On Feedback Linearization of Nonlinear Time-Delay Systems

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Abstract: We propose a result on the stabilization of nonlinear time-delay systems via the feedback linearization method. Using the predictor based control and the parametric coordinate transformation, we introduce a stabilizing controller to compensate time delay. Specifically, we present the delay-dependent stability analysis to makes the considered system stable. Also, an illustrative example is provided

Keywords: feedback linearization, time delay, reduction method, parametric coordinate transformation.

1. Introduction

Feedback linearization method is recognized as useful tool for designing nonlinear systems. Also, feedback linearization requires the accurate plant model to achieve exact linearization of the closed loop system. However, there exist inevitable time delays in many engineering systems. In other words, in real world, there are time delays because of transport process, computation process and other effects. In particular, an input time delay is one of the most common forms of time delay. It is well known that the existence of time delay degrades the controller performance. Worst of all, a time delay makes systems unstable. The research of control theory for time delay systems has established various types of approaches for linear systems with time delays. However, the nonlinear time delay systems was rarely analyzed. Thus, the time delay problem still remains as open problems [1]. In [2], the Smith predictor is usually used to improve the control performance of linear time delay systems. Thus, the Smith predictor framework is not available for nonlinear systems. The control scheme for nonlinear time-delay systems has been presented [3],[4]. In [5], using the extended Lie derivative for functional differential equations, the nonlinear systems with the state delay have been analyzed. Several theses deal with the control problem of the time delay systems via predictor based controller [6]-[8]. Therefore, a time delay system can be transformed into a delay free system. An input delay system based on a transformation that converts the original system into a delay free form. It is shown that the asymptotically stability of the transformed system guarantees the asymptotically stability of the original system.

In this paper, we use a predictor type transformation and the parametric coordinate transformation to compensate time delay. Also, based on the Lyapunov-based approach and the improved Razumikhin theorem, we propose a stabilizing controller and a stability analysis for the feedback linearizable system with time delay.

2. Preliminary

Consider the following single-input nonlinear system :

$$\dot{x}(t) = f(x(t), x(t-\tau)) + g(x(t))u(t-\tau) \tag{1}$$

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with $x \in R^n$ and $(f(x(t), x(t-\tau)), g(x(t)))$ being feedback linearizable where $f(x(t), x(t-\tau))$, $g(x(t))$ are smooth functions defined in a domain $D_x \subset R^n$ that contains the origin. Without loss of generality, it is assumed that $f(0) = 0$. The initial conditions are given as $x(0) = x_0$ and $x_0(\theta) = \phi(\theta)$, $-\tau \le \theta \le 0$ where $x_t(\theta) = x(t+\theta)$.

From an extension of the Lie derivative [5], the derivative of $\bar{\Phi}(x(t), x(t-\tau))$ along $f(x(t), x(t-\tau))$ is defined as

$$L_f \bar{\Phi}(x(t), x(t-\tau)) = \frac{\partial \bar{\Phi}}{\partial x(t)} f + \frac{\partial \bar{\Phi}}{\partial x(t-\tau)} f(\cdot) \tag{2}$$

where $f(\cdot) = f(x(t-\tau), x(t-2\tau))$. Since $L_f \bar{\Phi}(x(t), x(t-\tau))$ is a real-valued function with time delays, this operation can be respected for higher orders as

$$L_f^{k+1} \bar{\Phi}(x(t), x(t-\tau)) = \frac{\partial L_f^k \bar{\Phi}}{\partial x(t)} f + \frac{\partial L_f^k \bar{\Phi}}{\partial x(t-\tau)} f(\cdot) \tag{3}$$

where $f(\cdot) = f(x(t-\tau), x(t-2\tau))$. This operator is called the delayed state derivative.

From a coordinate transformation $z(t) = \bar{\Phi}(x(t), x(t-\tau))$, the system (1) is transformed into the following form

$$\begin{aligned} \dot{z}_i(t) &= z_{i+1}(t), \ for \ i = 1, ..., n-1 \\ \dot{z}_n(t) &= \beta^{-1}(z(t), z(t-\tau)) \\ &\quad \times [u(t-\tau) - \alpha(z(t), z(t-\tau))] \end{aligned} \tag{4}$$

where $\beta^{-1}(z(t), z(t-\tau)) = L_g L_f^{n-1} z(t)$ and $\alpha(z(t), z(t-\tau)) = -\frac{L_f^n z(t)}{L_g L_f^{n-1} z(t)}$. Moreover, the term $\alpha(z(t), z(t-\tau))$ can be written as $\alpha(z(t), z(t-\tau)) = \alpha_1(z(t), z(t-\tau)) + \alpha_2(z(t-\tau))$.

Assumption 1: There exist a nonzero constant m_0 and positive constants m_1, m_2 such that

$$\begin{aligned} L_g L_f^{n-1} z(t) &= m_0^{-1} \\ |\alpha_1(z(t), z(t-\tau))| &\le m_1 \sum_{k=1}^{n-1} |z_k(t)| + m_2 \sum_{k=1}^{n-1} |z_k(t-\tau)| \end{aligned}$$

Note that all norms are used in the sense of Euclidean 2-norm in this paper.

3. Main Results

From Assumption 1, the system (4) is written as

$$\begin{aligned} \dot{z}_i(t) &= z_{i+1}(t), \ for \ i = 1, ..., n-1 \\ \dot{z}_n(t) &= m_0^{-1} [u(t-\tau) - \alpha(z(t), z(t-\tau))] \end{aligned} \tag{5}$$

We introduce the control input $u(t) = u_1(t) + u_2(t)$. Using a linear transformation based on the reduction method [6], we obtain a simple transformation as follows:

$$\bar{z}(t) = z(t) + \int_{t-\tau}^{t} B_c m_0^{-1} u_1(\theta) d\theta \qquad (6)$$

Then, the system (5) is transformed to

$$\begin{aligned} \dot{\bar{z}}_i(t) &= \bar{z}_{i+1}(t),\ for\ i = 1, ..., n-2 \\ \dot{\bar{z}}_{n-1}(t) &= \bar{z}_n(t) - \int_{t-\tau}^{t} m_0^{-1} u_1(\theta) d\theta \\ \dot{\bar{z}}_n(t) &= m_0^{-1}[u_1(t) + u_2(t-\tau) \\ &\quad -\alpha_1(z(t), z(t-\tau)) - \alpha_2(z(t-\tau))] \qquad (7) \end{aligned}$$

Also, we introduce the control law $u_1(t) = \alpha_1(z(t), z(t-\tau)) + m_0 v(t)$. Then, the system (7) is obtained as follows

$$\begin{aligned} \dot{\bar{z}}_i(t) &= \bar{z}_{i+1}(t),\ for\ i = 1, \cdots, n-2 \\ \dot{\bar{z}}_{n-1}(t) &= \bar{z}_n(t) - \int_{t-\tau}^{t} (m_0^{-1}\alpha_1(z(\theta), z(\theta-\tau)) + v(\theta)) d\theta \\ \dot{\bar{z}}_n(t) &= v(t) + u_2(t-\tau) - \alpha_2(z(t-\tau)) \qquad (8) \end{aligned}$$

Using $u_2(t-\tau) = \alpha_2(z(t-\tau))$, the system (8) is written as follows

$$\begin{aligned} \dot{\bar{z}}(t) &= A_c \bar{z}(t) + B_c v(t) \\ &\quad -B[\int_{t-\tau}^{t} (m_0^{-1}\alpha_1(z(\theta), z(\theta-\tau)) + v(\theta)) d\theta] \qquad (9) \end{aligned}$$

where (A_c, B_c) is the Brunovsky controllable pair and $B = [0\ \cdots\ 0\ 1\ 0]^T$.

The parametric coordinate transformations are defined as [3]:

$$w_i(t) = \rho^{i-1} \bar{z}(t),\ i = 1, 2, \cdots, n \qquad (10)$$

for a positive constant $\rho \geq 1$. Then, we obtain the following equation:

$$\begin{aligned} \rho \dot{w}(t) &= A_c w(t) + \rho^n B_c v(t) - \rho^{n-1} B \\ &\quad \times[\int_{t-\tau}^{t} (m_0^{-1}\alpha_1(z(\theta), z(\theta-\tau)) + v(\theta)) d\theta] \qquad (11) \end{aligned}$$

Also, using the control law $v(t) = \rho^{-n} \sum_{i=1}^{n} k_i w_i(t)$,

$$\begin{aligned} \rho \dot{w}(t) &= A_c w(t) + B_c K w(t) - \rho^{n-1} B \\ &\quad \times[\int_{t-\tau}^{t} (m_0^{-1}\alpha_1(z(\theta), z(\theta-\tau)) + v(\theta)) d\theta] \qquad (12) \end{aligned}$$

where $K = [k_1,\ k_2,\ \cdots\ ,\ k_n]$.

From (10) and (12),

$$v(t) = \bar{K}(k, \rho) \bar{z}(t) \qquad (13)$$

where $\bar{K}(k, \rho) = [\rho^{-n} k_1,\ \rho^{1-n} k_2,\ \cdots\ ,\ \rho^{-1} k_n]$.

Let, P and Q be the positive-definite, symmetric matrices such that $(A_c + B_c K)^T P + P(A_c + B_c K) = -Q$ where K is chosen so that $(A_c + B_c K)$ is Hurwitz. Also, let $\sigma = \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}$.

Theorem 1: The control laws are defined as

$$\begin{cases} u_1(t) = \alpha_1(z(t), z(t-\tau)) + m_0 \bar{K}(k, \rho) \bar{z}(t) \\ u_2(t) = \alpha_2(z(t)) \end{cases} \qquad (14)$$

where $\bar{K}(k, \rho) = [\rho^{-n} k_1,\ \rho^{1-n} k_2,\ \cdots\ ,\ \rho^{-1} k_n]$.

The closed-loop system (7) with the control laws (14) is asymptotically stable if the following condition is satisfied

$$\tau < \frac{\sigma}{\rho^{n-1} m_0^{-1}(m_1 + m_2) + \rho^{-1}\|K\|} \qquad (15)$$

Proof: Let $V(w(t)) = \rho w^T(t) P w(t)$ be a Lyapunov function where $(A_c + B_c K)^T P + P(A_c + B_c K) = -Q$. Then,

$$\begin{aligned} \dot{V} &= \rho \dot{w}^T(t) P w(t) + w^T(t) P \rho \dot{w}(t) \\ &= -w^T(t) Q w(t) \\ &\quad -2w^T(t) P B[\int_{t-\tau}^{t} (a^{-1}\alpha_1(z(\theta), z(\theta-\tau)) + v(\theta)) d\theta] \end{aligned}$$

From (6), we can obtain as follows $[\bar{z}_1(t),\ \bar{z}_2(t),\ \cdots,\ \bar{z}_n(t)] = [z_1(t),\ z_2(t),\ \cdots,\ z_n(t) + \int_{t-\tau}^{t} m_0^{-1} u_1(\theta) d\theta]$. Also, from the equation (10) and the improved Razumikhin Theorem in [9], $\|\bar{z}_t(\theta)\| \leq q\|\bar{z}(t)\|,\ q > 1,\ -\tau \leq \theta \leq 0$, then we obtain

$$\begin{aligned} \|\alpha_1(z(t), z(t-\tau))\| &\leq m_1\|\bar{z}(t)\| + m_2\|\bar{z}(t-\tau)\| \\ &\leq (m_1 + q m_2)\|\bar{z}(t)\| \\ &\leq (m_1 + q m_2)\|w(t)\| \end{aligned}$$

Also, since $\|v(t)\| \leq \rho^{-n}\|K\|\|w(t)\|$, we obtain

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(Q)\|w(t)\|^2 + 2\rho^{n-1}\|w(t)\|\|P\| \\ &\quad \times[\int_{t-\tau}^{t} (m_0^{-1}(m_1 + q m_2)\|w(\theta)\| + v(\theta)) d\theta] \\ &\leq -\lambda_{min}(Q)\|w(t)\|^2 + 2\tau\|w(t)\|\|P\| \\ &\quad \times[\rho^{n-1} m_0^{-1}(m_1 + q m_2)\|w_t(\theta)\| + \rho^{-1}\|K\|\|w_t(\theta)\|] \end{aligned}$$

Again, from the improved Razumikhin Theorem, $\|w_t(\theta)\| \leq q\|w(t)\|,\ q > 1,\ -\tau \leq \theta \leq 0$, then we obtain

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(Q)\|w(t)\|^2 \\ &\quad +2\tau q[\rho^{n-1} m_0^{-1}(m_1 + q m_2) + \rho^{-1}\|K\|]\|P\|\|w(t)\|^2 \\ &= -b\|w(t)\|^2 \end{aligned}$$

where $b = \lambda_{min}(Q) - 2\tau q[\rho^{n-1} m_0^{-1}(m_1 + q m_2) + \rho^{-1}\|K\|]\|P\|$.

If the inequality (15) is satisfied, $q > 1$ exists such that $b > 0$. According to the improved Razumikhin Theorem [9], the system (7) is asymptotically stable. □

From (6), $\bar{z}(t) \to 0$ implies $z(t) \to 0$. Thus, the system (5) is asymptotically stable.

Remark 1: As the tuning parameter ρ is getting larger, the term the term $\rho^{-1}\|K\|$ can be effectively suppressed. However, the term $\rho^{n-1}(m_1 + m_2)$ will become large as a large value of the tuning parameter ρ. Thus, the tuning parameter ρ should be appropriately chosen.

4. Simulation Results

Example 1: Consider the following nonlinear system :

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_1^2(t-\tau) - 0.1x_1(t)\sin x_2(t-\tau) + u(t-\tau) \end{aligned}$$

where $\tau = 0.25$.

Let $z_1 = x_1$ and $z_2 = x_2$, then the control laws are defined as

$$\begin{cases} u_1(t) = 0.1z_1(t)\sin z_2(t-\tau) + \bar{K}(k,\rho)\bar{z}(t) \\ u_2(t) = -z_1^2(t) \end{cases} \tag{16}$$

where $\bar{K}(k,\rho) = [\rho^{-2}k_1,\ \rho^{-1}k_2]$. We select $\rho = 10$, $k_1 = -5$ and $k_2 = -6$.

For comparison, we use the conventional controller with the same parameter values [5]. As shown in Figs. 1-2, the proposed controller makes the considered system asymptotically stable while the conventional controller makes unstable.

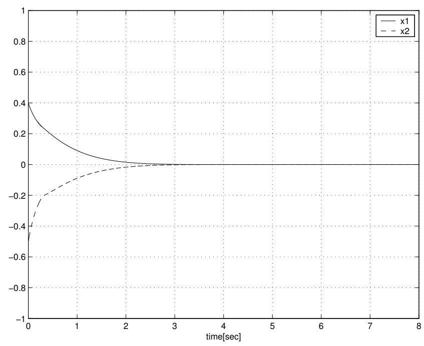

Figure 1: Result via the proposed controller

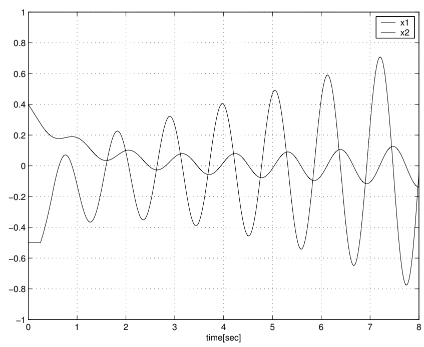

Figure 2: Result via the conventional controller [5]

5. Conclusions

In this paper, we propose a controller for a class of feedback linearizable nonlinear systems with time delay. To compensate time delay, we introduce the stabilizing controller based on the predictor type transformation and the parametric coordinate transformation. Specifically, we derive the delay-dependent sufficient condition to guarantee the stability for the considered system.

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