# A Study on Multi-Axial Fatigue Model Based on Structural Stress

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## 1. Introduction

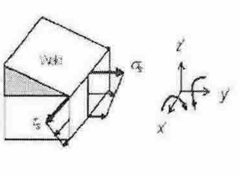
In nuclear components, cyclic loadings that cause complex states of stress are common. Through a reference review, four sources of the multi-axial fatigue data were collected from LBF [1], University of Illinois [2], EPRI [3], and TWI [4]. All these tests were conducted using tube to flange specimens with a circumferential fillet welds. The loading conditions were mostly bending/ torsion combinations, except that TWI used tension/ torsion combinations. None of fatigue correlation parameters have been demonstrated to be satisfactory in correlating the multi-axial fatigue data outside of their own. In this paper, we proposed the characterizing multi-axial fatigue behavior in terms of the structural stress methods by using some of the well-known multi-axial fatigue data available in the references.

## 2. Structural Stress Based Effective Stresses - In-Phase Loading

Once the structural stresses according to Fig. 1 are calculated for single loading modes (either bending or torsion) or in-phase combined loading, that generate either $\sigma_s$ or $\tau_s$, or both, an effective stress measure for $\sigma_s$ or $\tau_s$, or both can be in the form of von Mises, Tresca, or maximum principal stress definitions, as:

$$\Delta\sigma_e = \sqrt{(\Delta\sigma_s)^2 + 3(\Delta\tau_s)^2} \quad \text{(von Mises)} \quad (1)$$

$$\Delta\sigma_e = \sqrt{(\Delta\sigma_s)^2 + 4(\Delta\tau_s)^2} \quad \text{(Tresca)} \quad (2)$$




| | | SCF membrane | SCF bending | SCF total |
|---|---|---|---|---|
| UIUC stress relieved | Bending | 0.795 | 0.623 | 1.418 |
| | Torsion | 0.856 | 0.251 | 1.107 |
| Sonsino stress relieved | Bending | 0.887 | 0.747 | 1.634 |
| | Torsion | 0.91 | 0.197 | 1.107 |
| EPRI As-welded | Bending | 0.946 | 0.551 | 1.497 |
| | Torsion | 0.954 | 0.129 | 1.083 |
| TWI As-welded | Torsion | 0.887 | 0.249 | 1.136 |

(a) Multi-axial structural stress state (b) Structural stress based SCFs

Figure 1. Structural stress state considering both nomal and in-plane shear with respect a crack plane at weld toe and summary of SS-based SCFs calculated for the four sources of test.

$$\Delta\sigma_e = \dagger\left(\Delta\sigma_s + \sqrt{(\Delta\sigma_s)^2 + 4(\Delta\tau_s)^2}\right) \quad \text{(Max. Principle)} \quad (3)$$

Obviously, for a given pair of $\Delta\sigma_s$ and $\Delta\tau_s$, the difference between the von Mises and Tresca based effective stress definitions is small. Once the structural stress based SCFs are calculated (tabulated in Fig. 1), the effectiveness of three sffective stress definitions above can be tested in terms of their abilities to consolidate the data.

## 3. Equivalent Cycles for Out-of-Phase Loading

A simple treatment of out-of-phase loading is considered here. Miner's rule is assumed to be valid for out-of-phase cyclic loading by treating the bending/ tension and torsion as two separate damage events. Once an effective stress definition from Eqs. (1)-(3) is chosen, the Miner's rule damage summation can be expressed as:

$$\frac{n_{\sigma_e}}{N_{\sigma_e}} + \frac{n_{\tau_e}}{N_{\tau_e}} = 1 \quad (4)$$

In Eq. (4), $n_{\sigma_e}$ and $n_{\tau_e}$ represent the actual number of cycles under bending (or tension) and torsion and $N_{\sigma_e}$ represent the cycles to failure for a given effective stress range as described by Eqs. (1)-(3), as shown in Fig. 2. To compare the out-of-phase S-N data with the in-phase data, an equivalent cycle to failure can be derived from Eq. (4) For instance, after choosing the effective stress range due to bending ( $\Delta\sigma_e$ ) under out-of-phase combined bending and torsion, the equivalent cycles can be expressed as:

$$N_{eq} = n_{\sigma_e} + n_{\tau_e}\left(\frac{\Delta\tau_e}{\Delta\sigma_e}\right)^{-1/h} \quad (5)$$

In the above, the same slope of (-1/h) was derived based on Fig. 2 for respective effective stress measures to be used and $\Delta\tau_e$ signifies the effective stress calculated due to torsion only by choosing one of the three effective definitions from Eqs. (1)-(3).

Figure 2. Comparison of three structural stress based effective stress range definitions-S-N EPRI and LBF under pure bending, pure torsion and in-phase combined bending and torsion.

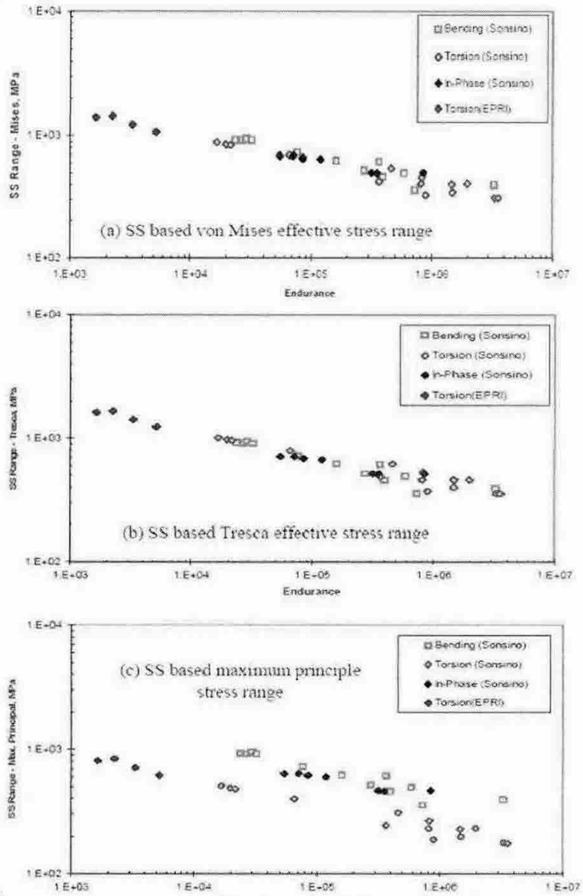



Figure 3. Comparison of multi-axial fatigue data from EPRI, LBF, UIUC, and TWI using various effective stress definitions.

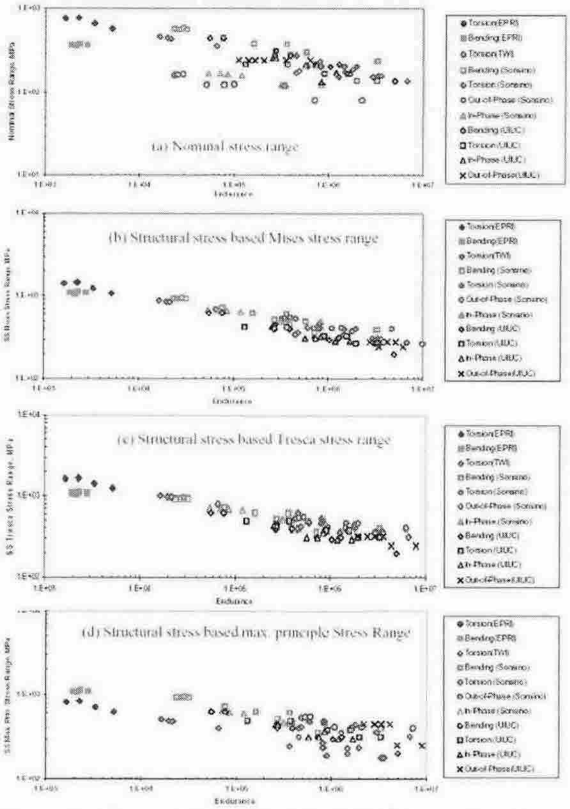



Figure 3. Comparison of multi-axial fatigue data from EPRI, LBF, UIUC, and TWI using various effective stress definitions.

## 4. Consolidation of S-N data

In Fig. 3(a), all the four data sets are plotted in terms of nominal stress range versus cycles to failure. In both in-phase and out-of-phase loading cases, the norminal stress ranges for the bending components were used, as typically done in [1~4]. Ths scatter band is very large in Fig. 3(a), as expected.

Once the structural stresses corresponding to the nomal and in-phase shear components are used the form of von Mises effective stress definition Eq (1), Fig. 3(b) shows that same data are effectively consolidated into a single narrow band. Tresca's effective stress defition Eq. (2) yields the similar results as shown in Fig. 3(c), although slightly less effective than the von Mises stress definition. Surprisingly, the maximum principle stress definition Eq. (3) is not effective in correlating the same S-N data, even though the structural stresses are used Eq. (3), with its scatter being rather similar to that in Fig. 3(a).

## 5. Conclusion

(1) von Mises effective definition and Tresca effective definition can effectively consolidate into a single narrow band for multi-axila fatigue data.

(2) Maximum principle stress definition is not effective in correlating the same S-N data, even though the structural stresses are used.

## REFERENCES

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