# Forced Structured Coarse Mesh Finite Difference Method for the Unstructured Mesh Transport Problems

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## 1. Introduction

The Coarse Mesh Finite Difference Method (CMFD) has been used to accelerate the nodal methods of the diffusion calculations and the characteristics method [1] of the transport calculations for the eigenvalue problems.[2] The CMFD has been applied mainly to the structured mesh diffusion or transport problems including regular triangle, square and regular hexagon meshes. The characteristics method can be easily applied to the transport problem including the complicated unstructured meshes. Since the transport sweepings in the characteristics method are very time consuming, an acceleration scheme is indispensable for the unstructured mesh problem. It is very complicated to derive a general type of the CMFD equations for the unstructured mesh problems. Therefore, we developed a new ray tracking method so that the structured CMFD can be applied to the unstructured mesh transport problems. This method has been implemented in the transport lattice code LIBERTE [3,4].

A new ray tracking method is introduced to consider the regular structure coarse mesh grids. We performed sample calculations to see the effectiveness and efficiency of this forced structured CMFD for the unstructured mesh problems.

## 2. Methods and Results

### *2.1 Characteristics method*

The characteristics method [1] adopts the advantages of the integral transport method and the discrete ordinate method. The former one is better in treating a complex geometry, and the latter is better in treating the anisotropic scattering problems and has no huge matrix. The characteristics method is used for the spatial discretization, which is primarily an approximation for the streaming plus collision operator on the left hand side of the transport equation.

In the characteristics method, the exiting angular flux for each track passing through a mesh cell $i$ is

$$\psi_{mik}(s_{mik}) = \psi_{mik}(0)e^{-\sigma_i s_{mik}} + \frac{1}{4\pi}\frac{Q_i}{\sigma_i}\left[1 - e^{-\sigma_i s_{mik}}\right], \qquad (1)$$

where $s_{mik}$ denotes the tracking segment, $\sigma_i$ the total cross section, and $Q_i$ the neutron source.

The average current at the surface *'s'* can be calculated from the average angular flux:

$$J^s_{i,g} = \sum_{m=1}^{M} w_m \Omega_{s,m} \psi_{m,i,g} \quad , \qquad (2)$$

where $w_m$ denotes the weighting, and $\Omega_{s,m}$ the directional cosine angle perpendicular to the surface.

### *2.2 Coarse Mesh Finite Difference Method*

The current correction coefficient can be calculated from the surface average current at the coarse mesh edges as follows:

$$\hat{D}^s_{i,g} = -\frac{J^s_{i,g} + \widetilde{D}^s_{i,g}(\phi_{i-1,g} - \phi_{i,g})}{\phi_{i-1,g} + \phi_{i,g}} \quad . \qquad (3)$$

The obtained scalar fluxes from the CMFD are used to update the local scalar fluxes and the incident angular fluxes at the boundaries in the characteristics method.

$$\phi^{(l+1/2)}_{k,g} = \phi^{(l)}_{k,g}\frac{\phi^{(l+1/2)}_{i,g}}{\phi^{(l)}_{i,g}} \quad and \quad \psi^{(l+1/2)}_{m,g} = \psi^{(l+1/2)}_{m,g}\frac{J^{(l+1,2)}_{i,g}}{J^{(l)}_{i,g}}, \qquad (4)$$

where '*k*' denotes the fine mesh cell index and '*i*' the coarse mesh cell index.

### *2.3 Ray Tracking Method*

The interface current at the structured coarse mesh edges should be obtained so that the structured CMFD can be applied for the unstructured mesh transport problem. Therefore, a ray tracking should be performed to obtain the required information in the transport sweeping.

We perform two independent ray trackings for the complicated unstructured meshes and for the coarse mesh grids as shown in Figure 1. By a comparison of the tracking segments from the two ray trackings, some of the fine mesh segments can be divided into two sub-segments. Since some of the track segments are divided into sub-segments, some of the subcells are divided into sub-subcells. In Figure 2 the complex unstructured meshes are overlapped with the structured coarse mesh grids. Region '*i*' denotes the flat source region in the transport calculation which is sub-divided into four different flat source sub-subcells due to the coarse mesh grids. Although this region includes the same compositions and cross sections, the scalar flux could be different. These sub-subcells are included in the different coarse mesh cells in the CMFD formulations, and are included in the same cells in the transport calculations.

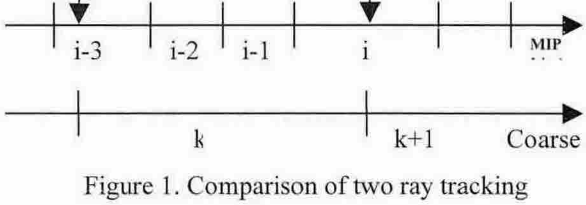


Figure 1. Comparison of two ray tracking

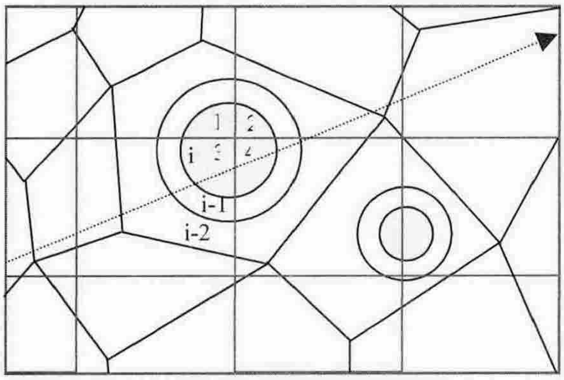


Figure 2. Division of flat source region

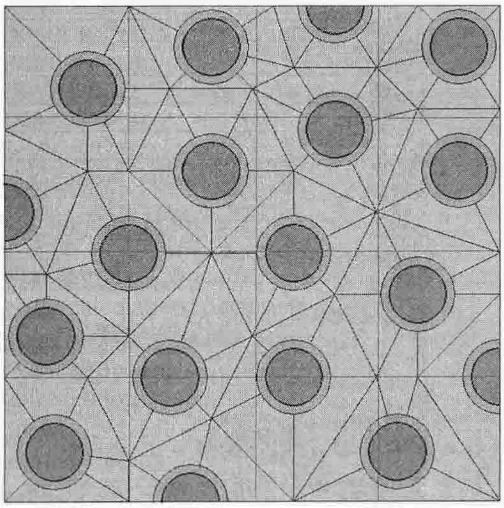

Figure 3. Geometrical Structure of Sample Problem

### *2.4 Sample calculation*

We constructed an arbitrary complex geometry with the rectangular domain boundary and various types of poygon cells as shown in Figure 3. Fuel pins are randomly distributed in the moderator. This figure was drawn by the geometry visualizing program LIBGRAF [4] for LIBERTE. We performed the eigenvalue transport calculations without and with the CMFD accelerations, and compared the multiplication factors and pin power distributions. Various coarse meshes were used to see the effects due to the coarse mesh size. Table 1 shows that there are slight differences in the multiplication factors and pin power distributions. Considering the computing time and the accuracy, it is reasonable to select a coarse mesh size corresponding to the average pin pitch. It seems that the slight difference comes from the uncertainty of the sub-subcell volume and the currents at the cell edges.

Table 1. Comparison of the multiplication factors, pin powers and the number of sweepings

| Case | keff | Pin Power Difference | Number of Sweeping |
|---|---|---|---|
| Reference | 1.44664 | - | > 1000 |
| 1x1 meshes | 1.44665 | 0.001 | 30 |
| 2x2 meshes | 1.44665 | 0.001 | >30 |
| 3x3 meshes | 1.44636 | 0.004 | 23 |
| 4x4 meshes | 1.44650 | 0.001 | 12 |
| 5x5 meshes | 1.44624 | 0.006 | 13 |

## 3. Conclusion

We developed a forced structured mesh CMFD to accelerate the unstructured mesh transport problems through a comparison of two independent ray trackings. We implemented this technique into the transport lattice code LIBERTE. The computational results showed that this method is effective and efficient in accelerating the unstructured mesh transport calculations using the characteristics method with slight errors in the multiplication factors and pin powers.

## REFERENCES

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