# The MOC Neutron Transport Calculations Accelerated by Coarse-Mesh Angular Dependent Rebalance

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## 1. Introduction

As the nuclear reactor core becomes more complex, heterogeneous, and geometrically irregular, the method of characteristics (MOC)[1] is gaining its wide use in the neutron transport calculations. However, the long computer times require good acceleration methods. In this paper, the concept of coarse-mesh angular dependent rebalance (CMADR)[2] acceleration is described and applied to the MOC calculations. The method is based on angular dependent rebalance factors defined on the coarse-mesh boundaries; a coarse-mesh consists of several fine meshes that may be (1) heterogeneous and (2) of mixed geometries with nonregular or unstructured mesh shapes. In addition, (3) the coarse-mesh boundaries may not coincide with the structural interfaces of the problem and can be chosen artificially for convenience. The CMADR acceleration method on the MOC scheme that enables the very desirable features (1), (2), and (3) above is new in the neutron transport literature to the best of the authors' knowledge.

## 2. CMADR Method

### *2.1 CMADR Equations*

In MOC calculations, for a computational mesh $i$ with flat source approximation, the outgoing angular flux along a ray $l$ and the average flux are given as follows:

$$\psi^{l,\alpha+1/2}_{m,n,i+1/2} = \exp(-\frac{\sigma_i L^l_{m,n,i}}{\sin\theta_n})\psi^{l,\alpha+1/2}_{m,n,i-1/2} + \frac{q^{\alpha}_{m,n,i}}{\sigma_i}(1-\exp(-\frac{\sigma_i L^l_{m,n,i}}{\sin\theta_n})), \quad (1)$$

$$\overline{\psi}^{\alpha+1/2}_{m,n,i} = \frac{q^{\alpha}_{m,n,i}}{\sigma_i} + \frac{\sin\theta_n}{\sigma_i A_i}\sum_{l\in i}\delta^l_m(\psi^{l,\alpha+1/2}_{m,n,i-1/2} - \psi^{l,\alpha+1/2}_{m,n,i+1/2}), \quad (2)$$

where $\alpha$ is the iteration index, $m$ and $n$ are azimuthal and polar angle indices respectively, $\theta_n$ is $n$-th polar angle, $L^l_{m,n,i}$ is the track length of $l$-th ray of mesh $i$ in $(m,n)$ direction, $A_i$ is the area of the mesh $i$, and $\delta^l_m$ is the ray spacing of ray $l$ in $m$-th azimuthal angle.

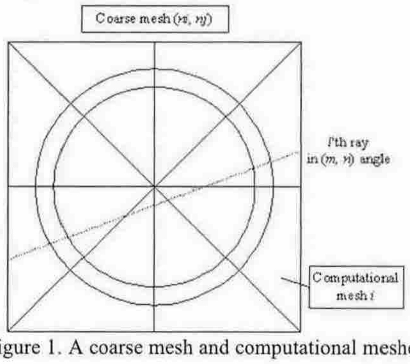


Figure 1. A coarse mesh and computational meshes

To obtain the CMADR equations, a coarse mesh $(ni,nj)$ composed of several computational meshes is considered as in Fig. 1. In this paper, coarse mesh index $(ni,nj)$ will be omitted for simplicity. Using Eq. (1), the outgoing angular flux of the coarse mesh along the $l$-th ray is given as follows[3]:

$$\psi^{l,\alpha+1/2}_{m,n,t} = T^l_{m,n,u\to t}\psi^{l,\alpha+1/2}_{m,n,u} + \sum_{i=1}^{N_m(u,t)} T^l_{m,n,i\to t} q^{\alpha}_{m,n,i}, \quad (3)$$

where $u$ and $t$ are incoming and outgoing edge indices, $N_m(u,t)$ is the number of meshes that affect outgoing edge $t$ along the $l$-th ray in incoming edge $u$. Coefficients $T$ are given as follows:

$$T^l_{m,n,u\to t} = \exp(-\sum_{i=1}^{N_m(u,t)}\sigma_i L^l_{m,n,i}/\sin\theta_n), \quad (4)$$

$$T^l_{m,n,i\to t} = \frac{[1-\exp(-\sigma_i L^l_{m,n,i}/\sin\theta_n)]}{\sigma_i}\exp(-\sum_{p=i+1}^{N_m(u,t)}\sigma_p L^l_{m,n,p}/\sin\theta_n). \quad (5)$$

Then, $x$- and $y$-direction outgoing angular fluxes of the coarse mesh are given by

$$\sum_{l\in t_x}\delta^l_m\psi^{l,\alpha+1/2}_{m,n,t_x} = \sum_{l\in(u_x,t_x)}\delta^l_m T^l_{m,n,u_x\to t_x}\psi^{l,\alpha+1/2}_{m,n,u_x} + \sum_{l\in(u_y,t_x)}\delta^l_m T^l_{m,n,u_y\to t_x}\psi^{l,\alpha+1/2}_{m,n,u_y} + \sum_{i=1}^{N_m(x)}\sum_{l\in(i,t_x)}\delta^l_m T^l_{m,n,i\to t_x} q^{\alpha}_{m,n,i}, \quad (6)$$

$$\sum_{l\in t_y}\delta^l_m\psi^{l,\alpha+1/2}_{m,n,t_y} = \sum_{l\in(u_x,t_y)}\delta^l_m T^l_{m,n,u_x\to t_y}\psi^{l,\alpha+1/2}_{m,n,u_x} + \sum_{l\in(u_y,t_y)}\delta^l_m T^l_{m,n,u_y\to t_y}\psi^{l,\alpha+1/2}_{m,n,u_y} + \sum_{i=1}^{N_m(y)}\sum_{l\in(i,t_y)}\delta^l_m T^l_{m,n,i\to t_y} q^{\alpha}_{m,n,i}, \quad (7)$$

where $N_m(x) = N_m(u_x,t_x) + N_m(u_u,t_x)$ and $N_m(y) = N_m(u_x,t_y) + N_m(u_u,t_y)$.

Now the angular dependant rebalance factors are defined on the coarse mesh boundaries:

$$f_{x,m,n,t_x} = \frac{\psi^{l,\alpha+1}_{m,n,t_x}}{\psi^{l,\alpha+1/2}_{m,n,t_x}} \cong f^{\gamma}_{x,t_x}, \quad f_{x,m,n,t_x} = \frac{\psi^{l,\alpha+1}_{m,n,t_x}}{\psi^{l,\alpha+1/2}_{m,n,t_x}} \cong f^{\gamma}_{x,t_x}, \quad (8)$$

where direction $(m,n)$ is in quadrant $\gamma$.

The CMADR equations are obtained by replacing iteration indices in Eqs. (6) and (7) to $\alpha+1$, introducing rebalance factors, and summing over each quadrant with weight $W_{m,n}$ as follows:

$$f^{\gamma}_{x,t_x} = AX^{\alpha+1/2,\gamma}f^{\gamma}_{x,u_x} + BX^{\alpha+1/2,\gamma}f^{\gamma}_{y,u_y} + CX^{\alpha+1,\gamma},$$
$$f^{\gamma}_{y,t_y} = AY^{\alpha+1/2,\gamma}f^{\gamma}_{x,u_x} + BY^{\alpha+1/2,\gamma}f^{\gamma}_{y,u_y} + CY^{\alpha+1,\gamma}. \quad (9)$$

In this paper, we use $\mu = \sin\theta_n\cos\varphi_m$ and $\eta = \sin\theta_n\sin\varphi_m$ as the weights of $x$- and $y$-direction edges, respectively.

In addition to Eq. (9) the mesh averaged angular flux can be expressed as follows:

$$\overline{\psi}^{\alpha+1/2}_{m,n,i} = \sum_{l\in(u_x,i)}\delta^l_m P^l_{m,n,u_x\to i}\psi^{l,\alpha+1/2}_{m,n,u_x} + \sum_{l\in(u_y,i)}\delta^l_m P^l_{m,n,u_y\to i}\psi^{l,\alpha+1/2}_{m,n,u_y} + \sum_{j=1}^{N_m(i)}\sum_{l\in(j,i)}\delta^l_m P^l_{m,n,j\to i} q^{\alpha}_{m,n,i}, \quad (10)$$

where $N_m(i) = N_m(u_x,i) + N_m(u_u,i)$ and coefficients $P$ are given as follows:

$$P^l_{m,n,u\to i} = \frac{\sin\theta_n}{\sigma_i A_i}[1-\exp(-\sigma_i L^l_{m,n,i}/\sin\theta_n)]\exp(-\sum_{k=1}^{N_m(u,i-1)}\sigma_k L^l_{m,n,k}/\sin\theta_n), \quad (11)$$

$$P^l_{m,n,j\to i} = \frac{\sin\theta_n}{\sigma_j\sigma_i A_i}[1-\exp(-\sigma_i L^l_{m,n,i}/\sin\theta_n)] \times[\exp(-\sum_{k=j+1}^{N_m(u,i-1)}\sigma_k L^l_{m,n,k}/\sin\theta_n) - \exp(-\sum_{k=j+1}^{N_m(u,i)}\sigma_k L^l_{m,n,k}/\sin\theta_n)], \quad (12)$$

$$P^l_{m,n,i\to i} = \frac{1}{\delta_{m,i}\sigma_i} - \frac{\sin\theta_n}{\sigma_i^2 A_i}[1-\exp(-\sigma_i L^l_{m,n,i}/\sin\theta_n)], \quad (13)$$

The update equations of CMADR are obtained by changing iteration indices in Eq. (10) to $\alpha+1$, introducing rebalance factors, and summing over each quadrant as follows:

$$\overline{\phi}^{\alpha+1,\gamma}_{m,n,i} = DM^{\alpha+1/2,\gamma}f^{\gamma}_{x,u_x} + EM^{\alpha+1/2,\gamma}f^{\gamma}_{y,u_y} + FM^{\alpha+1,\gamma},$$
$$\overline{\phi}^{\alpha+1}_{m,n,i} = \sum_{\gamma}\overline{\phi}^{\alpha+1,\gamma}_{m,n,i}. \quad (14)$$

The resulting CMADR equations [Eqs. (7) and (14)] resemble $S_2$ transport equations and we can solve these equations by transport-like sweep or the Krylov subspace method[4]. Also, the coefficients $T$ and $P$

are independent of iteration index and flux shape so that these coefficients can be calculated and stored before the iteration. Moreover, if modular ray tracing is used in calculation, only several coefficients are stored according to cell types. In this paper we use the CRX[5,6,7] code for MOC, which uses modular ray tracing and BiCGSTAB method to reduce computing time to solve the CMADR equations.

*2.2 Numerical Results*

2.2.2 Test Problem I

Test problem I is a homogeneous medium with vacuum boundaries but the source whose density is 1.0 $cm^{-3}sec^{-1}$ is located at the inner square only as shown in Fig. 2. The problem consists of $16\times16$ coarse meshes and a coarse mesh contains 24 computational meshes. The radii of circles are 0.45*cm* and 0.35*cm*. Scattering ratio is 0.999, angles are (8,4), and the number of rays is 50. Convergence criteria for high- and low-order calculations are $10^{-5}$. Table I shows that CMADR is about 36 times faster in the number of iterations and 11 times faster in computing time than the original CRX code.

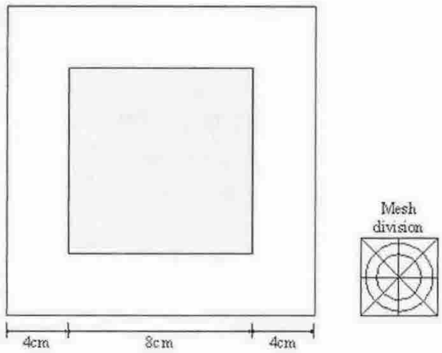


Figure 2. Configuration of test problem I

Table I. Results of test problem I

| | CRX | CRX-CMADR | Speedup |
|---|---|---|---|
| Number of iterations | 332 | 9 | 36.89 |
| Computing time (sec) | 1551.67 | 134.41 | 11.54 |

2.2.3 Test Problem II

Test problem II is a modified Kavenoky's problem[8] with vacuum boundaries as shown in Fig. 3. The problem consists of $7\times7$ coarse meshes and a coarse mesh is heterogeneous and contains 24 computational meshes. The size of the coarse mesh is 1.25*cm* and radii of circles are 0.45*cm* and 0.35*cm*. (8,4) angles and 50 rays per coarse mesh are used to solve the problem. Convergence criterion is $10^{-4}$. Material properties are given in Table II. Table III shows the results of calculation. CMADR is 7 times faster in the number of iterations and 3 times faster in computing times. In this problem, the fuel and burnable poison (BP) regions have small scattering ratios so that this diminishes the acceleration effect than in the previous problems.

Table II. Material properties for test problem II

| | Moderator | Fuel | BP |
|---|---|---|---|
| Source density ($cm^{-3}sec^{-1}$) | 1.000 | 0.000 | 0.000 |
| $\sigma(cm^{-1})$ | 1.250 | 0.625 | 14.000 |
| $\sigma_s(cm^{-1})$ | 1.242 | 0.355 | 0.000 |

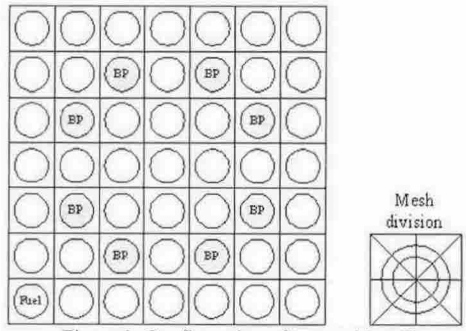


Figure 3. Configuration of test problem II

Table III. Results of test problem II

| | CRX | CRX-CMADR | Speedup |
|---|---|---|---|
| Number of iterations | 49 | 7 | 7.00 |
| Computing time (sec) | 42.56 | 11.94 | 3.56 |

**3. Conclusions**

In this paper, the MOC transport calculation was accelerated by the coarse-mesh angular dependent rebalance (CMADR) method. The CMADR method is based on the ADR factor concept, in which the rebalance factors are angular dependent and defined only on the coarse-mesh boundaries. The coarse mesh can be overlayed on a collection of fine meshes that may be heterogeneous and of mixed geometries with nonregular or unstructured mesh shapes. This is possible due to the capability of the MOC. Furthermore, the coarse-mesh boundaries may or may not coincide with the structural interfaces of the problem and can be chosen flexibly for the convenience of analysis. The CMADR method on MOC was tested successfully on several test problems and the results showed that it is very effective in reducing the number of iterations and computing time.

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