Throughflow Analysis by Newton-Raphson Method for Axial Flow Gas Turbines

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1. Introduction

The Brayton cycle is used for power conversion system in the gas-cooled reactor and it is very important to estimate the performance of compressors and turbines. A method has been developed for calculating the design point performance of multi-stage gas turbines. Axisymmetric throughflow calculations are essential part of the design and analysis of turbomachinery and the method of solving the streamline curvature equation is rather specified and many approaches exist. One of the most obvious deficiencies in current throughflow calculations, however, is the stability constraints. In this work, the Newton-Raphson method which is newly adapted to solve the throughflow calculation and its results are presented.

2. Methods and Results

Throughflow calculation methods are the most used of all the design and analysis procedures, being applied to predict thermodynamic properties and flow velocity everywhere inside the gas turbines. From the assumption of axisymmetric and inviscid flow it is possible to define a series of meridional stream surfaces of revolution through the turbomachine. The description and approach that is adapted here is that given by J.D. Denton (1978) with applicability to a wide range of geometries.

In the view of the meridional plane, Figure 1, the typical quasi-orthogonal lines and stream surface are illustrated.

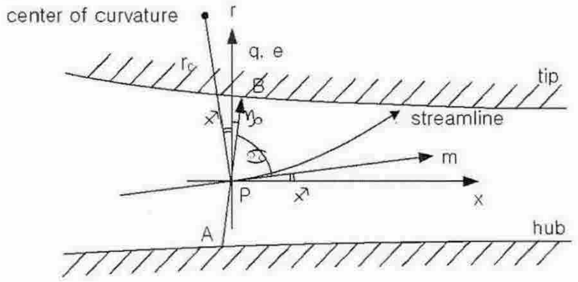

Figure 1 Meridional plane

The acceleration of a fluid particle at P can be built up from the following components: $V_m \partial V_m / \partial m$ in the m direction, $-V_\theta^2 / r$ in the radial direction, and V_m^2 / r_c in the normal to the m direction. With the blade loading to be neglected, these three acceleration components can be combined to give the total acceleration a_q in the direction of the unit vector e,

$$a_q = V_m \frac{\partial V_m}{\partial m} \cos\alpha + \frac{V_m^2}{r_c} \sin\alpha - \frac{V_\theta^2}{r} \sin(\phi + \alpha) \qquad (1)$$

The momentum equation and energy equation applied in the stream surface in the direction of e is then

$$\frac{1}{2} \frac{d}{dq} V_m^2 = \frac{dh_0}{dq} - T \frac{ds}{dq} - \frac{1}{2r^2} \frac{d\left(r^2 V_\theta^2\right)}{dq} + \frac{V_m^2}{r_c} \sin\alpha + V_m \frac{dV_m}{dm} \cos\alpha \qquad (2)$$

Equation (2) gives the gradient of meridional velocity and this must be solved in conjunction with the continuity equation across the quasi-orthogonal neglecting blockage factor.

$$\dot{m} = \int_A^B 2\pi r \rho V_m \sin\alpha \, dq \qquad (3)$$

The method of solving these two equations is to begin with a initial guess or previous iteration for the position of the streamline in the meridional plane and the distributions of V_m along the quasi-orthogonals. From this V_θ can be determined from the blade geometry using correlations and the entropy is assumed to be conserved along streamlines in calculations where losses are ignored and allowed to increase by amount of the losses more generally. The stagnation enthalpy is calculated from the Euler turbomachine equation. From this the terms in the radial equilibrium equation (2) are calculated. After calculating the distribution of change V_m across the annulus, then the integral of V_m is compared with the total mass flow rate and the overall level of V_m is adjusted. The mass flow rate between streamlines is used to determine the position of the streamline positions on the quasi-orthogonal lines to be used for the next iteration.

The existing methods have shortcomings of the stability constraints due to the separated calculation process of equation (2) and equation (3). The relaxation factors are necessarily required in velocity level and gradient in the radial equilibrium iteration in order to maintain stability.

Also the streamline shape must be adapted by a small fraction of its predicted change for each iteration if the process is to converge.

In order to overcomes the deficiencies as stated above while improving convergence and retaining the original calculation results, the calculation process of the radial equilibrium equation (2) and the continuity equation (3) has been completely changed by Newton-Raphson method.

Figure 2 shows the schematic of the grid generations by *i*-nodes for quasi-orthogonals and *j*-nodes for streamlines.

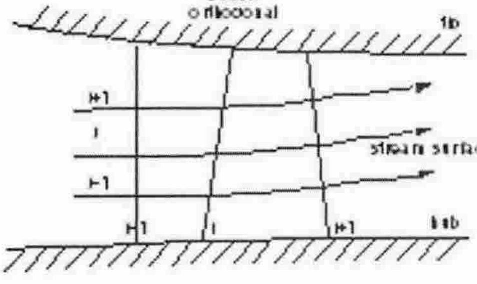

Figure 2 Grid generations

These two equations can be expressed by only two variables: the meridional velocity V_m and the radius of the streamline position on the quasi-orthogonals r_{qo}.

$$F_j\left((V_m)_j,\ (V_m)_{j+1},\ (r_{qo})_j,\ (r_{qo})_{j+1}\right) = \delta(V_m)_j^{\ 2} - (V_m)_{j+1}^{\ 2} + (V_m)_j^{\ 2} \tag{4}$$

$$\begin{aligned} &G_j\left((V_m)_j,\ (V_m)_{j+1},\ (r_{qo})_j,\ (r_{qo})_{j+1}\right) \\ &= \left((\rho)_{j+1}(r_{qo})_{j+1}(V_m)_{j+1}\sin(\alpha_{qm})_{j+1} + (\rho)_j(r_{qo})_j(V_m)_j\sin(\alpha_{qm})_j\right) \\ &\quad \times\pi\left((r_{qo})_{j+1} - (r_{qo})_j\right)/\sin(\alpha_{qo}) - \dot{m}/(j_{\max}-1) \end{aligned} \tag{5}$$

Therefore it is possible to derive the partial derivatives of equation (2) and equation (3) for each streamlines as follows:

$$\begin{aligned} &\frac{\partial F_j}{\partial (V_m)_{j+1}} d(V_m)_{j+1} + \frac{\partial F_j}{\partial (V_m)_j} d(V_m)_j \\ &+ \frac{\partial F_j}{\partial (r_{qo})_{j+1}} d(r_{qo})_{j+1} + \frac{\partial F_j}{\partial (r_{qo})_j} d(r_{qo})_j + F_j = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{\partial G_j}{\partial (V_m)_{j+1}} d(V_m)_{j+1} + \frac{\partial G_j}{\partial (v_m)_j} d(V_m)_j \\ &+ \frac{\partial G_j}{\partial (r_{qo})_{j+1}} d(r_{qo})_{j+1} + \frac{\partial G_j}{\partial (r_{qo})_j} d(r_{qo})_j + G_j = 0 \end{aligned} \tag{7}$$

The linearization of two equations yields a Jacobian matrix and the inverse matrix is calculated by Gauss elimination. The meridional velocity V_m and the streamline position r_{qo} are calculated simultaneously so that the relaxation factors and iterations are not necessary retaining the identical results. Therefore, the velocity distribution can be obtained at once and the streamline curvature converges quickly. This method is not only much stable but also potentially faster than the current calculation methods.

Comparisons of maximum errors in velocity calculation are presented in Figure 3. The 4-stage high-pressure turbine, the 8-stage high-pressure compressor and the 14-stage generator turbine of MPBR-MIT reference were analyzed by previously existing method and new method.

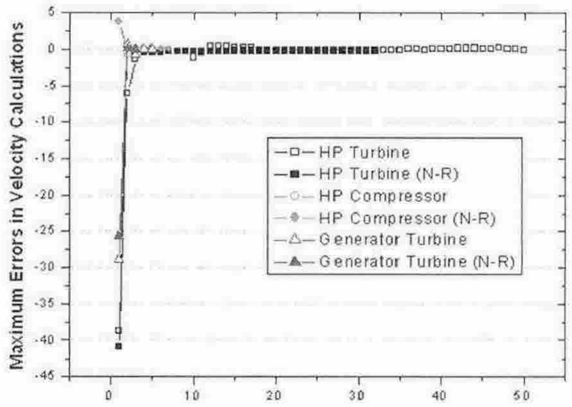

Figure 3 Total streamline curvature iterations vs. Maximum errors in velocity calculation

3. Conclusions

Newton-Raphson method is successively adapted to improve the current throughflow calculations. As a result of this method the faster and accurate results were achieved without relaxation factor. And the convergence limitation of the existing throughflow calculation method is also much improved. This method is useful for much stable and accurate result and substantially shows better convergence.

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