불확실성이 포함된 비선형 시스템에 대한 전역적 접근의 지능형 디지털 재설계

Intelligent Digital Redesign of Uncertain Nonlinear Systems : Global approach

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Abstract - This paper presents intelligent digital redesign method of global approach for hybrid state space fuzzy-model-based controllers. For effectiveness and stabilization of continuous-time uncertain nonlinear systems under discrete-time controller, Takagi-Sugeno(TS) fuzzy model is used to represent the complex system. And global approach design problems viewed as a convex optimization problem that we minimize the error of the norm bounds between nonlinearly interpolated linear operators to be matched. Also by using the power series, we analyzed nonlinear system's uncertain parts more precisely. When a sampling period is sufficiently small, the conversion of a continuous-time structured uncertain nonlinear system to an equivalent discrete-time system have proper reason. Sufficiently conditions for the global state-matching of the digitally controlled system are formulated in terms of linear matrix inequalities (LMIs). Finally, we prove the effectiveness and stabilization of the proposed intelligent digital redesign method by applying the chaotic Lorentz system.

Keywords - uncertain nonlinear systems, intelligent digital redesign, T-S fuzzy model, power series, Linear matrix inequalities(LMIs), chaotic Lorentz system

1. Introduction

Control of the complex dynamical systems which has uncertainty term is too difficult. So we have many technical problems to stabilize these whole systems. Generally, the uncertainty of the plant roots from unmodelled dynamics, sensor noises, parameter variations, etc. And complex dynamic systems should be described by a continuous-time and/or discrete-time uncertain framework. As advanced digital implements, represented computer and microprocessor, many analog systems are converted to digital systems. Since digital device has more merit, such as better performance, more flexibility, and lower cost. For digital simulation, digital control and digital implementation of a continuous-time uncertain linear system, it is necessary to find an equivalent discrete-time uncertain model which matched analog control.

The efficient approach to design digital controller is digital redesign, which was first proposed by Kuo. And these methods are developed, now we apply digital redesign method to complex nonlinear system. But because of uncertain exponential parts, it is too difficult to apply intelligent digital redesign technique which includes uncertain parts.

In this brief, we use power series to develop a systematic method for the intelligent digital redesign. Power series help us to solve uncertain complex parts. More precisely, the conversion of the hybrid state space TS fuzzy-model-based controller for sampled-data control of continuous time complex dynamical systems are analyzed by Power series. Finally by applying to chaotic Lorenz system, show the efficiency of these method.

2. Preliminaries

Consider a class of continuous-time nonlinear systems, contained parametric uncertainties, are

$$x(t) = f(x(t)) + \Delta f(x(t)) + (g(x(t)) + \Delta g(x(t)))u(t) \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $f(x(t)) \in R^n$ and $g(x(t)) \in R^n$ are nonlinear vector functions, and $\Delta f(x(t)), \Delta g(x(t))$ are uncertain vector functions. The i th rule of T-S fuzzy system is that

IF-THENForm

R^i : *IF* $x_i(t)$ *is about* Γ_1^i *and...and* $x_n(t)$ *is about* Γ_n^i

THEN $\dot{x}_c(t) = (A_i + \Delta A_i)x_c(t) + (B_i + \Delta B_i)u_c(t), \qquad i = 1, 2, \ldots, q,$

Defuzzified Form :

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)),$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma^i_j(x_j(t)), \qquad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))}, \quad (2)$$

where $\Gamma_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in Γ_j^i. We use the following fuzzy-model-based controller structure[6],

$$\dot{x}_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))\{(A_i + \Delta A_i) + (B_i + \Delta B_i)K_{cj}\}x_c(t). \quad (6)$$

Next we discuss the discretization of the continuous-time T-S fuzzy system. Consider a class of T-S fuzzy system governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q} \mu_i(z(t))\{(A_i + \Delta A_i)x_d(t) + (B_i + \Delta B_i)u_d(t)\}. \quad (7)$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT + T)$, where $T > 0$ is a sampling period. For the digital control, the digital fuzzy-model-based controller is

R^i : IF $z_1(kT)$ is about Γ_1^i and ... and $z_n(kT)$ is about Γ_1^i

THEN $u_d(t) = K_d^i x_d(kT) \quad (8)$

for $t \in [kT, kT + T)$, where K_d^i is the digital control gain matrix to be redesigned for the i th rule. The overall control law is given by

$$u_d(t) = \sum_{i=1}^{q} \mu_i(z(kT))K_d^i x_d(kT) \quad (9)$$

for $t \in [kT, kT + T)$.

For state matching, we need appropriate assumption because of time-varing polytopic system and approximated discretization

Assumption 1[3]: Assume that the firing strength of the i th rule, $\mu_i(z(t))$ is approximated by its value at time kT, that is

$$\mu_i(z(t)) \approx \mu_i(z(kT))$$

for $t \in [kT, kT + T)$. If a sufficiently small sampling period T is chosen, Assumption 1 is reasonable. We efficiently derive the discretization of T-S fuzzy system(7),

$$x_d(kT + T) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(kT))\mu_j(z(kT))(\hat{G} + \hat{H}_i k_{dj})x_d(kT) \quad (10)$$

where

$$\hat{G} = \exp(A_i + \Delta A)T, \hat{H} = \int_0^T e^{(A+\Delta A)\tau}(B + \Delta B)d\tau = (\hat{G} - I_n)(A_0 + \Delta A)^{-1}(B + \Delta B)$$

The pointwise dynamical behavior of the continuous-time closed-loop T-S fuzzy system can also be approximately as followed

$$x_c(KT + T) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(z(t))\mu_j(z(t))\Phi_{ij}x_c(kT) \quad (11)$$

where $\Phi_{ij} = \exp\{((A_i + \Delta A) + (B_i + \Delta B)K_c^j)T\}$

And $\Delta A_i, \Delta B_i$ are the unknown and possibly time-varing matrices. To find these gain matrices, K_i, we should remove the uncertain matrices under some reasonable assumptions.

Assumption 2: The uncertainty matrices ΔA_i and ΔB_i are norm bounded and have the following structures:

$$[\Delta A_i \quad \Delta B_i] = D_i F_i(t)[E_{i1} \quad E_{i2}]$$

where D_i, E_{i1}, and E_{i2} are predetermined constant real matrices of appropriate dimensions, which represent the structures of the system uncertainties, and $F_i(t) \in R^{i \times j}$ is an unknown matrix function satisfies

$$F_i^T(t)F_i(t) \le I.$$

3. Main Result

In this paper, we take a intelligent digital redesign approach of global approach. That is, a digital redesign algorithm for the uncertain nonlinear system is proposed, which is used for the design of the local digital control gains of the digital fuzzy−model−based controller.

Problem (γ − Suboptimal Global Intelligent Digital Redesign Problem) [3] : Given a well−constructed gain matrices K_{ci} for the stabilizing analog fuzzy−model−based controller (13), find gain matrices K_{di} for the digital fuzzy−model−based control law (11) such that the following constraints are satisfied.

Minimize γ subject to

$\|\Phi_{ij} - G_i - H_i K_d^j\| < \gamma,\ i, j = 1, 2, \dots, q,$ in the sense of the induced 2−norm distance measure.

$$\begin{bmatrix} -\gamma Q & * \\ \Phi_{ij}Q - \hat{G}Q - \hat{H}U_j & -\gamma I \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} -Q + (q-1)O & * \\ \hat{G}_i Q + \hat{H}_i U_i & -Q \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} -Q - O & * \\ \dfrac{\hat{G}_i Q + \hat{H}_i U_j + \hat{G}_j Q + \hat{H}_j U_i}{2} & -Q \end{bmatrix} < 0 \tag{15}$$

To solve matching problem, we have following Theorem comprise power series.

Theorem 1 : The exponential uncertainty terms which included the equation (13) are solved by Power Series,

$$\hat{G}_i \approx I_n + (A_i + \Delta A_i)T \tag{16}$$

$$\hat{H}_i \approx (B_i + \Delta B_i)T \tag{17}$$

$$\Phi_{ij} \approx I_n + (A_i + \Delta A_i)T + (B_i + \Delta B_i)K_c^j T \tag{18}$$

Proof) The general power series are

$$\exp(A_i T) = I_n + A_i T + A_i^2 \frac{T^2}{2} + \cdots \tag{19}$$

Applying these theorem to above Problem,

$$\begin{aligned} \hat{H}_i &= (\hat{G}_i - I_n)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i) \\ &\approx (I_n + (A_i + \Delta A_i)T - I_n)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i) \\ &\approx (A_i + \Delta A_i)(A_i + \Delta A_i)^{-1}(B_i + \Delta B_i)T \\ &= (B_i + \Delta B_i)T \end{aligned}$$

The exponential terms are easily defined by power series, but the uncertainty term $\Delta A_i, \Delta B_i$ are still discussed. Following Lemma can help us to solve these difficulties.

Lemma 1 [2] : Given constant symmetric matrices $N, O,$ and L of appropriate dimensions, the following two inequalities are equivalent:

(a) $O > 0,\ \ N + L^T O L < 0,$

(b) $\begin{bmatrix} N & L^T \\ L & -O^{-1} \end{bmatrix} < 0$ or $\begin{bmatrix} -O^{-1} & L \\ L^T & O \end{bmatrix} < 0.$

Lemma 2 [2] : Given constant matrices D and E, and a symmetric constant matrix S of appropriate dimensions, the following inequality holds, $S + DFE + E^T F^T D^T < 0$, where F satisfies $F^T F \le I$, if and only if for some $\varepsilon > 0$,

$$S + [\varepsilon^{-1}E^T \ \ \varepsilon D]\begin{bmatrix} \varepsilon^{-1}E \\ \varepsilon D^T \end{bmatrix} < 0.$$

Applying above Lemma 1 and 2 to (13), we have another Theorem, which contain the LMI approach.

Theorem 2(Globally state matching): If there exist symmetric positive definite matrix Q, symmetric positive−semidefinite matrix O, constant matrices F_i and a small positive scalar such that the following generalized eigenvalue problem (GEVP) has solutions:

Minimize

Q, O, F_i γ subject to

$$\begin{bmatrix} -\gamma Q & * & * & * \\ B_i T(K_c^j - F_j) & -\gamma I & * & * \\ (E_{2i}T(K_c^j - F_j))^T & 0 & -\varepsilon_{ii} I & * \\ 0 & D_i^T & 0 & -\varepsilon_{ii} I \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix} -Q + (q-1)O & * & * & * \\ A_i TQ + A_i TF_i + B_i K_c^j TF_i & -Q & * & * \\ (E_{1i}TQ)^T + (E_{1i}TF)^T + (E_{1i}TF)^T & 0 & -\varepsilon_{ii} I & * \\ 0 & D^T & 0 & -\varepsilon_{ii} I \end{bmatrix} < 0 \tag{21}$$

Remained stability theorem is similar to above equation (21), the difference are added j terms which are connected $\hat{G}_j$, $\hat{H}_j$ and U_j.

4. Simulation of the chaotic Lorenz System

We discuss the exact TS fuzzy modeling of the chaotic Lorenz system and prove the effectiveness and stabilization of the proposed intelligent digital redesign method. The dynamics of the controlled chaotic Lorenz system is described by

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sigma x + \sigma y \\ \gamma x - y - xz \\ xy - bz \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (22)$$

where $\sigma, \gamma, b > 0$ are parameters (σ is the Prandtl number, γ is the Rayleigh number, and b is a scaling constant). We further assume that system parameters σ, γ and b have additive complex uncertainties, i.e., $\sigma = \sigma_0 + \Delta\sigma, \gamma = \gamma_0 + \Delta\gamma, and\ b = b_0 + \Delta b$ and the initial condition parameter is that $(\sigma_0, \gamma_0, b_0) = (10, 28, (8/3))$, where the uncertain parameters are all bounded within 30% of their nominal values.

The corresponding TS fuzzy model of the system is expressed as follows [ ]:

IF x *is* F_1^i,

THEN $\dot{x}_c(t) = (A_{0i} + \Delta A_i)\, x_c(t) + Bu_c(t), \quad i = 1, 2 \quad (23)$

where $x_c = [\, x\ y\ z\,]^T$

$$A_{01} = \begin{bmatrix} -\sigma_0 & \sigma_0 & 0 \\ \gamma_0 & -1 & -M_1 \\ 0 & M_1 & -b_0 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -\sigma_0 & \sigma_0 & 0 \\ \gamma_0 & -1 & -M_2 \\ 0 & M_2 & -b_0 \end{bmatrix}$$

and $\Delta A_i = D_i F_i E_{i1}$ is given by

$$D_1 = D_2 = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \quad E_{11} = E_{21} = \begin{bmatrix} \sigma_0 & -\sigma_0 & 0 \\ \gamma_0 & 0 & 0 \\ 0 & 0 & b_0 \end{bmatrix}$$

or

$$\Delta A_i \in \Delta A_i' = \begin{bmatrix} \pm 0.3 & \pm 0.3 & 0 \\ \pm 8.4 & 0 & 0 \\ 0 & 0 & \pm 0.8 \end{bmatrix}, \quad i = 1, 2.$$

The membership functions are

$$F_1^1 = \frac{-x + M_2}{M_2 - M_1}, \quad F_1^2 = \frac{x - M_1}{M_2 - M_1} \qquad (24)$$

and $(M_1, M_2) = (-20, 30)$.

The object of intelligent digital redesign are two; the one is global state matching and the other is stabilize the whole system. To achieve these objects, we first find digital gain by using LMI frame work. The simulation results are these Table 1.

F{1}	−54.3053	−21.5040	0.7227
F{2}	−54.5675	−21.4878	1.6663

Table 1. Digital Gain

5. Conclusion

In this paper, we represent intelligent digital redesign method of global approach for uncertain nonlinear which analyzed by hybrid state space fuzzy−model−based controllers. For effectiveness and stabilization of continuous−time uncertain nonlinear systems under discrete−time controller, we use TS fuzzy model. Also, by applying power series to whole system, the uncertainty terms are easily defined.

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