Transient MOC Calculation within the CMFD Framework

Jin-Young Cho,[a] Kang-Seog Kim,[a] Chung-Chan Lee,[a] Han-Gyu Joo,[b] Won-Sik Yang,[c] T. A. Taiwo,[c] J. Thomas,[d]
a Korea Atomic Energy Research Institute, 150 Deokjin-dong, Yuseong-gu, Daejon, 305-353, Korea, jyoung@kaeri.re.kr
b Seoul National University, San 56-1 Sillim-Dong, Kwanak-Gu, Seoul, Korea
c Argonne National Lab., Argonne, Illinois, 60439, USA
d Purdue University, West Lafayette, Indiana, 47907, USA

1. Introduction

Recently, a transient calculation capability was implemented into the DeCART code[1] employing a MOC transient calculation scheme within the framework of the cell-wise CMFD formulation. The transport methods have already been applied to the transient problem in the nodal transport form [2,3] based on the cell or assembly homogenized cross sections. The VARIANT-K code [2] solves the transport transient problem by using the variational nodal method for the space domain, the P_N approximation for the angular domain and the fully implicit scheme for the time domain. The PARCS code [3] solves the transport transient problem by introducing the nodal expansion method or finite difference method for the space domain, the simplified P_N approximation for the angular domain and the theta method for the time domain. However, the MOC based transient transport calculation for the heterogeneous geometry itself is undertaken as a first attempt in this paper. Therefore, to verify the DeCART results, the DeCART code is first applied to the same homogeneous problems that were used for the verification of the transient transport codes. The transient capability of the DeCART code for the heterogeneous problems will be examined in the future.

2. MOC Transient Calculation Method

The time dependent form of the planar transport problem for the 2-D plane with Energy Group g and Angle m can be written as follow:

$$\frac{1}{v_g}\frac{\partial \varphi_g^m}{\partial t} = \frac{1}{4\pi}\left(\chi_{pg}(1-\beta)\psi + \chi_{dg}S_d + \sum_{g'=1}^{G}\Sigma_{gg'}\phi_{g'}\right) - \frac{\mu_m}{h_z}\left(\varphi_{gT}^m - \varphi_{gB}^m\right) - \left(\eta_m\frac{\partial \varphi_g^m}{\partial x} + \varepsilon_m\frac{\partial \varphi_g^m}{\partial y} + \Sigma_{tg}\varphi_g^m\right) \quad (1)$$

where φ_g^m, ϕ_g, ψ, and S_d are the angular flux, scalar flux, total fission source, and the delayed neutron source respectively, which are all axially averaged, whereas φ_{gT}^m and φ_{gB}^m are the angular fluxes at the top and bottom of the plane.

The temporal discretization of Eq. (1) can be made in the same way as the conventional transient equation that is also applied to the DeCART CMFD formulation. The resulting transient fixed source problem (TFSP) which has the transient fixed source in addition to the fission and scattering sources reads at time step n:

$$\left(\eta_m\frac{\partial \varphi_g^{m,n}}{\partial x} + \varepsilon_m\frac{\partial \varphi_g^{m,n}}{\partial y}\right) + \left(\Sigma_{tg} + \frac{1}{\theta v_g \Delta t_n}\right)\varphi_g^{m,n} = \frac{1}{4\pi}\left(\left(\chi_{pg}(1-\beta)+\chi_{dg}\omega\right)\psi^n + \sum_{g'=1}^{G}\Sigma_{gg'}\phi_{g'}^n\right) - \frac{\mu_m}{h_z}\left(\varphi_{gT}^m - \varphi_{gB}^m\right) + \frac{\varphi_g^{m,n-1}}{\theta v_g\Delta t_n} + \frac{1}{4\pi}\chi_{dg}\tilde{S}_d^{n-1} + \Theta R_g^{m,n-1} \quad (2)$$

Here the residual term appearing last represents the imbalance between the neutron production and loss in Direction m at a point in the case of the continuous form or in a flat source region in the case of the discretized form. In principle, Eq. (2) can be solved by MOC as long as the right hand side (RHS) is exactly known for each flat source region. However, there are several practical difficulties in solving Eq. (2) as is. First of all, the total cross section is augmented by the 1-over-vΔt term. This augmentation changes the ray attenuation characteristics in the MOC solution since all the exponential terms have to be evaluated with the augmented cross section. Secondly, since the angular flux of the previous step appears on the RHS, all the angular flux should be stored at every flat source region which would cause a significant increase in the memory.

In order to avoid these problems, an approximate solution approach is applied. The 1-over-vΔt term of the current time step is first moved to the RHS so that the left hand side (LHS) becomes identical to the steady-state form. The angular dependence of the 1-over-vΔt term is then neglected by treating this term as isotropic. This approximation would have a negligible impact since the isotropy assumption is applied to the difference term. Furthermore the angular dependency of the residual term is neglected. Regarding the residual term, due to the irregular surface shape of a flat source region, a further approximation is introduced to use the cell based residual term which is determined for the multi-group CMFD TFSP. The final equation becomes:

$$\left(\eta_m\frac{\partial \varphi_g^{m,n}}{\partial x} + \varepsilon_m\frac{\partial \varphi_g^{m,n}}{\partial y}\right) + \Sigma_{tg}\varphi_g^{m,n} = \frac{1}{4\pi}\left(\chi_g\psi^n + \sum_{g'=1}^{G}\Sigma_{gg'}\phi_{g'}^n\right) - \frac{\mu_m}{h_z}\left(\varphi_{gT}^m - \varphi_{gB}^m\right) + \frac{1}{4\pi}S_{tr}^{g,n}, \quad (3)$$

where the transient specific source is defined as:

$$S_{tr}^{g,n} = \left(\chi_{pg}(1-\beta)+\chi_{dg}\omega-\chi_g\right)\psi^n + \chi_{dg}\tilde{S}_d^{n-1} + \Theta\bar{R}_g^{n-1} - \frac{\phi_g^n - \phi_g^{n-1}}{\theta v_g\Delta t_n} \quad (4)$$

The bar above designates that these parameters are obtained from the multi-group CMFD so that there is no spatial variation within a cell.

3. Computational Results

The validity of the final form of Eqs. (3) and (4) for the MOC transient calculation is examined for the MOX core problem [4] and the mini-core problems[5]. The original MOX core problem, which was used for the examination of the VARIANT-K solution, is a 2-dimensional full core and homogeneous problem, and simulates a control rod ejection in one of the assemblies. In this paper, this problem is modified to a quarter core problem by simulating the control rod ejection in the four assemblies with a similar ejected rod worth to the original problem. The mini-core problems, which were applied to the analysis of the PARCS transient results, consists of the 2-D and 3-D problems by loading 4 MOX and 5 UOX assemblies in a checkerboard type using the radial reflective boundary conditions and the homogenized cross sections with the T/H feedback. This problem is modified to the no T/H feedback problem because of the incapability of the T/H calculation of the VARIANT-K code. This problem simulates a total of 1.0 second transient by completely ejecting the control rod from the initial position of a midway of the core in 0.1 seconds.

Table 1 shows the keff and the ejected rod worth (ERW) comparison. DeCART and VARIANT-K shows less than 5 pcm differences in the keff, and less than 0.005 $ in the ERW. In the 2-D problems, the DeCART results are essentially the same as the VARIANT-K. However, in the mini-core 3-D problem, DeCART shows a little difference from the VARIANT-K. Fig. 2 shows the transient core power comparison for the three codes. In the 2-Dimensional problems, DeCART shows a very similar behavior to the other codes. However, in the mini-core 3-Dimensional problem, DeCART shows a little difference which comes from the ERW difference.

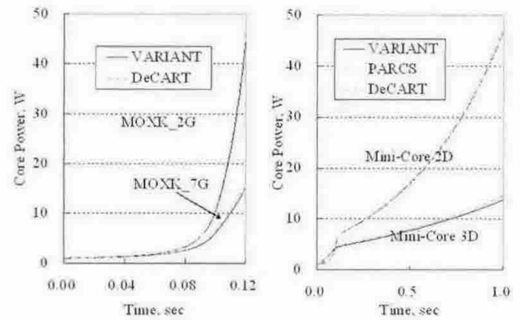
4. Conclusion

In this paper, the transient calculation capability was implemented into the DeCART code and the computational results were analyzed by solving the benchmark problems that were used for the VARIANT-K or PARCS transport transient verification. The computational results proved that the newly implemented transient calculation feature of the DeCART code works very well and produces sound results.

Table 1. keff and Ejected Rod Worth Comparison

Problems			VARIANT	DeCART	PARCS
MOXK	2G	keff	1.22041	1.22041	
		ERW*	0.984	0.984	
	7G	keff	1.22167	1.22166	
		ERW*	0.879	0.879	
Mini-	2D	keff	1.06332	1.06331	1.06332
		ERW*	0.804	0.805	0.804
	3D	keff	1.08664	1.08660	
		ERW*	0.715	0.719	

* Ejected Rod Worth, $



(a) MOXK Problem (b) Mini-Core Problem

Figure 2. Transient Power Comparison

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