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Three Dimensional Analytical Model for The Prediction of The Transport of Dissolved Pollutants
오염물질 이동 예측을 위한 3차원 해석해 모델

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1. INTRODUCTION

The problem of predicting the transport of solutes such as dissolved toxic matters, radionuclides and suspended sediments on the basis of a convection-diffusion equation appear in various disciplines such as hydrology, environmental and chemical engineering and oceanography. Since the basic equation is nonlinear, numerical methods are usually adopted as a predictive tool. A very limited number of analytical solutions are available in idealized conditions when the depth is spatially constant and the convection velocity and eddy diffusivity are constants or take simple functional forms with specific initial and boundary conditions (for example, Domenico and Schwartz, 1990; Prakash, 1977; Smith, 1982; Wilson and Okubo, 1978; Yasuda, 1989; Zoppou and Knight, 1997). For some collection of solutions, see Noye (1987). There is obviously a continuing need to develop analytical solutions of the convection-diffusion equation because of its fundamental and practical importances; analytical solutions are valuable not only for the better understanding on the transport processes but the verification of the numerical schemes.

In this study an analytical solution of time-integral form has been derived to predict the transport of solute in a homogeneous open sea, combining the horizontally two-dimensional solution, derived by Jung et al (2003a) for the build-up of the heat field due to a point source in coastal sea regions with an oscillatory cross-flow, with the vertically one-dimensional solution derived by Jung et al (2003b), for the determination of local distribution of suspended sediment. The model incorporates spatially uniform, uni-directional horizontal convection, anisotropic horizontal diffusion, the vertical convection due to settling velocity, and the vertical diffusion. The presence of a point source at the sea surface is assumed which represents the dumping rate of solute. No net flux condition is applied at the sea surface (except for the source point), while the downward net flux is considered through the introduction of a depositional velocity. In the model by Yasuda(1989) the horizontal diffusion was neglected and no net flux condition was applied at the sea surface and sea bottom boundaries. The solution is sought as an salient feature of this study by applying the Galerkin method in time domain via the eigenfunction expansion over the water

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column. In applications the direct time-integration of the solution is made rather than the calculation of Arie's moments (1956) to investigate the solution behaviors. A series of calculations are carried out to examine the contribution of settling velocity, mean and oscillatory currents along with the role of longitudinal turbulent diffusion to the determination of the solute distribution.

2. RESULTS

A total of three sets of calculations have been performed. The first set is the calculations in stagnant water, the second set is the calculations with mean flow. The final set includes the calculations with mean and oscillatory flows. Throughout the calculations the water depth is assumed to be $70m$. Calculations have continued over a month with source strength $F_s = 100\,kg/m^2/s$.

Three calculations denoted by Cal-T1 to Cal-T3 have been carried out in the presence of the horizontal convection via the mean and oscillatory currents. In these calculations, the mean current, u_m, is set to $0.02m/s$, and the settling and depositional velocities are set to $-4\times10^{-5}m/s$ and $2\times10^{-5}m/s$, respectively. In Cal-T1, the amplitude of the oscillatory currents($u_{\max}$) and the horizontal diffusivity (k_x) are given as $0.2m/s$ and $0.2m^2/s$, respectively. In Cal-T2, $u_{\max} = 0.2\,m/s$ with $k_x = 2\,m^2/s$, and in Cal-T3 $u_{\max} = 0.4m/s$ with $k_x = 2\,m^2/s$. Results shown in Fig.1 are cross-sectional concentrations when the excursion in the x direction reaches its maximum (that is, just at the time when the oscillatory current changes signs to the negative x direction).

It is noted that Cal-T1 gives a very complicated pattern; wiggles are pronounced and an unrealistic rise of concentration appears in an isolated form near the tidal excursion distance. It has been found that use of a relatively high value of k_x almost eliminates the pattern (Cal-T2). However, with the use of increased $u_{\max}$ the concentration rise in an isolated form reappears (Cal-T3).

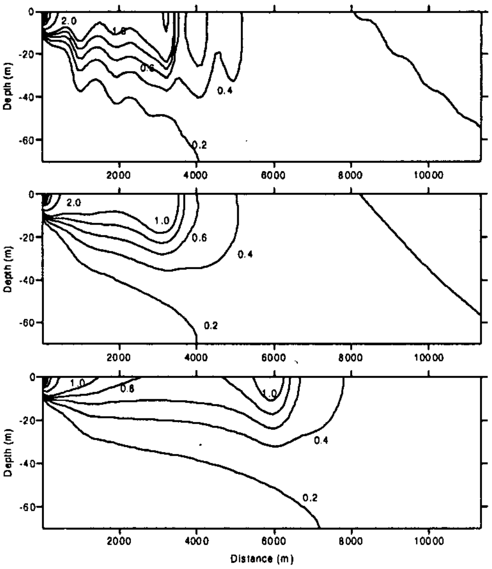

Fig. 1. Concentration computed in the presence of mean and oscillatory flows with $u_m = 0.02\,m/s$, and with: (Top) $u_{\max} = 0.20\,m/s$, $k_x = 0.2\,m^2/s$, (Middle) $u_{\max} = 0.40\,m/s$, $k_x = 2.0m^2/s$, (Bottom), $u_{\max} = 0.40\,m/s$, $k_x = 2.0\,m^2/s$.

3. CONCLUSION

By adopting the approaches developed by Junge et al (2003a) and Jung et al (2003b), an analytical model for predicting the convection-diffusion of solute dumped in a homogeneous open sea of constant water depth has been developed in a time-integral form. The model incorporates spatially uniform, uni-directional, mean and oscillatory currents for horizontal convection, the settling velocity for the vertical convection and the anisotropic horizontal turbulent diffusion. The convection-diffusion equation has been reduced to Fickian type diffusion equation and then the Galerkin method is then applied via the expansion of eigenfunctions over the water column derived from the well-known Sturm-Liouville system.

A series of calculations has been performed to demonstrate the applicability of the model, including the calculations in stagnant water, the calculations with mean water flow, and the calculations with mean and oscillatory flows.

In the course of sensitivity calculations it has been found that the most tricky problem is to define the horizontal and the vertical eddy diffusivity coefficients, particularly in the presence of the oscillatory flow. Wiggles are pronounced in the concentration field and an unrealistic rise of concentration appears in an isolated form near the tidal excursion distance when the tidal convection dominates over the dispersion process. No spurious wiggles appear in the presence of mean current. More thorough comparison with field and laboratory experiments are required in the future but it might be needed to develop a variable horizontal diffusion model in which the coefficeint increases downstream direction $k_x = \varepsilon u x$ as proposed by Hunt (1999).

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