Combining approach in Fault Detection and Isolation for GPS applications

Jaywon Chey*, Gyu-In Jee** and Jang Gyu Lee*

* School of Electrical Engineering and Computer Science, Seoul National University, Seoul, Korea
(Tel: +82-2-880-7317, E-mail: j1chey@korea.com, jgl@snu.ac.kr)
**Department of Electronics Engineering, Kunkuk University, Seoul, Korea
(Tel: +82-2-450-3070; E-mail: gijee@kkucc.kunkuk.ac.kr)

Abstract: GPS is widely used for outdoor positioning in many applications. But it is not suitable for positioning in an obstacle environment such as urban area, tunnels and so on, due to variable signal level. So new technology of the positioning is required to provide the consistent error level regardless of any changes in any environment. Abrupt changes of GPS signal can be detected by various fault detection and isolation methods. Conventional FDI (Fault Detection and Isolation) methods are categorized into two approaches. One approach is the snapshot method that uses measurements only at present step. The other approach is the filtering method that uses measurements stacked from previous step to present step. The FDI result of the snapshot method can be considered reliable independently with previous results and the FDI result of the filtering method is more reliable and detection time is a little longer. Therefore combining approach of two methods is proposed for increasing FDI performance in this paper. Three approaches that are the snapshot method, the filtering method and the combining method are compared to show the probability of correct FDI in simulations. The combining approach presents best result of FDI among them and shows the consistent accuracy irrespective of any changes in outdoor environment.

Keywords: Fault, Detection, Isolation, GPS

1. INTRODUCTION

Navigation system usually employs multiple sensors such as GPS, gyroscope, and accelerometer. GPS is widely used for outdoor positioning in many applications such as land applications, aviation applications and marine applications. But it is not appropriate for the positioning in an obstacle environment such as urban area, tunnels and so on, due to variable signal level. The accuracy of the positioning error is up to the order of tens to hundreds meters when GPS is applied in urban region. Recently the reliability is also considered importantly for positioning with the accuracy. So new technology of the positioning is required to provide the consistent error level regardless of any changes in any environment.

Abrupt changes of GPS signal can be detected by various fault detection and isolation methods. Conventional FDI (Fault Detection and Isolation) methods are categorized into two approaches. One approach is the snapshot method that uses measurements only at present step to decide the existence of a fault [1-2]. The other approach is the filtering method that uses measurements stacked from previous step to present step to determine a fault or not. Residual test is a basic approach in fault detection and isolation. Using the residual that is composed of differences between a measurement and a predicted measurement performs residual test. The predicted measurement is calculated by multiplying observation matrix and predicted estimate of position through least square method. Test statistic is defined by the residual and is compared with predetermined threshold, which is calculated by the probability of false alarm. Fault can be detected with the information of comparison between test statistic and threshold. A fault is detected if test statistic is not less than threshold. In the filtering method, residual test is performed in a same way but predicted estimate of position is produced from filter. The snapshot method uses only present measurement and the result of FDI can be considered reliable independently with previous results. But the filtering method uses many stacked measurements and the result of FDI is more reliable and detection time is a little longer.

It is important to increase FDI performance for increasing reliability with accuracy [3]. Combining these two methods can increase a performance of FDI, that is the probability of correct FDI. Therefore combining approach is proposed for increasing FDI performance in this paper.

The paper is organized as follows. Snapshot method that is a one basic FDI approach is presented in section 2 and filtering method is presented in section 3. Section 4 describes combing method of snapshot method and filtering method and the performance of FDI presented by simulations. A concluding remark is given in section 5.

2. SNAPSHOT METHOD

A linearized measurement equation for GPS including a faulty satellite is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{f} + \boldsymbol{\varepsilon} \tag{1}$$

where n is the number of a satellite measurement from a receiver. $\mathbf{y}$ is a measurement vector (n × 1 vector) and $\mathbf{x}$ is a state vector (4 × 1 vector) that is composed of position and additional receiver clock bias. $\boldsymbol{\varepsilon}$ is a measurement error vector (n × 1 vector) and $\mathbf{H}$ is a observation matrix between the receiver and the satellites (n × 4 matrix). $\mathbf{f}$ is a faulty vector that is zero vector except a faulty satellite's element.

The components of $\boldsymbol{\varepsilon}$ have a Gaussian distribution with zero mean, equal variance and no correlation.

$$\begin{aligned} \mathbf{m}_{\varepsilon} &= E\{\boldsymbol{\varepsilon}\} = \mathbf{0} \\ \mathbf{Q}_{\varepsilon} &= E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\} = \sigma^2 \mathbf{I} \end{aligned} \tag{2}$$

where $\mathbf{T}$ represent the transpose of a matrix or a vector and $\mathbf{I}$ represent the identity matrix.

It is appropriate to normalize the measurement equation with the inverse square root of $E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\}$ which is a n × n matrix. Now define,

$$\mathbf{U} \equiv \mathbf{Q}_{\varepsilon}^{-1/2} = \left(E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}\}\right)^{-1/2} \tag{3}$$

and multiply the measurement equation (1) by it.

$$\mathbf{Uy} = \mathbf{UHx} + \mathbf{Uf} + \mathbf{U\varepsilon} \tag{4}$$

Therefore we redefine the measurement equation using new vectors $\mathbf{y_u}$ and $\mathbf{\varepsilon_u}$ and matrix $\mathbf{H_u}$. $\mathbf{y_u}$ is a remeasurement vector that is composed of original measurement vector multiplied by $\mathbf{U}$ matrix and $\mathbf{\varepsilon_u}$ is a remeasurement error vector that is composed of original measurement error vector multiplied by $\mathbf{U}$ matrix. $\mathbf{H_u}$ is a new observation matrix that is composed of original observation matrix multiplied by $\mathbf{U}$ matrix. In addition, $\mathbf{f_u}$ is a new faulty vector that is composed of original faulty vector multiplied by $\mathbf{U}$ matrix.

$$\mathbf{y_u} = \mathbf{H_u x} + \mathbf{f_u} + \mathbf{\varepsilon_u} \tag{5}$$

Therefore the components of $\mathbf{\varepsilon_u}$ have a Gaussian distribution with zero mean and unity variance

$$\begin{aligned} \mathbf{m}_{\varepsilon_u} &= \mathbf{E}\{\mathbf{\varepsilon_u}\} = \mathbf{0} \\ \mathbf{Q}_{\varepsilon_u} &= \mathbf{E}\{\mathbf{\varepsilon_u \varepsilon_u}^{\mathbf{T}}\} = \mathbf{I} \end{aligned} \tag{6}$$

We abbreviate a subscript $\mathbf{u}$ in Eqs. (5) ~ (6) for a simpler form derivation afterwards.

Three methods have been widely used for GPS fault detection. First method is the range-comparison method suggested by Lee and second method is the least-squares-residual method suggested by Parkinson and Axelrad. Third method is the parity space method suggested by Sturza and A. Brown [4]. All three methods are snapshot schemes that use measurements only at a sample point in time and assume that redundant measurements are available. A fault is detected using redundant measurement information when more than four satellites are visible and making consistency checks between measurement information. In this paper we deal with only least-square-residual method.

We can use the predicted estimate of the position solution by the least squares method to make the predicted estimate of the measurements after assuming that there is no fault.

$$\hat{\mathbf{x}} = \left(\mathbf{H^T H}\right)^{-1} \mathbf{H^T y} \tag{7}$$

$$\hat{\mathbf{y}} = \mathbf{H}\hat{\mathbf{x}} = \mathbf{H}\left(\mathbf{H^T H}\right)^{-1} \mathbf{H^T y} \tag{8}$$

And the residual vector that is obtained by subtracting the measurement vector and the predicted estimate vector of the measurement has the linear relationship with the measurement vector and measurement error vector.

$$\begin{aligned} \mathbf{w} &= \mathbf{y} - \hat{\mathbf{y}} \\ &= \left[\mathbf{I} - \mathbf{H(H^T H)^{-1} H^T}\right]\mathbf{y} \\ &= \left[\mathbf{I} - \mathbf{H(H^T H)^{-1} H^T}\right]\mathbf{\varepsilon} \\ &= \mathbf{S\varepsilon} \end{aligned} \tag{9}$$

where $\mathbf{S} = \left[\mathbf{I} - \mathbf{H(H^T H)^{-1} H^T}\right]$ (10)

Therefore it has a Gaussian distribution with zero mean and variance $\mathbf{S}$.

$$\begin{aligned} \mathbf{m_w} &= E\{\mathbf{w}\} = \mathbf{0} \\ \mathbf{Q_w} &= E\{\mathbf{ww^T}\} \\ &= E\left\{\mathbf{S\varepsilon\varepsilon^T S^T}\right\} \\ &= \mathbf{S}E\left\{\mathbf{\varepsilon\varepsilon^T}\right\}\mathbf{S^T} \\ &= \mathbf{SS^T} \\ &= \mathbf{S} \end{aligned} \tag{11}$$

We define test statistic using the residual vector like Eq. (13). So it has the property of the nonnegative value that is compared with the threshold to check a fault.

$$\begin{aligned} {TS_{LS}}^2 &= \mathbf{w^T Q_w}^{-1}\mathbf{w} \\ &= \mathbf{\varepsilon^T S^T S^{-1} S\varepsilon} \\ &= \mathbf{\varepsilon^T S^T \varepsilon} \\ &= \mathbf{\varepsilon^T S\varepsilon} \end{aligned} \tag{12}$$

This test statistic is compared with the threshold that is calculated by the probability of false alarm. Fault is detected if test statistic is not less than threshold.

Detection time is fast and independent reliability is guaranteed since snapshot method detects a fault in one step. Limitation of snapshot method is that FDI performance by using only one-step information is relatively conservative.

3. FILTERING METHOD

Filtering method uses many measurements instead of using only one-step measurement. Residual test is also used in same way with snapshot method except predicted estimate of position is produced from filter. Kalman filter is usually used for GPS applications. Standalone GPS dynamic models for Kalman filtering uses extended Kalman filter. There are three models that are position model, position velocity model, and position velocity acceleration model. Appropriate model is selected to reflect observer's dynamic situation. Position model is selected in case that observer is near stationary and position velocity model is selected in case that observer is not stationary but moving with nearly constant velocity. Position velocity acceleration model is selected when assumption of near constant velocity is not hold.

The PV dynamic process can be described by the following vector differential equation [5]

$$\dot{\mathbf{x}} = \mathbf{Fx} + \mathbf{u} \tag{13}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ u_2 \\ 0 \\ u_4 \\ 0 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix} \tag{14}$$

where x_1 and x_2 represent east position and east velocity. x_3 and x_4 represent north position and north velocity. x_5 and x_6 represent altitude and altitude rate. x_7 and x_8 represent clock range bias error and clock range drift error.

The process noise covariance matrix is as follows

$$\mathbf{Q} = \begin{bmatrix} S_p\frac{\Delta t^3}{3} & S_p\frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ S_p\frac{\Delta t^2}{2} & S_p\Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_p\frac{\Delta t^3}{3} & S_p\frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_p\frac{\Delta t^2}{2} & S_p\Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_p\frac{\Delta t^3}{3} & S_p\frac{\Delta t^2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_p\frac{\Delta t^2}{2} & S_p\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_f\Delta t + S_g\frac{\Delta t^3}{2} & S_g\frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_g\frac{\Delta t^2}{2} & S_g\Delta t \end{bmatrix} \tag{15}$$

and the state transition matrix can be derived as

$$\boldsymbol{\varphi} = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

Detection time is a little longer and FDI result is more reliable than the snapshot method. But limitation of filtering method is that filter tends to be corrupted with a fault if this fault is not detected in proper time. Independent reliability is not guaranteed in the filtering method.

4. COMBINING METHOD

The limitation of each FDI method is mentioned in last end of section 2 and section 3. The limitation of snapshot method is that FDI performance is relatively conservative and the limitation of filtering method is that filter tends to be corrupted with an undetected fault. Snapshot method and filtering method has a limitation but complementary limitation. Therefore combining these two methods to complement each limitation will show better performance than any of them.

Simulation is performed to find out the FDI performance of three methods that is snapshot method, filtering method and combining method. After FDI process of three methods, horizontal position error is compared to consider reliability and accuracy.

Simulation is iterated for 100 times and mean error is presented. Simulation condition is as follows. Position of observer is Seoul, Korea and number of visible satellite is 8 for 100 seconds that is simulation time. The probability of false alarm is 0.001. The mask angle set to 5 degrees. Added error to pseudorange as a fault is represented in Fig. 1. Faulty error is injected from 30 seconds to 80 seconds. Maximum magnitude of it is 100 meters.

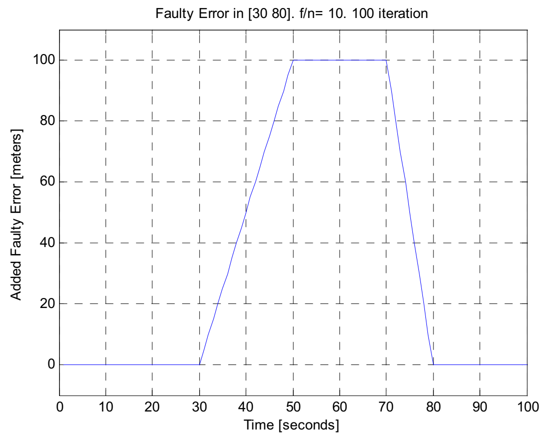

Fig. 1 Faulty error

Simulation result is presented in Fig. 2. In this figure solid line represents the positioning error after applying snapshot FDI method and dotted line represents the positioning error after applying filtering FDI method. Dashed line represents positioning error after applying combining FDI method of snapshot method and filtering method.

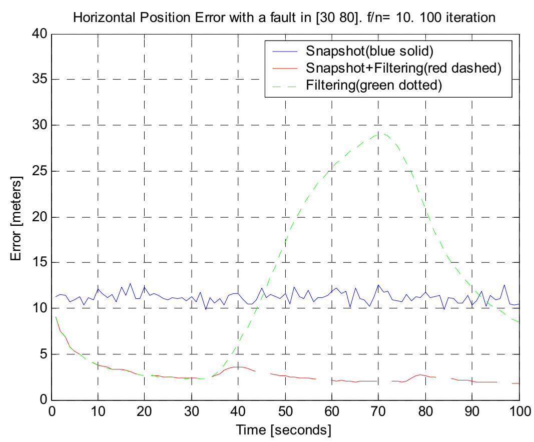

Fig. 2 Horizontal Position error

Combining method show best performance of three methods from simulation result. Comparing the result of snapshot method and filtering method, filtering method is better for time zone of no fault and snapshot method is better for time zone of a fault. This fault is not detected in proper time to recover from corrupting filter state. So there is long time to recover in the result of filtering method.

5. CONCLUSION

Conventional two FDI methods are presented in this paper. One FDI method is the snapshot method that uses only one present measurement. The other FDI method is the filtering method that uses many measurements. FDI performance of them is presented and limitation of them is also mentioned. The FDI result of the snapshot method can be considered reliable independently with previous results and the FDI result of the filtering method is more reliable and detection time is a little longer. The limitation of snapshot method is that FDI performance is relatively conservative and the limitation of filtering method is that filter tends to be corrupted with an undetected fault.

Therefore combining these two methods to complement each limitation is proposed for increasing FDI performance and accuracy performance in this paper. Three approaches that are the snapshot method, the filtering method and the combining method are compared in simulations. The combining approach presents best result among them and shows the consistent accuracy irrespective of any changes.

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