등각 사상법을 이용한 매입형 영구자석 전동기의 코깅토크 해석

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Conformal Mapping for Cogging Torque computation in IPM motor

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Abstract – This paper deals with magnetic field analysis and computation of cogging torque in IPM motor with an analytical method, which is based on the Conformal Mapping technique. The magnetic field is analyzed by solving space harmonic field analysis due to inserted PM magnetizing distribution. Conformal Mapping method is then used for considering the slot opening effect and rotor saliency effect on the air-gap field magnetic distribution. Then, by integrating the field over the stator surface, cogging torque is calculated. The validity of the proposed analytical method is confirmed by comparing the results with 2-D FEA results.

1. INTRODUCTION

Interior Permanent Magnet(IPM) motor is appearing in a variety of important applications for its superior advantages, such as high torque and power density, and reduce PM requirements. The IPM motor generates the reluctance torque due to saliency of the rotor, on the other hand, IPM motor has significant cogging torque due to the unique IPM structure [1].

The magnetic field and cogging torque characteristics are quite sensitive to the geometry of rotor and the stator due to the small air-gap [1]. Therefore, it is necessary to have an analytical method which can precisely consider the details of motor geometry for computation. To improve the accuracy in the direct calculation, conformal mapping method is used in this analytical method. Using this method, the initial motor geometry with complicated boundaries is transformed into another equivalent system with simpler boundary shape, where the magnetic field can be solved analytically [2]. The field characteristics of the two systems are identical because of the conformal property of the transformation.

In this paper, flux density distribution produced by the PMs is calculated by space harmonic analysis method. In air gap field analysis, stator slot opening and rotor interior saliency effect on the flux density distribution are considered by using conformal mapping method in term of relative permeance. Then, cogging torque of the IPM motor can be precisely computed.

2. MAGNETIC FIELD ANALYSIS

An IPM motor model is shown in Fig.1. for analysis in this paper. It has 4 poles and 16 slots, with inserted PMs and buried flux barrier regions in the rotor. In the IPM motor, flux dispersion is developing in air-gap because the magnetic flux passes the iron core and leakage flux in the buried flux barrier areas. In the presented analytical method, fringing and leakage effect on flux distribution is involved in PM equivalent magnetization.
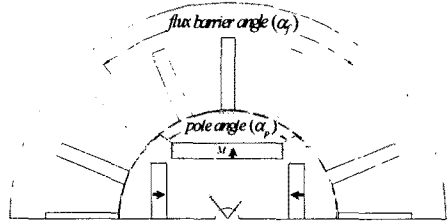

Fig.1. IPM motor model configuration

The IPM magnetization $\vec{M}$ represented by a Fourier series expansion is given as following[1]:

$$\vec{M} = M_r \vec{r} = \sum_{n=1,3,5...}^{\infty} M_n \sin(np\theta) \cdot \vec{r} \tag{1}$$

$$M_n = \frac{-2M_e}{bn^2\pi}[\sin an\{1-(-1)^n\} + \sin(n[a+b])\{-1+(-1)^n\}] \tag{2}$$

$$\vec{M}(\theta) = \sum_{n=1,3,5...}^{\infty} \frac{-4M_e}{bn^2\pi}[\sin(an) - \sin(n[a+b])] \cdot \sin(np\theta) \tag{3}$$
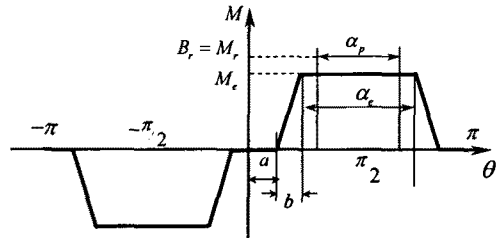

Fig.2. Magnetizing distribution of IPM

where $\alpha_e = (\alpha_f / \alpha_p) \cdot \alpha_p$ and $b = \alpha_p \cdot (\psi_l / \psi_r)$, $a = \frac{\pi - \alpha_e}{2} - b$,

B_r is the residual flux density and P is the pole pair, ψ_r is magnet flux, and ψ_l is the leakage flux in the flux barrier areas, at which the fields are assumed to be full saturation.

The equivalent magnetization of PM as Fig.2. shows, indicates that the effect of flux dispersion and leakage are considered to increase pole angle and to decrease residual flux density of PM [1].

The magnetic field is obtained from equivalent magnetization, and radial component of flux density in air-gap is obtained from Poisson equation [1],[3]. The air gap flux distribution are given by:

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0 \quad \text{(in the air-gap region)} \tag{4}$$

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = \frac{1}{r}\frac{M_r}{\mu_r} \quad \text{(in the magnet region)} \tag{5}$$

$$B_{gr}(r,\theta) = \sum_{n=1,3,5...}^{\infty} \frac{\mu_0 M_n}{\mu_r} \frac{np}{(np)^2 - 1} R_r^{-(np-1)} \cdot A$$
$$\cdot [r^{(np-1)} + R_s^{2np} r^{-(np+1)}]\sin(np\theta) \qquad (6)$$

$$B_{g\theta}(r,\theta) = \sum_{n=1,3,5...}^{\infty} (-1) \cdot \frac{\mu_0 M_n}{\mu_r} \frac{np}{(np)^2 - 1} R_r^{-(np-1)} \cdot A$$
$$\cdot [r^{(np-1)} - R_s^{2np} r^{-(np-1)}]\cos(np\theta) \qquad (7)$$

$$R_m = R_r - h_m$$

$$A = \frac{\left[(np-1)R_r^{2np} + 2R_m^{np+1}R_r^{np-1} - (np+1)R_m^{2np}\right]}{\left[\frac{\mu_r+1}{\mu_r}[R_s^{2np} - R_m^{2np}] - \frac{\mu_r-1}{\mu_r}[R_r^{2np} - R_s^{2np}(R_m/R_r)^{2np}]\right]} \qquad (8)$$

where, B_{gr} and $B_{g\theta}$ are flux density normal and tangential component, μ_r is the relative permeability of PM, R_r is the radius of rotor, h_m is magnetizing length of inserted PM, and R_s is the stator inner radius.

3. CONFORMAL MAPPING METHOD

The conformal mapping method is a powerful analytical tool for the determination of 2-D (two-dimensional) magnetic field which is based on the complex analysis theory [2]. The general idea of conformal mapping is that by choosing an appropriate transformation function $z = f(w)$, the given configuration placed in original complex plane Z is transformed to another complex plane W, where the field can be solved analytically [2]. In this paper, flux density distribution is analytically computed according to the product of magnet flux density distribution and corresponding relative permeance. Therefore, in the following parts, conformal mapping is used for calculating relative permeance due to rotor saliency effect and slot opening effect. The total relative permeance expressed as: $\lambda_{total} = \lambda_{rotor-saliency} \times \lambda_{slot-opening}$.

3.1 Rotor Saliency Effect

For rotor saliency effect consideration, a conformal mapping is used here, by which the cylindrical rotor can be mapped to a square so that the accurate air gap length can be determined.

The conformal mapping function $z(u,v) = f[t(x,y)]$ satisfies the Cauchy-Riemann equation [5], giving as :
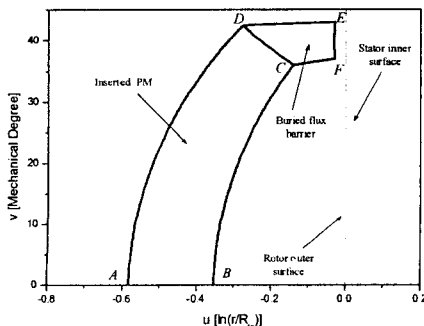$$\begin{cases} u = \frac{1}{2}\ln(x^2 + y^2) \\ v = \arctan(y/x) \end{cases} \qquad (9)$$

Fig.3. Mapping of the cross section of the IPM rotor

Here, 1/8 part of the rotor is transformed into a square.

According to this conformal mapping, the boundaries of inserted PM and buried air region are represented by the curves are shown in Fig.3. By using these boundary curves equations, the effective air-gap length can be calculated as the following function[4]:

$$G(\theta) = g_0 + \frac{\{\exp[G_{out}(v)] - \exp[G_{in}(v)]\}}{\mu_r} \qquad (10)$$

where, $G(\theta)$ is the effective air gap length, $G_{out}(v)$ responds with inserted parts outer sider boundary (BC, CF, EF) curves equations, $G_{in}(v)$ responds with inner side boundary (AD, DE, CD) curves equations in the mapping region. So, according to the obtained effective air-gap length, a new relative permeance function that includes the effect of rotor saliency approximately is expressed as:

$$\lambda_{rotor-saliency}(\theta) = \frac{G_0}{G(\theta)} = \frac{g_0 + h_m/\mu_r}{G(\theta)} \qquad (11)$$

3.2 Slot Opening Effect

In a simplified calculation, by assuming that the fluxes pass on each side of tooth circularly, the relative permeance is computed depending on the effective flux path length from the rotor to the stator [1]. The more refined relative permeance is derived from conformal mapping by calculating the waveform directly from the transformed field distribution which described by the transformation equation.

(a). Initial slot opening configuration in Z plane

(b). Simplified slot opening configuration in Z plane

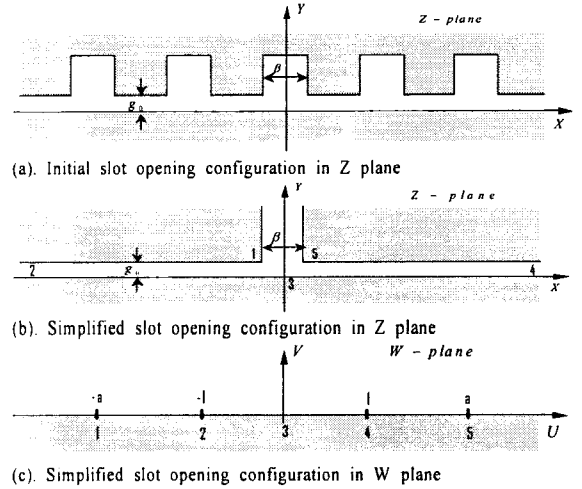(c). Simplified slot opening configuration in W plane

Fig.4. Mapping of IPM motor slot opening configuration

This conformal mapping will "open" the slot area. By this conformal mapping, the complex upper half-plane Fig.4 (c) can be transformed to an interior of polygon area (as be shown in Fig.4 (b), created by the path 1-2-3-4-5). A simplified infinite permeability, infinitely deep rectilinear single slot area is mapped from Z plane to the W plane, where a simple configuration that consists of only one ideal iron half-plane, as be shown in Fig.4 (c). This conformal transformation equation $z = f(w)$ is obtained from Schwarz-Christoffel transform theory [2], giving as:

$$z = f(w) = \frac{2g_0}{\pi}\left[\frac{\beta}{2g_0}\arcsin\frac{w}{a} + \frac{1}{2}\log\frac{\sqrt{a^2 - w^2} + \frac{2g_0}{\beta}w}{\sqrt{a^2 - w^2} - \frac{2g_0}{\beta}w}\right] \qquad (12)$$

where, $a = \sqrt{1 + \left(\dfrac{2g_0}{\beta}\right)^2}$ (13)

Now, the final configuration in the W plane can be determined. Then a unit potential is assumed between the rotor and stator surfaces in the W plane. Therefore, the magnetic field distribution in the W plane can be easily solved. So, the slot opening effect relative permeance can be calculated according to the transformation equation and flux density distribution on slot surface. The slot opening effect relative permeance can be computed as following:

$$\lambda_{slot-opening} = \frac{1}{\beta \sqrt{a^2 - w^2}} \quad (14)$$

4. COGGING TORQUE CALCULATION

Cogging torque is the motor parasitic torque component created by the PM magnetic field, being a consequence of the stator slotting [2]. It is a circumferential component of attractive force that attempt to maintain the alignment between the stator teeth and the rotor with IPM [6].

First, according to the electromagnetic theory (Maxwell's tensor) [2], the reluctance pressure on the boundary between stator surface and air gap magnetic field can be calculated as:

$$p_R = \frac{B_n^2}{2\mu_0} \quad (15)$$

the reluctant pressure acts perpendicularly on the boundary air-iron. Then, the cogging force $\overrightarrow{F_R}$ and the cogging torque $\overrightarrow{T_R}$, acting on the stator surface S, are determined by the integration of the pressure p_R over the surface S as:

$$\overrightarrow{F_R} = \int_S p_R d\overrightarrow{S} \quad (16)$$

and, $\overrightarrow{T_R} = \int_S p_R \overrightarrow{r} \times d\overrightarrow{S} = \frac{1}{2\mu_0} \int_S B_n^2 \overrightarrow{r} \times d\overrightarrow{S}$ (17)

where, $\overrightarrow{r}$ is the radius-vector, and $d\overrightarrow{S} = dS\overrightarrow{n}$ (where $\overrightarrow{n}$ is the unit normal vector).

5. ANALYSIS RESULTS

The appropriateness of the presented analytical method in IPM motor is verified through comparison with 2-D FEA (finite element analysis) results. Fig.5, compares The flux density in the air-gap produced by the parallel magnetized PM. Both of proposed analytical method and FEA results.
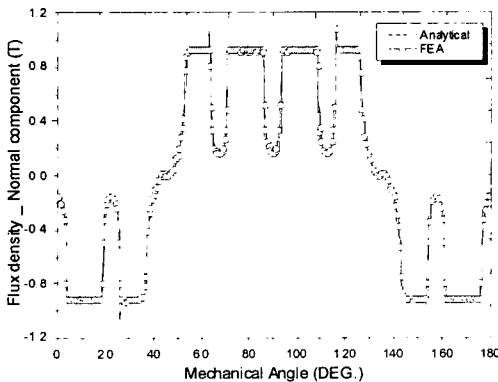

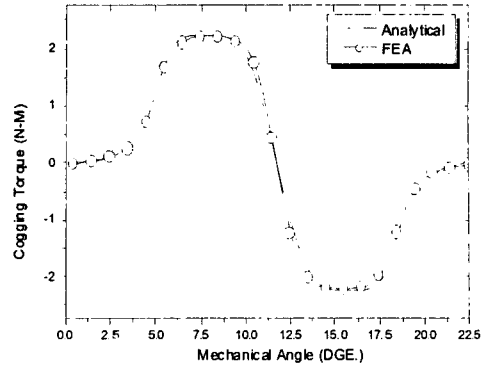Fig.5. Flux density distribution comparison

Fig.6. Cogging Torque comparison

As Fig.6, shows the comparison of cogging torque in the IPM motor. By fully considering slot opening effect and rotor saliency effect in the proposed analytical method, both of the comparisons show good agreement. The proposed analytical method is well verified. Due to saturation, local differences are apparent as the flux density distribution on the rotor surface where responding with flux barrier areas.

6. CONCLUSION

The magnetic field distribution and cogging torque in IPM motor are computed by the presented analytical method which is based on Space harmonic analysis. The effect of stator slot-opening and rotor interior saliency are well considered by the conformal mapping technique. Then, the results computed by the analytical method have been shown to be in good agreement with 2-D FEA. According to the verified analytical method, magnetic field can be easily analyzed, furthermore it can give helps in motor optimal design.

ACKNOWLEDGEMENT

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