Feedback Error Learning and H^{∞}-Control for Motor Control

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Abstract: In this study, the basic motor control system had been investigated. The controller for this study consists of two main parts, a feedforward controller part and a feedback controller part. Each part will deals with different control problems. The feedback controller deals with robustness and stability, while the feedforward controller deals with response speed. The feedforward controller, used to solve the tracking control problem, is adaptable. To make such a tracking perfect, an adaptive law based on Feedback Error Learning (FEL) is designed so that the feedforward controller becomes an inverse system of the controlled plant. The novelty of FEL method lies in its use of feedback error as a teaching signal for learning the inverse model. The theory in H^{∞}-control is selected to be applied in the feedback part to guarantee the stability and solve the robust stabilization problems. The simulation of each individual part and the integrated one are taken to clarify the study.

Keywords: Feedback Error Learning, H^{∞}-Control Theorem, Chain-Scattering Representation, Adaptive Inverse Model.

1. Introduction

Production machines in industry have played a significant role in Thailand. They have been improved in order to increase both quantity and quality of the production. Unfortunately, these machines, e.g. CNC machine, are normally imported from foreign countries. They are high-cost, over functional and, due to the use of static PID controllers, have moderate performance. Meanwhile, there are some locally made ones, but they are low-performance, even though they have moderate cost. The objective of this study is to find a cheap controller that can achieve the desired system performance.

For simplicity, the simple DC-servo motor, instead of the expensive CNC machine, is used as a controlling plant. Normally, one controller scheme alone can not respond well. So, more controller schemes are usually proposed. The controller for this study consists of two main parts, a feedforward controller part and a feedback controller part. The Feedback Error Learning (FEL) is applied to the feedforward part. The FEL novel architecture combines learning and control efficiently. The novelty of the FEL method lies in its use of feedback error as a teaching signal for learning the inverse model, which is essentially new in control literature. Originally, FEL is adopted from the concept of brain motor control[3]. From this study, it is shown that the system with FEL controller have fast response speed.

Kimura[1] proposed a unified framework of H^{∞}-control theory based on the chain-scattering representation of the plant and the $(J\,,J\,')$-lossless factorization. This paper is concerned with the application of this approach for motor control systems. The chain-scattering representation of systems enables us to treat the feedback connection as a cascade connection. Furthermore, this property of the chain-scattering representation has been used in a variety of engineering fields; e.g. a method of representing the reletionship between the power port, the scattering properties of a physical system, etc. In this study, this approach is applied to the feedback part. The stability of the overall system can also be guaranteed.

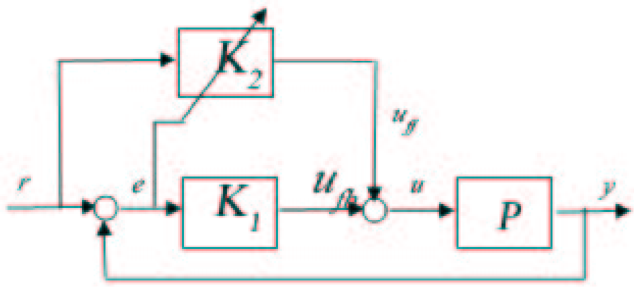

Fig. 1. Feedback error learning scheme

This paper is organized as follows. In section 2 some theoretical backgrounds are briefly reviewed. This section also consists of two subsections. The first one will concern about the FEL method and the other one is about chain-scattering approach to H^{∞}-control. Section 3 shows and discusses about the simulation results. Section 4 gives the conclusion of the whole study.

2. Theoretical Background

To briefly explain the key concept of FEL, consider its architecture shown in Fig. 1. The objective of control is to minimize the error e between the command signal r and the plant output y. The input u to the plant P is composed of the output u_{ff} of feedforward controller K_2 and u_{fb} of the feedback controller K_1. If P is known and P^{-1} exists and is stable, choosing $K_2 = P^{-1}$ makes the tracking perfect. Indeed, from the relations $u_{ff} = P^{-1}r$, $u_{fb} = K_1(r-y)$ and $y = P(u_{ff} + u_{fb})$, it is easily to see that $y = r$. Thus, the novelty of the FEL method lies in its way to learn the inverse model of P. That is the parameter be adapted so that $K_2 = P^{-1}$. Unfortunately, in practice, most of the plant is not invertible or has unstable inverse. However, a method to solve this problem is simply proposed by adding a prefilter. As mention earlier, the feedback controller K_1 is used to solve stability problems, such as robust stabilization or sensitivity reduction problems. The robustness in the system with uncertainty is the main objective in this study. To deal with this kind of problem, modern control theory has provided many powerful solving methods. The chain-scattering approach to H^∞-control is one of them and will be applied in this study. This approach is very suitable and easy to utilize for controling K_1 in the feedback path of FEL system.

This section will then be divided into two subsections. The first one will analyze the FEL mathematically and the last one will explain how H^∞-control theory can be applied to such a system.

2.1. Feedback Error Learning(FEL)

Now, the FEL, one of the newest powerful method for solving control tracking problems, is introduced. The FEL, as an adaptive control for two-degree-of-freedom control schemes, was proposed firstly by Kawato and his group[7]. In this control scheme the adaptive controller must become an inverse system of plant. Inversion is a key notion of the FEL method. In practice, most of the plant is not invertible or has an unstable inverse. A method to solve this problem is proposed by using a prefilter to utilize to an invertible system.

2.1.1 Feedback adaptive control method for invertible plant case

The Feedback error learning[3] can be shown in the system diagram Fig. 1. The system is simply composed of three main parts. First, the feedback controller K_1. Second, the plant P. Last, the feedforward controller K_2 which is the most significant part in FEL. The main role that Feedforward controller K_2 plays is to act like the inversion of the plant P. Therefore, the feedforward controller K_2 is actually the inversion of the plant P, which can be consided to be a parameterization of an unknown system. There are many research topics concentrated on this adaptation algorithm[3][4][5].

In Miyamura's paper[3] the following assumptions are given

1. The plant P is stable and has stable inverse P^{-1}.
2. The upper bound of the order of P is known.
3. The high frequency gain

$$k_o = \lim_{s\to\infty} P(s)$$

is assumed to be positive,

and the parameterization of the unknown system K_2 is :

$$\frac{d\xi_1(t)}{dt} = F\xi_1(t) + gr(t) \tag{1}$$

$$\frac{d\xi_2(t)}{dt} = F\xi_2(t) + gu(t) \tag{2}$$

$$u(t) = c^T(t)\xi_1(t) + d^T(t)\xi_2(t) + k(t)r(t) \tag{3}$$

where F is any stable matrix, g is any vector being controllable and ξ is any state vector of the system. $c(t)$, $d(t)$ and $k(t)$ are unknown parameters to be estimated. $u(t)$ is the desired output of the system. The appropriate selection of parameters $c(t) = c_o$, $d(t) = d_o$ and $k(t) = k_o$ can yield an arbitrary transfer function from $r(t)$ to $u(t)$. Let the matrix F and vector g be in a controllable canonical form :

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ -f_1 & -f_2 & -f_3 & \ldots & -f_n \end{bmatrix}, g = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

The transfer function can be rewritten as the following equation :

$$\frac{U(s)}{R(s)} = \frac{k_o s^n + (f_n k_o + c_n)s^{n-1} + \ldots + (f_1 k_o + c_1)}{s^n + (f_n - d_n)s^{n-1} + \ldots + (f_1 - d_1)} \tag{5}$$

$$\begin{aligned} c_o &= [c_1 \quad c_2 \quad \cdots \quad c_n]^T \quad , \\ d_o &= [d_1 \quad d_2 \quad \cdots \quad d_n]^T \end{aligned} \tag{6}$$

Hence, the construction of any transfer function of degree n is adaptively done by continually updating appropriate parameters $c(t)$, $d(t)$ and $k(t)$ to the true values c_o, d_o and k_o, respectively. This is equivalent to tune K_2 to P^{-1}. The stability has already been proved for a system using the following adaptation law :

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \alpha K_1(s)e(t)\xi(t) \tag{7}$$

where

$$\xi(t) := \begin{bmatrix} \xi_1(t)^T & \xi_2(t)^T & r(t) \end{bmatrix}^T,$$

$$\theta(t) := [c(t)^T \quad d(t)^T \quad k(t)^T]^T \ ,$$

and a parameter α is to adjust adaptation speed. The proof says that the FEL algorithm is stable and the error $e(t)$ tends to 0, by choosing *sufficiently large positive constant* K_1.

2.1.2 Feedback adaptive control method for non-invertible plant case

In the previous section, it is assumed that the plant $P(s)$ has a stable inverse $P^{-1}(s)$. But most plants, including DC-servo motors, do not have stable inverses. This section considers the case where $P(s)$ is strictly proper, i.e., $P(s)$ has positive relative degree. This section also proposes a method in dealing with this problem by introducing a prefilter $W(s)$.

When the plant $P(s)$ does not have a stable inverse, an approximated inverse P_a^{-1} is introduced as,

$$P(s)P_a^{-1}(s) = W(s) \tag{8}$$

$$P_a^{-1}(s) = P^{-1}(s)W(s) \tag{9}$$

Using this approximation, the relative degree of $P(s)$, which is the cause of non-invertibility, is compensated by the relative degree of $W(s)$.

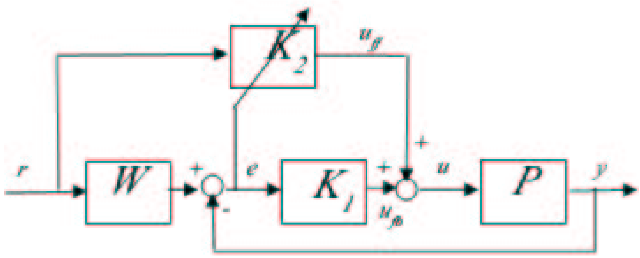

Fig. 2. The system block diagram described by FEL with prefilter

Then, consider the system illustrated in Fig. 2.This section aims to construct $P_a^{-1}(s) = W(s)P^{-1}(s)$ as a feedforward controller by the scheme of the feedback error learning method. In other words, an adaptive scheme, of the case when a part of the adapted controller is known, is proposed. Throughout this section, the following assumptions are made:

Assumptions :

1. The plant P is stable.
2. The upper bound of the order of P is known.
3. The high frequency gain

$$k_o = \lim_{s\to\infty} P(s)$$

is assumed to be positive.

4. Prefilter $W(s)$ is given and known.
5. The upper bound of relative degree of P, i.e. $max\{rd[P(s)]\}$ is known.

The parameterization of the unknown system for feedforward controller K_2 is the same as the previous section.

Since the dynamics of $W(s)$ is known, parameter in K_2 are subject to some constraints. In other words, because of the information from $W(s)$, *the dimension of unknown parameters is reduced.*

To show the constraints in the case of relative degree 1 which corresponds to the concerned servo plant system, $P(s)$ would be written generally as :

$$P(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n} \tag{10}$$

Select a prefilter with relative degree 1 as :

$$W(s) = \frac{v_o}{s + w_1} \tag{11}$$

where w_1 and v_o are known.

Since $P^{-1}W(s)$ is represented by Equation (5), then :

$$\begin{aligned} K(s) &= \frac{U(s)}{R(s)} \\ &= \frac{k_o s^n + (f_n k_o + c_n)s^{n-1} + \ldots + (f_1 k_o + c_1)}{s^n + (f_n - d_n)s^{n-1} + \ldots + (f_1 - d_1)} \\ &= \frac{v_o}{s + w_1} \cdot \frac{s^n + a_1 s^{n-1} + \ldots + a_n}{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n} \end{aligned} \tag{12}$$

Comparing the coefficients of both sides, the following equations are obtained :

$$\begin{aligned} f_1 - d_1 &= \frac{b_n}{b_1} w_1 \\ f_2 - d_2 &= \frac{b_{n-1}}{b_1} w_1 + \frac{b_n}{b_1} \\ &\vdots \\ f_{n-1} - d_{n-1} &= \frac{b_2}{b_1} w_1 + \frac{b_3}{b_1} \\ f_n - d_n &= w_1 + \frac{b_2}{b_1} \end{aligned} \tag{13}$$

From these relations, it is easy to derive the relation :

$$\sum_{k=0}^{n-1} (-w_1)^k (d_{k+1} - f_{k+1}) = (-w_1)^n \tag{14}$$

This relation is written as :

$$[h_o \quad h_1 \quad h_2 \quad \ldots \quad h_{n-1}] \cdot [d(t) - f] = h_n \tag{15}$$

where h_k are defined recursively as :

$$\begin{aligned} h_o &= 1 \\ h_{j+1} &= -w_1 h_j \\ h_m &= 0, m < 0 \end{aligned}$$

where f and d are defined as :

$$\begin{aligned} f &:= [f_1 \quad f_2 \quad f_3 \quad \ldots \quad f_n]^T \\ d &:= [d_1 \quad d_2 \quad d_3 \quad \ldots \quad d_n]^T \end{aligned}$$

The relationship written as Equation (15) tells that one element of $d(t)$ is determined by other elements of $d(t)$, i.e. there exists a function $\xi_1(d_2, d_3, \ldots, d_n)$ such as :

$$d_1(t) = \xi_1(d_2, d_3, \ldots, d_n) \tag{16}$$

So in this case of $rd[P(s)] = 1$, the number of free parameters decreases by one, i.e. the dimensions in which parameters can move decreases by one.

Up to this point, by using the results from the previous section, the construction of an adaptation law and the stability proof can be taken similarly and will be skipped.

2.2. Chain-Scattering Approach to H^∞-Control

2.2.1 Formulation of H^∞-Control

Let the controlled plant be described as :

$$\begin{bmatrix} z \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{17}$$

where $\dim(z) = m,\ \dim(y) = q,\ \dim(w) = r,\ \dim(u) = p$

z : errors to be reduced
y : observation output
w : exogenous input
u : control input

If a controller is given by :

$$u = K\, y$$

The closed-loop system corresponding to this controller is shown in Fig. 3. The closed-loop transfer function is given

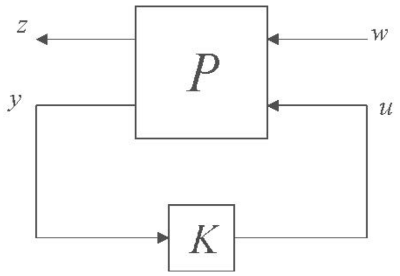

Fig. 3. Closed-Loop System.

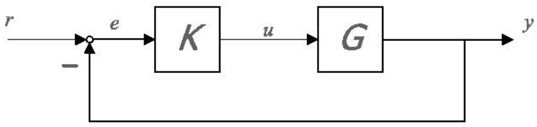

Fig. 4. Unity Feedback Scheme

by :

$$\Phi = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

H^∞-Control Problem is to find a controller K such that the closed-loop system is internally stable and the closed-loop transfer function Φ satisfies

$$||\Phi||_\infty < \gamma \tag{18}$$

for a positive number $\gamma > 0$. In this study, for simplicity, the normalize H^∞problem, where $\gamma = 1$, is considered.

Several Classical synthesis problems of practical importance can be reduced to the H^∞-control problem. These problems are concerned with the synthesis of a controller K of the unity feedback scheme which is obviously analogous to find K_1 in FEL system.

The robust stabilization problem H^∞-control problem are usually concerned with the synthesis of a controller K of the unity feedback scheme described in Fig. 4.

Consider the case where the plant P contains uncertainties in the sense that G is represented by :

$$G(s) = G_0(s) + \Delta(s)W(s) \tag{19}$$

where $G_0(s)$ is a given nominal plant model, $W(s)$ is a given weighting function, and $\Delta(s)$ is an unknown function that is only known to be stable and satisfies :

$$||\Delta||_\infty < 1.$$

The class of plants that can be represented in (19) is often referred to as the plant with *additive unstructured uncertainty.* It is well known that a controller K stabilizes the closed-loop system if and only if K stabilizes G_0 and satisfies :

$$||WQ||_\infty < 1,$$

$$Q := K(I + G_0K)^{-1}$$

The problem is reduced to an H^∞control problem by choosing P in (17) for which Φ in (18) coincides with :

$$WQ = WK(I + G_0K)^{-1}$$

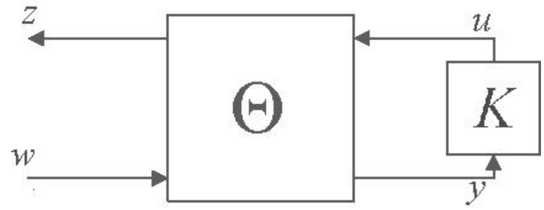

Fig. 5. Chain-Scattering Representation of the Closed-Loop System.

e.g.

$$P = \begin{bmatrix} 0 & W \\ I & -G_0 \end{bmatrix} \tag{20}$$

2.2.2 Chain-Scattering Representations of Plants and H^∞-Control

There are many methods to solve such H^∞-control problems. The chain-scattering is one of the most powerful tools. To deal with this approach, it must be first assumed that P_{21} in (17) is square and invertible. This implies $q = r$. Then, the plant in (17) can also be represented as :

$$\begin{bmatrix} z \\ w \end{bmatrix} = \Theta \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{21}$$

where Θ is the chain-scattering representation of the plant P given by :

$$\begin{aligned} \Theta &= CHAIN(P) \\ &:= \begin{bmatrix} P_{12} - P_{11}P_{21}^{-1}P_{22} & P_{11}P_{21}^{-1} \\ -P_{21}^{-1}P_{22} & P_{21}^{-1} \end{bmatrix} \end{aligned}$$

With the same controller as the previous section, Fig. 5 illustrates the chain-scattering representation of the same system as in Fig. 3. The closed-loop transfer function Φ becomes :

$$\Phi = HM(\Theta; K) := (\Theta_{11}K + \Theta_{12})(\Theta_{21}K + \Theta_{22})^{-1}$$

Note that the symbol HM stands for "Homographic Transformation". The graphical representation of $HM(\Theta; K)$ is just the same as shown in Fig. 5.

Take a look at the H^∞-control problem statement, Φ in (18), now be in terms of Θ, instead of P. To apply H^∞-control theory to the robust stabilization problem, similar to (20), choose Θ such that Φ is coincide with (18),e.g.

$$\Theta = \begin{bmatrix} W & 0 \\ G_0 & I \end{bmatrix}.$$

Then, the following theorem will conveniently be used. Not only to answer whether the controller K exists or not, but this theorem also find out what it is.

Theorem 1: [1] Assume that the plant P has a chain-scattering representation $\Theta = CHAIN(P)$ such that Θ is left invertible and has no poles nor zeros on the $j\omega$-axis. Then, the normalized H^∞ problem is solvable for P if and only if Θ has a $(J\,, J\,')$-lossless factorization[1]

$$G = CHAIN(P) = \Theta\Pi$$

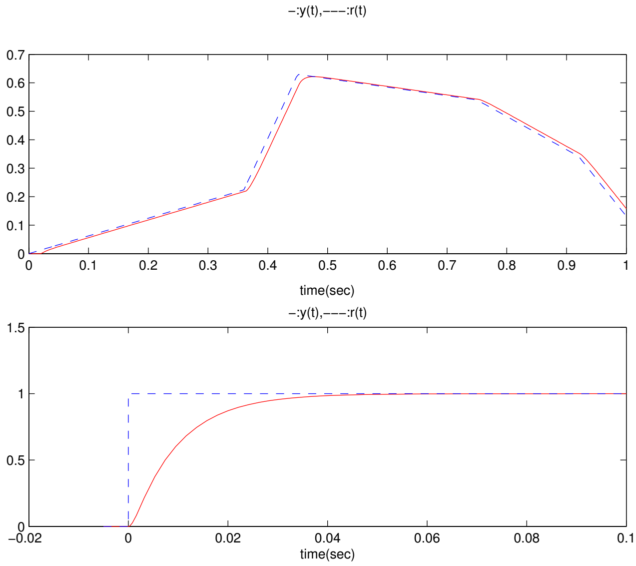

Fig. 6. The result of the system with H^{∞}-controller

where Π is a unimodular matrix. In that case, K is a desired controller if and only if

$$K = HM(\Pi^{-1}; S)$$

for an $S \in \mathbf{BH}_{\infty}^{\mathbf{p}\times\mathbf{r}}$[1].
Now, all mathematical descriptions and proof of convergence for all concerned controlled mechanisms have been explained successfully. In the next section, all of these concepts will be applied in the simulation.

3. Simulation Results

Three main simulations have been done in order to illustrate the improved system. The first simulation was the operation of H^{∞}-controller itself as shown in Fig. 6. In this case, the feedforward path K_2, in Fig. 2, have been removed and the feedback controller K_1 is just replaced with H^{∞}-controller. The second simulation is based on the FEL method where the adaptive law in the feedforward path is the only consideration. In other words, K_1 is just the constant gain chosen to be sufficiently large to guarantee the stability of the system. This simulation diagram is shown in Fig. 7. In comparison of results from the first and second simulation (Fig. 6 and Fig. 7 accordingly), it is observed that the first simulation is more robust to the system disturbance than the second simulation. While, the tracking ability of the second one is clearly better.
For the last simulation,the H^{∞}-controller in the first simulation system is integrated into the feedback path of the original FEL system in the second one. When the system uncertainty is low, the simulation result is almost the same as that of Fig. 7 and will not be shown. However, when the uncertainty of the plant increases to some level, FEL alone is not stable any more while the integrated one has much better stability (see Fig. 8). The tracking performance also remains the same. In another words the FEL with H^{∞}-controller is more robust to the system disturbance while keeping the FEL tracking ability.

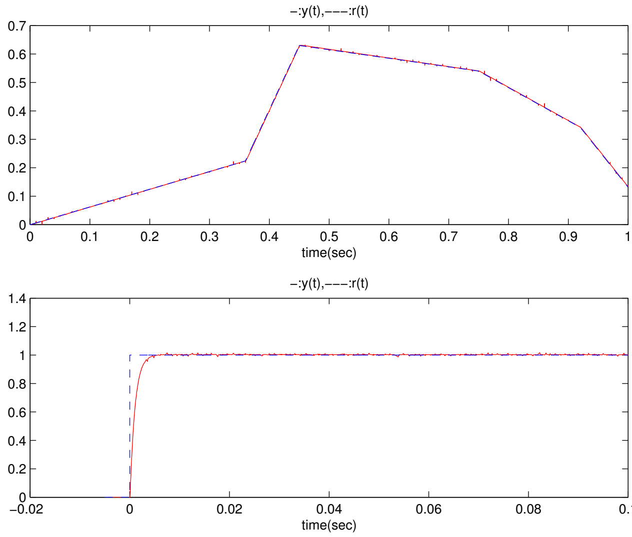

Fig. 7. The result of the FEL system without H^{∞}-controller

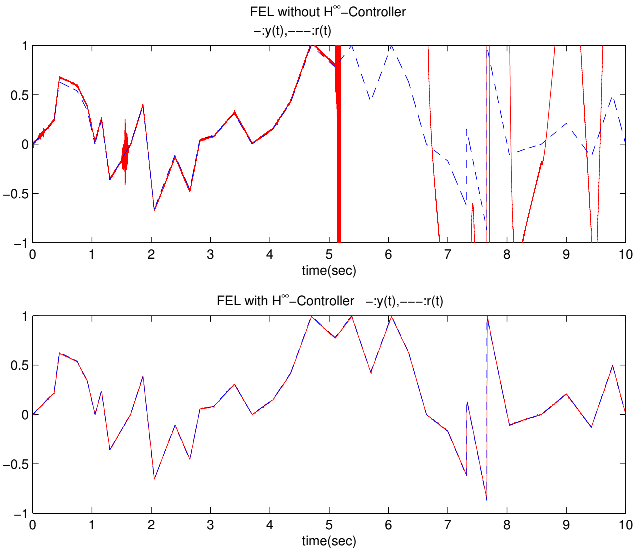

Fig. 8. The Comparison between the FEL system with and without H^{∞}-controller when the system uncertainty is large

4. Conclusions

In this study, it is concerned with the extention of the FEL mechanisms for control DC-servo motor. Firstly, FEL with adaptive learning scheme, in feedforward part, alone is concentrated. Then, the H^{∞}-control theory is applied via the feedback path, to increase the robustness of the controlled system to the uncertainty of the plant. The results are quite satisfactory. The analysis of each can be done separately because the adaptive controller in feedforward path is obviously outside the loop. It is clear that all of the results in this study can be easily applied to another system. Those controlled systems will have a good tracking response together with robustness to system disturbances. Furthermore, FEL itself can also have the ability to solve the instability due to the time-delay[6] in the feedback and feedforward path. This obviously coincides with the human motor control that has very fast responses although the peripheral sensors have very slow rate of transmission response. This time-delay is the key to the new study and opens problems concerning the FEL method.

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