,\mu)}$ of X is **IFS** in a set Xdefined by for each $y \in X$

$$x_{(\lambda,\mu)}(y)=\begin{cases}(\lambda,\mu) & \text{if } y=x \\ (0,1) & \textit{otherwise}\end{cases}$$

In this case, x is called the *support* of $x_{(\lambda,\mu)}$ and μ are called the *value* and the *nonvalue* of $x_{(\lambda,\mu)}$, respectively. An **IFP** $x_{(\lambda,\mu)}$ is said to *belong* to an **IFS** $A=(\mu_A,\nu_A)$ in X, denoted by $x_{(\lambda,\mu)} \in A$, if

$\lambda \leq \mu_A(x)$ and $\mu \geq \nu_A(x)$.

Clearly an intuitionstic fuzzy point can be represented by an ordered pair of fuzzy points as follows:

$x_{(\lambda,\mu)} = (x_\lambda, 1 - x_{1-\mu})$

We will denote the set of all **IFPs** in a set X $\mathbf{IF}p(X)$.

Definition 2.5[9]. Let A be an **IFS** in a set X and let $(\lambda,\mu) \in I \times I$ with $\lambda + \mu \leq 1$. Then the set $A^{(\lambda,\mu)} = \{x \in X: \mu_A(x) \geq \lambda \text{ and } \nu_A(x) \leq \mu\}$ is called a (λ,μ)*-level subset* of A

Result 2.1[18, Theorem 2.4]. Let X be a set and let $A \in IFS(X)$. Then

$A = \bigcup \{x_{(\lambda,\mu)}: x_{(\lambda,\mu)} \in A\}$.

In fact, it is not difficult to see that

$A = \bigcup_{x \in A^{(0,1)}} x_{A(x)}$.

3. Intuitionstic ideals generated by intuitionistic fuzzy sets

Lets S be a semigroup. By a S we mean a non-empty subset of A of such that

$A^2 \subset A$

and by a [resp] *ideal* of S we mean a non-empty subset of S such that

$SA \subset A$ $[resp.\ AS \subset A]$,

By tow-sided ideal or simply ideal we mean a subset A of S which is both a left and a right ideal of S. We well denote the set of all left ideals [resp right ideals and ideals] of S as **LI**(S) [resp. **RI**(S) and **I**(S)].

Definition 3.1[9]. Let S be a semigroup and let $0_\sim \neq A \in IFS(S)$. Then A is called an :

(1) *intuitionistic fuzzy subsemigroup* (in short, **IFSG**) of S if $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ for any $x,y \in S$,

(2) *intuitionistic fuzzy left ideal* (in short, **IFLI**) of S if $\mu_A(xy) \geq \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(y)$ for any $x,y \in S$,

(3) *intuitionistic fuzzy right ideal* (in short, **IFSG**) of S if $\mu_A(xy) \geq \mu_A(x)$ and $\nu_A(xy) \leq \nu_A(x)$ for any $x,y \in S$,

(4) *intuitionistic fuzzy (two-sided) ideal* (in short, **IFI**) of S if is both an in-tuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S.

We well denote the set of all **IFSGs** [resp. **IFLIs**, **IFRIs** and **LFIs**] of S as **IFSG**(S) [resp. **IFLI**(S), **IFRI**(S) and **IFI**(S)]. It is clear that $A \in IFI(S)$ if and only if $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \leq \nu_A \wedge \nu_A(y)$ for any $x,y \in S$, and if $A \in$ **IFLI**(S)[resp. **IFRI**(S) and **IFI**(S)], then $A \in$ **IFSG**(S).

Result 3.1[9, Proposition 3.7 and 14, Proposition 2.3]. Let S be a semi-group and let $(\lambda,\mu) \in I \times I$ with $\lambda + \mu \leq 1$. Then $A \in$ **IFSG**(S) [resp. **IFI**(S), **IFLI**(S) and **IFRI**(S)] if and only if $A^{(\lambda,\mu)}$ is a subdemigroup [resp. ideal, left ideal and right ideal] of S.

It is well-known[4] that I is complete completely distributive lattice. Thus we have the following result.

Proposition 3.1. Let S be a semigroup. Then **IFI**(S) is a complete completely distributive lattice with respect to the meet '$\cap$' and the union '$\cup$'.

Definition 3.2. Let S be a semigroup and let $A \in$ **IFS**(S). Then the least **IFLI**[resp. **IFRI** and **IFI**] of S containing A is called the **IFLI**[resp. **IFRI** and **IFI**] of S *generated* by A and is denoted by $(A)_L$ [resp. $(A)_R$ and (A)].

Lemma 3.1. Let X be a set, let $A \in$ **IFS**(X) and let $x \in X$. Then $A(x) = (\vee_{x \in A^{(\lambda,\mu)}} \lambda,\ \wedge_{x \in A^{(\lambda,\mu)}} \mu)$, where $\lambda,\mu \in I$ with $\lambda + \mu \leq 1$.

Proof. Let $\lambda_0 = \vee_{x \in A^{(\lambda,\mu)}} \lambda$, let $\mu_0 = \wedge_{x \in A^{(\lambda,\mu)}} \mu$ and let $\epsilon > 0$. Then $\vee_{x \in A^{(\lambda,\mu)}} \lambda > \lambda_0 - \epsilon$ and $\wedge_{x \in A^{(\lambda,\mu)}} \mu < \mu_0 + \epsilon$. Thus there exists $(s,t) \in \{(\lambda,\mu): x \in A^{(\lambda,\mu)}\}$ such that $s > \lambda_0 - \epsilon$ and $t < \mu_0 + \epsilon$. Since $x \in A^{(\lambda,\mu)}$, $\mu_A(x) \geq \lambda$ and $\nu_A(x) \leq \mu$. Then $\mu_A(x) > \lambda_0 - \epsilon$ and $\nu_A(x) < \mu_0$.. Since ε is an arbitrary real number, $\mu_A(x) \geq \lambda_0$ and $\nu_A(x) \leq \mu_0$. On the other hand, let $A(x) = (s,t)$. Then $x \in A^{(s,t)}$. Thus $(s,t) \in \{(\lambda,\mu): x \in A^{(\lambda,\mu)}\}$.

So $s \leq \vee_{x \in A^{(\lambda,\mu)}} \lambda$ and $t \geq \wedge_{x \in A^{(\lambda,\mu)}} \mu$,i.e.,

$\mu_A(x) = s \leq \lambda_0$ and $\nu_A(x) = t \geq \mu_0$.

Hence $A(x) = (\mu_A(x), \nu_A(x)) = (\lambda_0, \mu_0)$

Theorem 3.1. Let S be a semigroup, let $A \in$ **IFS**(S) and let $(\lambda,\mu) \in I \times I$ with $\lambda + \mu \leq 1$. We define a complex mapping $A^* = (\mu_{A^*}, \nu_{A^*}): S \to I \times I$ as follows for each $x \in S$

$$A^*(x) = \left(\bigvee_{x \in (A^{(\lambda,\mu)})} \lambda,\ \bigwedge_{x \in A^{(\lambda,\mu)}} \mu \right)$$

Then $A^* = (A)$, where $(A^{(\lambda,\mu)})$ denotes the ideal generated by $A^{(\lambda,\mu)}$

Proof. For each $x \in S$, let $(s,t) \in \{(\lambda,\mu): x \in A^{(\lambda,\mu)}\}$. Then $x = A^{(s,t)}$. Thus $x \in (A^{(s,t)})$. So $(s,t) \in \{(\lambda,\mu): x \in (A^{(\lambda,\mu)})\}$, I.e., $\{(\lambda,\mu): x \in (A^{(\lambda,\mu)})\} \subset \{(\lambda,\mu): x \in (A^{(\lambda,\mu)})\}$. Then, by Lemma 3.1,

$\mu_A(x) = \vee_{x \in A^{(\lambda,\mu)}} \mu \leq \vee_{x \in (A^{(\lambda,\mu)})} \mu = \mu_{A^*}(x)$

and $\mu_A(x) = \wedge_{x \in A^{(\lambda,\mu)}} \mu \geq \wedge_{x \in (A^{(\lambda,\mu)})} \mu = \nu_{A^*}(x)$.

So $A \subset A^*$. For each $(s,t) \in ImA^*$, let $s_n = s - \frac{1}{n}$ and $t_n = t - \frac{1}{n}$ for each $n \in N$. Let $x \in A^{*(s,t)}$. Then $\mu_{A^*}(x) \geq s$ and $\nu_{A^*}(x) \leq t$. Thus, for each $n \in N$

$\vee_{x \in (A^{(\lambda,\mu)})} \lambda \leq s > s - \frac{1}{n} = s_n$

and

$\wedge_{x \in (A^{(\lambda,\mu)})} \mu \leq t \leq t + \frac{1}{n} = t_n$.

So there exists $a(\lambda_n, \mu_n) \in (\lambda,\mu): x \in (A^{(\lambda,\mu)})$ such that $\lambda_n > s_n$ and $\mu_n < t_n$. Then $A^{(\lambda_n,\mu_n)} \subset A^{(s_n,t_n)}$. $x \in (A^{(\lambda_n,\mu_n)}) \subset (A^{(s_n,t_n)})$. Consequently, we have $x \in \cap_{n \in N} (A^{(s_n,t_n)})$. Now let $x \in \cap_{n \in N} (A^{(s_n,t_n)})$. Then clearly $(s_n,t_n) \in (\lambda,\mu): x \in (A^{(\lambda,\mu)})$ for each $n \in N$. Thus for each $n \in N$, $s - \frac{1}{n} = s_n \leq \vee_{x \in (A^{(\lambda,\mu)})} \lambda = \mu_{A^*}(x)$ and $t - \frac{1}{n} = t_n \leq \wedge_{x \in (A^{(\lambda,\mu)})} \lambda = \nu_{A^*}(x)$. Since n is an arbitrary positive interger, $s \leq \mu_{A^*}(x)$ and $t \geq \nu_{A^*}(x)$. Thus. So $A^{*(s,t)} \in \cap_{n \in N} (A^{(s_n,t_n)})$. It is clear that $\cap_{n \in N} (A^{(s_n,t_n)})$ is an ideal of S. So, by Result 3.1, $A^* \in$ IFI(S).

Now let $B\in$ IFI(S) such that $A\subset B$ and let $x\in S$. If $A^*(x)=(0,1)$, then clearly $\mu_{A^*}(x)<\mu_B(x)$ and $\nu_{A^*}(x)>\nu_B(x)$, I.e., $A^*\subset B$. If $A^*(x)(s,t)\neq(0,1)$, then $x\in A^{*(s,t)}=\cap_{n\in N}(A^{(s_n t_n)})$.

Thus $x\in(A^{(s_n t_n)})=A^{(s_n t_n)}S\cup SA^{(s_n t_n)}S\cup A^{(s_n t_n)}S\cup A^{(s_n t_n)}$ for each $n\in N$. We consider the following cases:

Case(i) : Suppose $x\in A^{(s_n t_n)}$. Then clearly for each $n\in N$ $s_n\le\mu_A(x)\le\mu_B(x)$ and .

Case(ii) : Suppose $x\in A^{(s_n t_n)}$. Then there exist $a\in A^{(s_n t_n)}$ and $b\in S$ such that $x=ab$. Thus for each $n\in N$

$s_n\le\mu_A(a)\le\mu_B(b)\le\mu_B(ab)=\mu_B(x)$ and

$t_n\ge\nu_A(a)\ge\nu_B(b)\ge\nu_B(ab)=\nu_B(x)$.

Case(iii) : Suppose $x\in SA^{(s_n t_n)}$. Then by the similar arguments of Case(ii), we have $\mu_B(x)\ge s_n$ and $\nu_B(x)\ge t_n$ for each $n\in N$

Case(iiii) : Suppose $x\in SA^{(s_n t_n)}$. Then there exist $a\in A^{(s_n t_n)}$ and $b\in S$ such that $x=abc$. Since $B\in$ IFI(S). for each $n\in N$

$s_n\le\mu_A(a)\le\mu_B(a)\le\mu_B(x)$ and

$t_n\ge\nu_A(a)\ge\nu_B(a)\ge\nu_B(x)$.

Since n is an arbitrary number in N, in all, $\mu_{A^*}(x)=s\le\mu_B(x)$ and $\nu_{A^*}(x)=t\ge\nu_B(x)$. Thus $A^*\subset B$. Hence $A^*=(A)$. This complete the proof.

Corollary 3.1. Let S be a semigroup and let $x_{(\lambda,\mu)}\in IFp(S)$. We define a complex mapping $(x_{(\lambda,\mu)}):S\to I\times I$ as follows : for each $x=S$,

$$(x_{(\lambda,\mu)})(y)=\begin{cases}(\lambda,\mu) & \text{if } y\in(x),\\(0,1) & \text{if } y\notin(x),\end{cases}$$

where (x) is the principal ideal of S generated by x. Then $(x_{(\lambda,\mu)})$ is the IFI generated by $x_{(\lambda,\mu)}$. In this case, $(x_{(\lambda,\mu)})$ is called the intuitionistic fuzzy principal ideal(in short, ***IFPI***)of S generated by $x_{(\lambda,\mu)}$.

Proof. By Theorem 3.1

$(x_{(\lambda,\mu)})(y)=(\vee_{z\in(A^{(s,t)})}s,\ \wedge_{z\in(A^{(s,t)})}t)$ for each $y\in S$.

Case(i) : Suppose $y\in(x)$. Let $(s,t)\in(0,\lambda]\times[\mu,1)$. Then $A^{(s,t)}=z\in S:\mu_{x_{(\lambda,\mu)}}(z)\ge s,\nu_{x_{(\lambda,\mu)}}(z)\le t=x$. Thus $y\in(x)=(A^{(s,t)})$. If $s>\lambda$ and $t>\mu$, then clearly $x_{(\lambda,\mu)}=(0,1)$, So

$$(x_{(\lambda,\mu)})(y)=(\bigvee_{z\in(A^{(s,t)})}s,\ \bigwedge_{z\in(A^{(s,t)})}t)=(\bigvee_{0<s\le\lambda}s,\ \bigwedge_{\mu\le t<1}t)=(\lambda,\mu).$$

Case (ii) : Suppose $y\notin(x)$. Assume that $(x_{(\lambda,\mu)})(y)\neq(0,1)$. Then there exists $(s,t)\in(0,\lambda]\times[\mu,1)$ with $s+t\le1$ such that $y\in(A^{(s,t)})$. Since $A^{(s,t)}\neq(0,1)$, by Case (i), $s\le\lambda$ and $t\ge\mu$. Thus $A^{(\lambda,\mu)}=x$. So $y\in(A^{(s,t)})=(x)$. This is a contradiction. Thus $(x_{(\lambda,\mu)})(y)=(0,1)$. Hence $(x_{(\lambda,\mu)})$ is well-defined.

The following is an easy modification of Theorem 3.1.

Theorem 3.2. Let S be a semigroup and let $A\in$ IFI(S). We define a complex mapping $(x_{(\lambda,\mu)}):S\to I\times I$ as follows : for each $x=S$,

$$A^*(x)=(\bigvee_{x\in(A^{(\lambda,\mu)})_L}\lambda,\ \bigwedge_{x\in(A^{(\lambda,\mu)})_L}\mu)$$

then $A^*=(A)_L$, where $(A^{(\lambda,\mu)})_L$ denotes the left ideal generated by $A^{(\lambda,\mu)}$.

Corollary 3.2. Let S be a semigroup and let $x_{(\lambda,\mu)}\in IFp(S)$. We define two complex mapping $(x_{(\lambda,\mu)})_L:S\to I\times I$ and $(x_{(\lambda,\mu)})_R:S\to I\times I$ as follows, respectively : for each $y\in S$,

$$(x_{(\lambda,\mu)})_L(y)=\begin{cases}(\lambda,\mu) & \text{if } y\in(x)_L\\(0,1) & \text{if } y\notin(x)_L\end{cases} \quad\text{and}$$

$$(x_{(\lambda,\mu)})_R(y)=\begin{cases}(\lambda,\mu) & \text{if } y\in(x)_R\\(0,1) & \text{if } y\notin(x)_R,\end{cases}$$

Then $(x_{(\lambda,\mu)})_L$[resp.$(x_{(\lambda,\mu)})_R$] is the IFLI[resp. IFRI] of S generated by $x_{(\lambda,\mu)}$ in S.

In this case, $(x_{(\lambda,\mu)})_L$[resp.$(x_{(\lambda,\mu)})_R$] is called the intuitionistic fuzzy principal [resp.] ideal(in short, IFPLI [resp.IFPRI]) generated by $x_{(\lambda,\mu)}$.

Proof. The proofs are similar to the case of Corollary 3.1. So we omit.

Remark 3.1. As the dual of Theorem 3.2, $(A)_R$ can be characterized by $(A)_R(x)=(\vee_{x\in(A^{(\lambda,\mu)})_R}\lambda,\ \wedge_{x\in(A^{(\lambda,\mu)})_R}\mu)$ for each $x=S$, where $(A^{(\lambda,\mu)})_R$ denotes the right ideal generated by $A^{(\lambda,\mu)}$.

A nonempty subset A of a semigroup S is called a bi-ideal of S if $A^2\subset A$ and $ASA\subset A$. We will denote the set of all bi-ideal of S as BI(S).

Definition 3.3[14]. Let S be a semigroup and let $0_\sim\neq A\in IFS(S)$. Then A is called an intuitionistic fuzzy bi-ideal(in short, IFBI) of S if it satisfies the following conditions : for any $x,y,z\in S$.

(i)$\mu_A(xy)\ge\mu_A(x)\wedge\mu_A(y)$ and $\nu_A(xy)\le\nu_A(x)\vee\nu_A(y)$

(ii)$\mu_A(xyz)\ge\mu_A(x)\wedge\mu_A(z)$ and $\nu_A(xyz)\le\nu_A(x)\vee\nu_A(z)$.

We will denote the set of all IFBIs of S as IFBI(S).

Result 3.2[14, Proposition 2.8]. Let S be a semigroup and let $A\in$ IFI(S).

Then $A\in$ IFI(S) if and only if $A^{(\lambda,\mu)}\in BI(S)$ for each $(\lambda,\mu)\in I\times I$ with $\lambda+\mu<1$.

Let A be a subset of a semigroup S. Then it is not difficult to see that the bi-ideal$(A)_B$ generated by A in S is $A\cup A^2\cup ASA$.

The following can be shown by the above comment, Result 3.2 and a moderate modification of Theorem 3.1.

Theorem 3.3. Let S be a semigroup and let $A\in IFS(S)$. We define a complex mapping $A^*:S\to I\times I$ as follows : for each $x=S$,

$A^*(x)=(\bigvee_{x\in(A^{(\lambda,\mu)})_B}\lambda,\ \bigwedge_{x\in(A^{(\lambda,\mu)})_B}\mu)$ then $A^*=(A)_B$, where $(A)_B$ denotes the IFBI generated by A.

Corollary 3.2. Let S be a semigroup and let $x_{(\lambda,\mu)}\in IFp(S)$. We define two complex mapping $(x_{(\lambda,\mu)})_B:S\to I\times I$ follows, respectively : for each $y\in S$,

$$(x_{(\lambda,\mu)})_B(y)=\begin{cases}(\lambda,\mu) & \text{if } y\in(x)_B\\(0,1) & \text{if } y\notin(x)_B\end{cases}$$

Then $(x_{(\lambda,\mu)})_B$ is the IFBI of S generated by $x_{(\lambda,\mu)}$ in S. In this case, $(x_{(\lambda,\mu)})_B$ is called the intuitionistic fuzzy principal bi-ideal(in short, IFPBI)generated by $x_{(\lambda,\mu)}$.

Proof. The proofs is similar to the case of Corollary 3.1. So we omit. It is well-known that every ideal of a semigroup S is the union of some principal ideals of S. Similarly, we have the following result.

Theorem 3.4. Let S be a semigroup. Then every IFI of S is the union of some IFPIs of S.

Proof. Let $A\in IFI(S)$. Then, by Result 2.1,

$A=\bigcup_{x_{(\lambda,\mu)}\in A} x_{(\lambda,\mu)} = \bigcup_{x\in A^{(0,1)}} x_{A(x)}$. Let $y\in S$.

Case (i) : Suppose $A(y)\neq(0,1)$. then

$\cup_{x\in A^{(0,1)}} x_{A(x)})(y)=(\cup_{y\in(z),z\in A^{(0,1)}}))(y)$

$=(\vee_{y\in(z),z\in A^{(0,1)}}\mu_{A(z)}, \wedge_{y\in(z),z\in A^{(0,1)}}\nu_{A(z)})$. If $z\neq y$, then there exist $a_1,a_2,b_1,b_2\in S$ such that $y=za_1$ or $y=a_2z$ or $y=b_1zb_2$. For any cases, since $A\in IFS(S)$, $\mu_A(y)\geq\mu_a(z)$ and $\nu_A(y)\leq\nu_a(z)$. Thus

$(\cup_{x\in A^{(0,1)}} x_{A(x)})(y)=(\vee_{y\in(z),z\in A^{(0,1)}}\mu_{A(z)}, \wedge_{y\in(z),z\in A^{(0,1)}}\nu_{A(z)})$
$=(\mu_{A(y)},\nu_{A(y)})=A(y)$.

Case (ii) : Suppose $A(y)=(0,1)$. Assume that there exists $z\in A^{(0,1)}$ such that $y\in(z)$. Then $\mu_{A(y)}\geq\mu_{A(z)}$ and $\nu_{A(y)}\leq\nu_{A(z)}$ as above. Thus $A(y)\neq(0,1)$. This is a contradiction. Then $y\notin(z)$ for each $z\in A^{(0,1)}$.

So $A(y)=(\cup_{x\in A^{(0,1)}}(x_{A(x)}))(y)=(0,1)$. Hence, in all,

$A=\cup_{x\in A^{(0,1)}} x_{A(x)}$. This completes the proof.

4 Some special cases

In this case, we study intuitionistic fuzzy ideal generated by an IFS A in S^1.

Theorem 4.2. Let S be a regular semigroup and let $A\in IFS(S^1)$. Then

$$A(a)=(\bigvee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2), \bigwedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\nu_A(x_2))$$ for each $a\in S$.

Proof. Let $a\in S$ such that $a=x_1x_2x_3$ for some $x_1,x_2,x_3\in S^1$ and $A(x_2)=(s,t)$. Then $x_2\in A^{(s,t)}$. Thus $a\in(A^{(s,t)})$. So $A(x_2)\in\{(s,t): a\in(A^{(s,t)})\}$.

By theorem 3.1,

$$\mu_A(a)=\bigvee_{a\in(A^{(s,t)})} s\geq \bigvee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2) \text{ and}$$

$$\mu_A(a)=\bigwedge_{a\in(A^{(s,t)})} t\leq \bigwedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2) \quad (1)$$

On the other hand, let $(\lambda,\mu)\in\{(s,t): a\in(A^{(s,t)})\}$. Then clearly $a\in(A^{(\lambda,\mu)})$. Thus there exist $x_1, x_3\in S^1$ and $x_2\in A^{(\lambda,\mu)}$ such that $a=x_1x_2x_3$. Since $x_2\in A^{(\lambda,\mu)}$, $\mu_A(x_2)\geq\lambda$ and $\mu_A(x_2)\leq\mu$. Then

$$\mu_A(a)=\bigvee_{a\in(A^{(s,t)})} s\leq \bigvee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2) \text{ and}$$

$$\mu_A(a)=\bigwedge_{a\in(A^{(s,t)})} t\geq \bigwedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2) \quad (2)$$

Hence, by(1) and (2),

$$A(a)=(\bigvee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\mu_A(x_2), \bigwedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}\nu_A(x_2)) .$$

This complete the proof.

Remark 4.1. By theorem 3.1 and its dual, we can easily obtain $(A)_L$[resp. $(A)_R$] generated by A in S^1 defined by $A_L(a)=(\bigvee_{\substack{a=x_1x_2\\ x_1,x_2\in S^1}}\mu_A(x_2), \bigwedge_{\substack{a=x_1x_2\\ x_1,x_2\in S^1}}\nu_A(x_2))$

[resp. $A_R(a)=\vee_{\substack{a=x_1x_2\\ x_1,x_2\in S_1}}\mu_A(x_1), \wedge_{\substack{a=x_1x_2\\ x_1,x_2\in S_1}}\mu_A(x_1)$],

for each $a\in S$

Theorem 4.2. Let S be a regular semigroup and let $A\in IFS(S^1)$.

Then

$$(A)_B(a)=(\vee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\mu_A(x_1)\wedge\mu_A(x_3)], \wedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\nu_A(x_1)\vee\nu_A(x_3)]$$

for each $a\in S$.

Proof. Let $a\in S$ such that $a=x_1x_2x_3$ for some $x_1,x_2,x_3\in S^1$

and let $(s,t)=(\mu_A(x_1)\wedge\mu_A(x_3),\nu_A(x_1)\vee\nu_A(x_3))$. Then clearly $x_1,x_3\in A^{(s,t)}$. Thus clearly $a\in(A^{(s,t)})_B$.

So $(\mu_A(x_1)\wedge\mu_A(x_3),\nu_A(x_1)\vee\nu_A(x_3))\in\{(s,t): a\in(A^{(s,t)})_B\}$.

By theorem 3.3

$$\mu_{(A^{(s,t)})_B}(a)=\vee_{a\in(A^{(s,t)})_B} s\geq\vee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\mu_A(x_1)\wedge\mu_A(x_3)] \text{ and}$$

$$\nu_{(A^{(s,t)})_B}(a)=\wedge_{a\in(A^{(s,t)})_B} s\leq\wedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\nu_A(x_1)\vee\nu_A(x_3)] \quad (3)$$

Now let $(\lambda,\mu)\in\{(s,t): a\in(A^{(s,t)})_B\}$. Then

$a=(A^{(s,t)})_B=A^{(s,t)}\cup A^{(s,t)}A^{(s,t)}\cup A^{(s,t)}S^1A^{(s,t)}=A^{(s,t)}\cup A^{(s,t)}S^1A^{(s,t)}$

since S^1 is regular, $A^{(s,t)}\subset A^{(s,t)}S^1A^{(s,t)}$.

Then $a\in(A^{(s,t)})_B=A^{(s,t)}S^1A^{(s,t)}$. Thus there exist

$x_1,x_3\in A^{(s,t)},\mu_A(x_1)\geq s,\mu_A(x_1)\leq t$ and

$\mu_A(x_3)\geq s,\nu_A(x_3)\leq t$.

Then $\mu_A(x_1)\wedge\mu_A(x_3)\geq s,\nu_A(x_1)\vee\nu_A(x_3)\leq t$. Thus

$$\mu_{(A^{(s,t)})_B}(a)=\vee_{a\in(A^{(s,t)})_B} s\geq\vee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\mu_A(x_1)\wedge\mu_A(x_3)] \text{ and}$$

$$\nu_{(A^{(s,t)})_B}(a)=\wedge_{a\in(A^{(s,t)})_B} s\leq\wedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\nu_A(x_1)\vee\nu_A(x_3)] \quad (4)$$

Hence, by (3) and (4),

$$(A^{(s,t)})_B(a)=(\vee_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\mu_A(x_1)\wedge\mu_A(x_3)], \wedge_{\substack{a=x_1x_2x_3\\ x_1,x_2,x_3\in S^1}}[\nu_A(x_1)\vee\nu_A(x_3)]$$

This completes the proof.

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