An Adaptive Multi-Echelon Inventory Control Model for Nonstationary Demand Process

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Abstract

In this paper, we deal with an inventory model of a multi-stage, serial supply chain system where a single product type and nonstationary customer demand pattern are considered. The retailer and suppliers place their orders according to an echelon-stock based replenishment control policy. We assume that the suppliers can access online information on the demand history and use this information when making their replenishment decisions. Using a reinforcement learning technique, the inventory control parameters are designed to adaptively change as the customer demand pattern is altered, in order to maintain a given target service level. Through a simulation based experiment, we verified that our approach is good for maintaining the target service level.

1. Introduction

Unpredictable customer demand pattern usually obstructs the entire supply chain efficiency, resulting in loosing selling chance or possessing excessive chain wide inventories. This phenomenon of whimsical amplification is known as the 'bullwhip effect' in supply chain management (Lee *et al.*, 1997). Each participant in supply chain will attempt to optimize its own preference, without the consideration of the interests of the other participants. These competitive behaviors do not lead entire supply chain optimization.

In this paper, we deal with an inventory model of a multi-stage, serial supply chain system as shown in figure 1. Customer demand pattern at the retailer is nonstationary with time. It is assumed that the suppliers can access online information on the demand history when making their replenishment decisions. The order quantity is assumed fixed size of Q. Our objective is to satisfy customer demands with maintaining target service level.

In order to maintain a given target service level, we control safety lead time as a decision variable. If the actual service level collected during operation is lower than the target service level, long safety lead time is set. In the contrary situation, short safety lead time is applied.

We first propose echelon-stock based inventory control model, and then we suggest self-control model in which every echelon controls decision variable shown in figure 1.

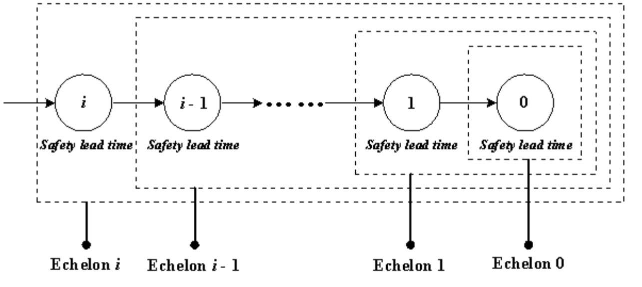


Figure 1. A multi-stage, serial supply chain system

Each echelon updates its performance measure (actual service level) based on which it selects next action (safety lead time level). In more detail, in nonstationary demand environment, it makes sense to weight recent rewards (observed service levels) more heavily than long past ones. Thus, our approach in updating actual service level uses a weighted time-average method. Then action-reward based reinforcement-learning technique, a kind of intelligent machine learning techniques (Sutton and Barto, 1998), is applied to select the next action weighted time-average method.

The rest of this paper is organized as follow. In the next section, we present a review of the related research works. In section 3, we formally propose adaptive control methodology. In section 4, we present the results of a simulation based experiment. Finally, in section 5, we summarized the major findings and contributions, and indicate some directions for future research areas.

2. Literature Review

The first approach to multi-echelon inventory theory was begun by Clark and Scarf (1960). Their theory is now with voluminous by demand characteristics, deterministic and stochastic, and various network structures such as series, assembly, and distribution.

The serial model for deterministic demands was studied by Roundy (1985, 86), Maxwell and Muckstadt (1985), and Atkins and Sun (1995). They showed that the so-called power-of-two policies were close to optimal solution in the deterministic environment.

There is a large body of research on (*R*, *Q*) policies in multi-echelon systems. De Bodt and Graves (1985) studied echelon-stock (*R*, *Q*) policies in serial system. Their assumption was that a batch size *Q* must be immediately sent to its downstream stage whenever the stage receives a shipment. Chen (1999) studied serial model with random demand and setup costs. He suggested echelon-stock (*R*, *nQ*) policies, cost-effective heuristic policies. Whenever the inventory position at a stage is at or below *R*, order *nQ* units where *n* is minimum integer required to increase the inventory position to above *R*. On the other hand, Badinelli (1992) considered installation-stock (*R*, *Q*) policies in serial system. Installation stock is local inventory information, whereas echelon stock requires centralized information. However, this advantage is disappearing as recent company industries are equipped with advanced information technologies.

Song and Zipkin (1993) considered nonstationary demand with single-item model. They assumed that the demand process was Poisson, but where a Markov process governed the rate. They formulated a dynamic program to characterize the optimal policy. Also, Chen and Song (2001) and Abhyankar and Graves (2001) considered nonstationary demand processes modeled as a Markov-modulated Poisson demand process where the demand process was governed by a discrete time Markov chain.

Finally, Pontrandolfo et al. (2002) applied a Markov decision model based reinforcement-learning technique to a coordination and integration problem of multinational corporations with emphasis on logistics and the production management.

3. The Echelon Inventory Control Model

3.1 Mathematical formulation

We consider a serial inventory system with *N* stages, that is, stage 0 is the retailer, and from stage 1 to stage *N* are suppliers. Stage 0, the retailer, places orders to stage 1, which places orders to stage 2 and so on. Finally, stage *N* issues orders to an outside supplier with unlimited capacity and no lead time. For convenience, the outside supplier is also referred to as stage *N* + 1. The customer demand process at the retailer is not assumed to know in advance, even their distribution forms. Besides, the backorders from the retailer are not allowed. Instead, if customer demands are not satisfied at sales points of time, the demands are treated as lost sales.

We assume that the delivery time from one stage to the next is constant and multiple batch sizes are not allowed in any stage. Also, we assume that the initial on-hand inventory at stage *i* is a batch size of *Q*.

The system has the following parameters. Let

i = index of stage (i = 0 for retailer, and i = 1, 2,…, N for suppliers),
t = index of time period,
$R_i(t)$ = echelon reorder point of stage i at time period t,
L_i = lead time of stage i,
$IL_i(t)$ = echelon inventory level of stage i at time period t
= on-hand inventory at stage i plus downstream inventories on-hand at, and in transit to, stage 0,…, i –1,
$P_i(t)$ = pipeline inventory of stage i at time period t,
$D(t)$ = customer demand at retailer during time period t.

3.1.1. Forecasting model

The linear time series model applied in this study is defined as $\hat{D}(t) = a_0 + a_1 t$ (Brown, 1962). This model continuously updates coefficients a_0 and a_1 as

$$a_0^{new} = \alpha IL_i(t) + (1-\alpha)(a_0^{old} + a_1^{old}) \quad \text{and}$$
$$a_1^{new} = \beta(a_0^{new} + a_0^{old}) + (1-\beta)a_1^{old}.$$

However, if customer demands show high nonstationary trends, this cannot be reflected to the time series model at the right time because the exponential smoothing method gives constant weight α and β to update coefficient a_0 and a_1. Therefore, the model tends to make a wrong estimation the true demand trend. In order to solve this difficulty, we apply tracking signal, which is defined as the mean deviation over mean absolute deviation of the forecast errors (Trigg and Leach, 1967). That is, tracking signal is defined as $TS = MD/MAD$. Also, the abbreviated terms, MD and MAD, are defined as

$$MD(t) = (1-\gamma)\cdot MD(t-1) + \gamma\{D(t) - \hat{D}(t)\} \quad \text{and}$$
$$MAD(t) = (1-\gamma)\cdot MAD(t-1) + \gamma\,|\,D(t) - \hat{D}(t)\,|.$$

γ is normally set to 0.1 (Trigg and Leach, 1967). Then, α and β are updated as

$$\alpha = |TS| \text{ and } \beta = 1 - \sqrt{1-\alpha}.$$

If forecast errors significantly occur, the tracking signal is also altered accordingly. As the result, weights α and β are updated automatically.

Brown (1962) and Trigg and Leach (1967) experimentally showed that the forecasting accuracy in the nonstationary demand patterns was effectively improved by the tracking signal.

3.1.2. Adaptive parameter control: safety lead time

For echelon i, let $S_i = \{s_{i1}, s_{i2} ..., s_{ik}\}$ is the set of candidate safety factors, in which some s_{ij}, $j = 1,\ldots, k$ may have negative values. For example, S_i may be {-3, -2, -1, 0, 1, 2, 3}.

Actual service level is a function of safety factor. For each safety factor s_{ij}, update the actual service level as

$$\overline{SL}_{new}(s_{ij}) = \overline{SL}_{old}(s_{ij}) + StepSize(t)[SL(lead\ time) - \overline{SL}_{old}(s_{ij})]$$

where *lead time* is the total lead time from stage i to the final stage, and *SL*(*lead time*) is the service level during the total lead time. *StepSize*(t) is expressed as follows.

$$StepSize(t) = (1-\eta)\cdot StepSize(t-1) + \eta \cdot MD(t)/MAD(t)$$

where *MD*(t) and *MAD*(t) are the *Mean Deviation* and *Mean Absolute Deviation* of the service level errors at period t, respectively, and they are defined as

$$MD(t) = (1-\gamma)\cdot MD(t-1) + \gamma\{SL(lead\ time) - \overline{SL}_{new}(s_{ij})\},$$

$$MAD(t) = (1-\gamma)\cdot MAD(t-1) + \gamma\,|\,SL(lead\ time) - \overline{SL}_{new}(s_{ij})\,|.$$

η and γ are normally set to 0.1.

If $\overline{SL}_{new}(s_{ij})$ is close to the target service level, then, s_{ij} can be regarded as an appropriate safety factor for the current demand pattern. Thus, the selection chance of s_{ij} at the next time should be increased. In this manner, the next safety factor is determined with the following softmax rule.

$$\Pr\{next\ safety\ factor = s_{ij}\} = \frac{e^{-1\cdot(SL(s_{ij})-TSL)^2}}{\sum_{j=1}^{k} e^{-1\cdot(SL(s_{ij})-TSL)^2}}$$

In order to maintain target service level, each stage controls its reorder point by adding safety lead time. According to the same probabilistic rule stated by the softmax rule, safety lead time st_i of the stage i is determined with the safety factor s_{ij}, estimated standard deviation of forecast errors $\hat{\sigma}_{\varepsilon}(t_c)$, and estimated mean of demand $\hat{D}(t_c)$ at the current time t_c as follows.

$$st_i = \frac{s_{ij}\hat{\sigma}_{\varepsilon}(t_c)}{\hat{D}(t_c)} \qquad \text{for } i = 0,\ldots, N$$

Hence, the lead time L_i for stage i is updated as

$$L_i^{new} = L_i + st_i.$$

Suppose that the current safety factor for stage i is s_{ij} and t_c is the current time. Then the stage i places its order to the upstream stage according to the following rule.

$$If \quad IL_i(t_c) - \sum_{j=0}^{i} \sum_{t=t_c}^{t_c+L_j+st_j} \hat{D}(t) + P_i(t_c) \le R_i,$$

then place an order of size Q to stage i + 1 at time t_c.

The service levels of unselected safety factors are also evaluated after customer demand data are collected. This is called retrospective evaluation.

4. Simulation Based Experiments

4.1 Experimental design

The run length of the simulation is 3,000 days and we exclude the results collected during the initial 100 days in order to minimize the effect of the transient behavior of the simulation. In our proposed model the set of safety factors for each echelon is set as

S_i = {-3, -2, -1, 0, 1, 2, 3} for i = 0, 1, 2, 3.

Target service level is set to 95%. We consider three and four stages supply chain model and set two factors for this experiment: customer demand pattern and lead time.

Nonstationary demand process is considered by changes of its mean and slope. The mean of normal distribution is designed to alter at every random interval T in accordance with the rule of $mean_j$ = $mean_{j-1}$ + *slope*. In this rule, *slope* and T are randomly devised by uniform distributions U(-*sm*, *sm*) and U(*tu*/2, *tu*) respectively. The nonstationarity of the demand process is represented by *sm* and *tu*. In this simulation, we set the two parameters as:

Low Mean Variance (LMV) : *sm* = 1.0 and *tu* = 20
Medium Mean Variance (MMV): *sm* = 2.0 and *tu* = 15
High Mean Variance (HMV) : *sm* = 4.0 and *tu* = 8

Also, three types of coefficient of variation (CV) are also taken into account in the performance evaluation, where the CV is defined as standard deviation separated by the mean. Since the mean of demand is varied with time, performance evaluation with a immovable CV implies the change of standard deviation of demand. We set two types of coefficient of variance: CV = 0.1, CV = 0.2, and CV = 0.3.

For lead time of stage i, two types of lead time for three and four stages are set as following days respectively:

Type Ⅰ : $[L_0, L_1, L_2] = [1, 2, 3]$ and $[L_0, L_1, L_2, L_3] = [1, 2, 3, 4]$

Type Ⅱ : $[L_0, L_1, L_2] = [2, 3, 4]$ and $[L_0, L_1, L_2, L_3] = [2, 3, 4, 5]$

4.2 Results and analysis

Table 1-(a) and (b) show the results for the case of 3-stage and 4-stage supply chain controlled by our proposed methodology.

We can firstly observe that actual service levels are higher than the target service level when lead time is type Ⅰ. The reason is that the value of safety lead time has a sensitive effect on order placement time in the relatively short lead time.

Table 1. Average service level

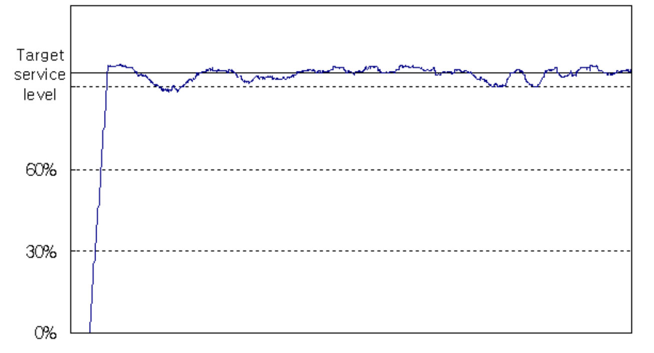
	Type Ⅰ (lead time)		
	LMV	MMV	HMV
CV = 0.1	97.319875	96.656652	95.706491
CV = 0.2	97.374512	96.689731	95.776816
CV = 0.3	97.282574	96.667876	95.691951
	Type Ⅱ (lead time)		
	LMV	MMV	HMV
CV = 0.1	94.326516	92.745149	90.825460
CV = 0.2	94.287592	92.682617	90.842906
CV = 0.3	94.054190	92.776245	90.795766

(a) 3 stages supply chain system

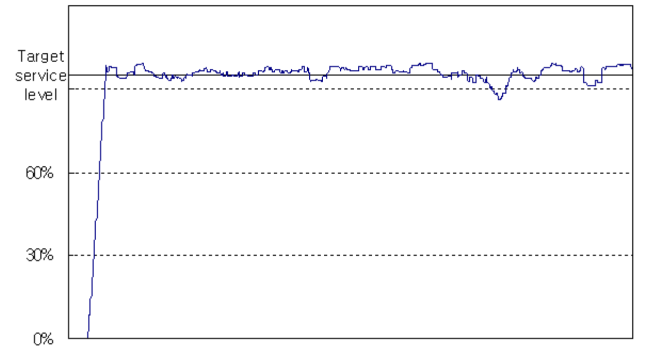
	Type Ⅰ (lead time)		
	LMV	MMV	HMV
CV = 0.1	97.283517	96.653778	95.808266
CV = 0.2	97.250578	96.682328	95.673849
CV = 0.3	97.260747	96.535048	95.668115
	Type Ⅱ (lead time)		
	LMV	MMV	HMV
CV = 0.1	94.102551	92.718796	90.716883
CV = 0.2	94.110715	92.528160	90.750061
CV = 0.3	93.888291	92.380234	90.560482

(b) 4 stages supply chain system

Secondly, we observe that the actual service levels in most cases scarcely any change though the lead time grows longer, the coefficient of variance increases in size, and the number of stages lengthens. These results show that our proposed methodology is robust control against the severe environment of the supply chain system. This can be also explained by the plotted data in a figure 2 that are the service levels of the retailer stage collected at inventory replenishment time. Figure 2-(a) represents a sample path of 4-stage supply chain having low mean variance (LMV) and CV = 0.1. In figure 2-(b), we set CV = 0.2. Both cases have almost same actual service level shown in table 1. We can observe that there are no remarkable differences of the fluctuation of the sample paths though they have different coefficients of variance.



(a) CV = 0.1



(b) CV = 0.2

Figure 2. Sample paths of the actual service level of the 4-stage supply chain system having low mean variance (LMV)

5. Conclusions

In this paper, we deal with an inventory model of a multi-stage, serial supply chain system where a single product type is considered. Due to unpredicted customer needs, their demands fluctuate with time, showing nonstationary patterns. To handle, with this situation, we propose an adaptive, intelligent echelon-stock based inventory control models having dynamic safety lead time at every stage of the supply chain. Applying a reinforcement-learning technique, the control parameters are designed to adaptively change as customer demands alter. An experiment based on simulation was performed to examine the performance of the echelon-stock based inventory control model.

There are number of important future extension to this work. One includes the analysis to the case when the order quantity is a multiple integer. In such case, our policy considers order costs as well as shortage and inventory holding costs. Also, we need to develop a measure for explanations between the service level and the total costs.

Another interesting extension is the study of an echelon-stock based inventory control system consisting of much more complicated systems such as assembly system and series system with multiple retailers having different nonstationary demands.

References

Abhyankar, H. S., S. C. Graves (2001), "Creating Inventory Hedge for Markov-Modulated Poisson Demand: An Application and a Model." *Manufacturing & Service Operations Management*, **3**, 306-320.

Atkins, D. and D. Sun (1995), "98%-Effective Lot-Sizing for Series Inventory System with Backlogging." *Operations Research*, **43**, 335-345.

Badinelli, R. (1992), "A Model for Continuous-Review Pull Policies in Serial Inventory Systems." *Operations Research*, **40**, 142-156.

Brown, R. G. (1962), *Smoothing, Forecasting, and Prediction of Discrete Time Series*. Englewood Cliffs, NJ: Prentice Hall.

Chen, F. (1999), "On (R, nQ) Policies in Serial Inventory Systems." *International Series in Operations Research and Management Science*, **17**, 71-110.

Chen, F., and J.-S. Song (2001), "Optimal Policies for Multiechelon Inventory Problems with Markov-Modulated Demand." *Operations Research*, **49**, 226-234.

Clark, A. and H. Scarf (1960), "Optimal Policies for a Multi-Echelon Inventory Problem." *Management Science*, **6**, 475-490.

De Botd, M. and S. Graves (1985), "Continuous review policies for a multi-echelon inventory problem with stochastic demand." *Management Science*, **31**, 1286-1295.

Lee, H. L., P. Padmanabhan and S. Whang (1997), "Information Distortion in a Supply Chain: the Bullwhip Effect." *Management Science*, **43**, 546-558.

Muckstadt, J. and R. Roundy (1993), "Analysis of Multi-Stage Production Systems." in *Handbook in Operations Research and Management Science, Vol. 4, Logistics of Production and Inventory*, Ed. By S. Graves, A. Rinnooy Kan and P. Zipkin, North Holland.

Pontrandolfo, P., Gosavi, A., Okogbaa, O. G., and Das, T. K. (2002), "Global Supply Chain Management: A Reinforcement Learning Approach." *International Journal of Production Research*, **40**, 1299-1317.

Roundy, R. O. (1985), "98% Effective Integer-Ratio Lot-Sizing for One-Warehouse Multi-Retailer Systems." *Management Science*, **31**, 1416-1430.

Roundy, R. O. (1986), "98% Effective Lot-Sizing Rule for a Multi-Product, Multi-Stage Production Inventory System." *Mathematics of Operations Research*, **11**, 699-727.

Song, J. and P. Zipkin (1993), "Inventory Control in a Fluctuating Demand Environment." *Operations Research*, **41**, 351-370.

Sutton, R. S. and Barto (1998), A. G., *Reinforcement Learning*. MIT Press.

Trigg, D. W. and Leach, A. G. (1967), "Exponential Smoothing with an Adaptive Response Rate." *Operational Research Quarterly*, **18**, 53-59.