퍼지 기법을 이용한 비선형 시스템의 카오스화

Chaotification of Nonlinear Systems Via Fuzzy Approach

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Abstract

This paper presents a simple methodology that makes a continuous-time nonlinear system chaotic using fuzzy control. The nonlinear system is represented by the T-S fuzzy model. Then, a fuzzy controller makes the T-S fuzzy model, which could be stable or unstable, bounded and chaotic. The verification of chaos in the closed-loop system is done by the following procedures. We establish an asymptotically approximate relationship between a continuous-time T-S fuzzy system with time-delay and a discrete-time T-S fuzzy system. Then, we verify the chaos in the closed-loop system by applying the Marotto theorem to its associated discrete-time T-S fuzzy system.

1. Introduction

The usefulness of chaos has already been recognized in various areas of biology, medicine, physics, engineering, and technology [2]. Recently, intentional generation of chaos (called anticontrol of chaos, or chaotification) has attracted increasing interest toward practical applications of chaos.

There are many studies of chaotifying a discrete-time system [4-5]. Chen and Lai first developed a simple yet mathematically rigorous anticontrol method in [4]. For a discrete-time Takagi-Sugeno (T-S) fuzzy system, Li et al. developed an anticontrol method in [5].

On the other hand, for a continuous-time system, chaotification is much more difficult and challenging because even a unified definition of chaos for a continuous-time system does not exist in the mathematical and scientific literatures. Nevertheless, there have been some attempts to chaotify continuous -time systems in recent years. Especially, Wang et al. proposed a systematic anticontrol method for both stable linear continuous-time systems and stable nonlinear continuous-time systems in [1]. But the proposed anticontrol method for stable nonlinear continuous-time systems uses a linearization technique, so it can only verify chaos in a neighborhood of the fixed point of the nonlinear system. The method for anticontrol of chaos for a stable continuous-time T-S fuzzy system proposed by Li et al. [3] also has the same weakness since the basic approach for chaotification is the same with Wang's approach.

In this paper, a simple and systematic anticontrol method is proposed for a continuous-time nonlinear system, which could be stable or unstable. This new method can overcome the aforementioned problem. Specifically, the nonlinear system is represented by the T-S fuzzy model. Then, the concept of parallel distributed compensation is used to determine the structure of a fuzzy controller from the T-S fuzzy model. The consequent part of the fuzzy control rules is composed of the feedback gain and the local time-delay of the state of the system. The feedback gain is used for making unstable nonlinear system asymptotically stable. If the nonlinear system is originally asymptotically stable, the feedback gain need not to be designed. The local time-delay controller is designed to make the nonlinear system chaotic. For the local time-delay controller, we use a simple sinusoidal function, which was selected for chaotifying a system in [1] and [5].

To verify chaos in the closed-loop system, the following procedure is done. First, an asymptotically approximate relationship between a continuous-time T-S fuzzy system with time-delay and a discrete-time T-S fuzzy system is established by using the theorem in [1], which establishes an asymptotically approximate relationship between the time-delay differential equation and the corresponding difference equation. Then, chaos in the closed-loop is verified by applying the Marotto theorem to its associated discrete-time T-S fuzzy system. Therefore, the generated chaos is in the sense of Li and Yorke.

The remainder of the paper is organized as follows: the continuous-time T-S fuzzy system is reviewed briefly in Section 2. In Section 3, the design of the fuzzy controller that makes a continuous-time T-S fuzzy system chaotic is presented and an asymptotically approximate relationship between a continuous-time T-S fuzzy system with time-delay and a discrete-time T-S fuzzy system is derived. The verification of chaos in closed-loop T-S fuzzy system is also presented in Section 3. Finally, some conclusions are drawn in Section 4.

In this paper, we denote by x_j or $(x)_j$ and $|x|_\infty$ as the jth element and the infinity norm of the vector x, respectively. For matrices, we denote by $(T)_{ij}$ and $(T)_j$ as the ijth element and the jth column of the matrix T, respectively.

2. Preliminary

The T-S fuzzy model is generally known as the universal approximator of nonlinear systems. We consider a nonlinear system represented by the following T-S fuzzy model:

Plant Rule i:

IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in},

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$ (1)

The final output of the fuzzy model is inferred by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_i x(t) + B_i u(t)\} \quad (2)$$

in which $\mu_i(z(t))$ can be regarded as the firing strength of the IF-THEN rules.

3. Chaotifying a continuous-time nonlinear system

3.1 The design of fuzzy controller

For a continuous-time T-S fuzzy Model, a simple fuzzy controller is designed to make it chaotic. The defuzzified form of the fuzzy controller is the following:

Control Rule i:

IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in}

THEN $u(t) = -Kx(t) + \nu_i(t)$ (3)

where $\nu_i(t)$ is the time-delay feedback,

$$\nu_i(t) = \sigma \sin\left(\frac{\beta\pi}{\sigma}(T_i^{-1}x)_1(t-\tau)\right) \quad (4)$$

where τ is the delay time, σ and β are constants and $(T_i^{-1}x)_1(t-\tau)$ is the first element of the vector $T_i^{-1}x(t-\tau)$The local time-delay controller $\nu_i(t)$ is bounded by σ. Later, we will see that this parameter determines the range of the generated chaos. The transformation matrices T_i are determined in the next subsection. The overall fuzzy controller is inferred by

$$u(t)=\sum_{i=1}^{r}\mu_i\{-Kx(t)+v_i(t)\} \qquad (5)$$

Substituting (5) into (2) we obtain the closed-loop system

$$\dot{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{G_ix(t)+B_i\nu_j(t)\} \qquad (6)$$

where $G_i=A_i-B_iK$

The equation (6) can be seen the final output of the following continuous-time TS fuzzy system, which has r^2 subsystems.

Rule ij:

IF $z_1(t)$ is M_{i1} and M_{j1} $\cdots$and $z_n(t)$ is M_{in} and M_{jn},

THEN $\dot{x}(t)=G_ix(t)+B_i\nu_j(t)$ (7)

where G_i is the system matrix of the subsystem of (7).

The feedback gain matrix K is designed to make that the system matrices of each subsystem of (7), G_i, are Hurwitz stable matrices. To this purpose we use LMI method.

Theorem 1 [8] The feedback gain matrix K is designed from the following LMI to make that G_i are Hurwitz stable matrices.

$$QA_i^T-M^TB_i^T+A_iQ-B_iM<0, \qquad i=1,\cdots,r$$
$$Q>0$$

where

$$Q=P^{-1},\ M=KQ,$$

where P and Q are positive symmetric matrices. K is solved by $K=MP$.

3.2 Asymptotically approximate relationship between the continuous-time T-S fuzzy system with time-delay and the discrete-time T-S fuzzy system

We establish an asymptotically approximate relationship between the continuous-time T-S fuzzy system with time-delay and the discrete-time T-S fuzzy system, as the first step to verify the generated chaos in the closed-loop system. The subsystems of (7) (the consequent part of the rules) are considered in the local area corresponding to each rule, respectively. It can be viewed by the different axis on each local area. Specifically, the subsystem of (7) in the ijth local area (or for the ijth rule) is the following:

$$\dot{x}(t)=G_ix(t)+B_i\nu_j(t)$$

Define a new state vector $\hat{x}_i$ by

$$\hat{x}_i=(T_i)^{-1}x$$

Then,

$$\begin{aligned}\dot{\hat{x}}_i(t)&=T_i^{-1}G_iT_i\hat{x}_i(t)+T_i^{-1}B_i\nu_j(t)\\&=E_i\hat{x}_i(t)+D_i\nu_j(t)\end{aligned} \qquad (8)$$

where

$$E_i=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&&\vdots\\0&0&0&\cdots&1\\-a_{i0}&-a_{i1}&-a_{i2}&\cdots&-a_{i(n-1)}\end{bmatrix},\quad D_i=\begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix}$$

We use the same transformation matrix for local areas that have the same i index.

Theorem 2 There is an asymptotically approximate relationship between the continuous-time T-S fuzzy system with time-delay (7) and the following discrete-time T-S fuzzy system, which is represented by the defuzzified form, if the delay time τ is sufficiently large and all elements of $(T_i)_1$ are nonzero.

$$x(k+1)\approx\sum_{i=1}^{r}\mu_i^2\begin{bmatrix}\frac{\sigma(T_i)_{11}}{a_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)_{11}}x_1(k)\right)\\\frac{\sigma(T_i)_{21}}{a_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)_{21}}x_2(k)\right)\\\vdots\\\frac{\sigma(T_i)_{n1}}{a_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)_{n1}}x_n(k)\right)\end{bmatrix}$$

$$+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}\mu_i\mu_j\begin{bmatrix}\frac{\sigma(T_i)_{11}}{a_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)_{11}}{\sigma(T_i)_{11}}x_1(k)\right)\\\frac{\sigma(T_i)_{21}}{a_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)_{11}}{\sigma(T_i)_{21}}x_2(k)\right)\\\vdots\\\frac{\sigma(T_i)_{n1}}{a_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)_{11}}{\sigma(T_i)_{n1}}x_n(k)\right)\end{bmatrix}$$

$$\equiv F(x) \qquad (9)$$

Proof: The proof is omitted due to lack of space. ■

By Theorem 2, if the discrete-time T-S fuzzy system (9) is bounded and chaotic and the delay time τ is sufficiently large, the closed-loop T-S fuzzy system (7) is also bounded and chaotic. We verify the boundedness and chaos of (7) by proving the boundedness and chaos of (9).

Theorem 3 The magnitude of the system response of the discrete-time T-S fuzzy system (9) is bounded by the following constant:

$$|x(k)|_{\infty} \le \max_{1 \le d \le n,\ 1 \le i \le r} \left\{ \frac{(T_i)_{d1}}{\alpha_{i0}} \sigma \right\}$$

Proof: The proof is omitted due to lack of space. ■

Therefore, the magnitude of the system response is determined by σ.

3.3 Verification of chaos in the closed-loop T-S fuzzy system

Theorem 4 Suppose that $\mu_i(k)\mu_j(k)$, $i, j = 1, 2, \cdots,$ are continuously differentiable in the neighborhood of the fixed point, $x^* = 0$, of the controlled system (13). Then there exists a positive constant $\bar{\beta}$ such that if $\beta > \bar{\beta}$, then the controlled discrete-time TS fuzzy system (9) is chaotic in the sense of Li and Yorke.

Proof: The proof is omitted due to lack of space. ■

Therefore, the nonlinear system is converted to a chaotic system by the fuzzy controller (5).

4. Conclusions

A systematic anticontrol method for a continuous-time nonlinear system has been proposed and discussed in this paper. The nonlinear system was first represented by T-S fuzzy model. The design method of the fuzzy controller based on the parallel distributed compensation technique is conceptually simple. To verify chaos in the closed-loop system, an asymptotically approximate relationship between the continuous-time T-S fuzzy system with time-delay and the discrete-time T-S fuzzy system was derived. Then, the Marotto theorem was applied to the discrete-time T-S fuzzy system for verifying the resulting chaos in the closed-loop system, showing that the generated chaos is in the sense of Li and Yorke.

5. Reference

[1] X. F. Wang, G. Chen, and X. Yu, "Anticontrol of chaos in continuous-time system via time-delay feedback", Chaos, Vol. 10, No. 4, pp. 771-779, 2000.

[2] G. Chen, "Chaos: control and anticontrol", IEEE Circuits Syst. Soc. Newslett. Vol. 9, pp. 1-5, 1998.

[3] Z. Li, W. Halang, G. Chen and L. F. Tian, "Chaotifying continuous-time Takagi-Sugeno fuzzy systems via time-delay feedback", Dynamics of Continuous, Discrete and Impulsive Systems, Vol. 10, pp. 813-832, 2003.

[4] G. Chen and D. Lai, "Making a dynamical system chaotic: feedback control of Lyapunov exponents for discrete-time dynamical systems", IEEE Trans. Circuits Syst.-I, Vol. 44, pp. 250-253, 1997.

[5] Z. Li, J. B. Park, Y. H. Joo, Y. H. Choi and G. Chen, "Anticontrol of choas for discrete TS fuzzy systems", IEEE Trans. Circuit and Systems.I, Vol. 49, pp.249-253, 2002.

[6] F. R. Marotto, "Snap-back repellers imply chaos in R^n", J. Math. Anal. Appl., Vol. 63, pp. 199-223, 1978.

[7] T. Y. Li and J. A. Yorke, "Period three implies chaos", Amer. Math. Monthly, Vol. 82, pp. 481-485, 1975.

[8] K. Tanaka and H. O. Wang, "Fuzzy control systems design and analysis: a linear matrix inequality approach", John Wiley & Sons, Inc., 2001.