CDM Controller Incorporating Friction Compensation for Rotational Inverted Pendulum

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Abstract: A controller designed by CDM for a servo type system which is an augmented system constructed from a rotational inverted pendulum with an integrator added to its arm, is presented in this paper. In order to be able to apply the CDM concept, the augmented system must be linearized and converted into controllable canonical form. Then, the controller consisting of the state feedback gain matrix and an integral gain in the sense of CDM can be obtained. This shows that design procedure for the proposed controller is easy. The experimental results obtained from the rotational inverted pendulum controlled by the proposed controller show that the system response has no steady-state error, however, the oscillation amplitude of the arm angle is still significant. Therefore, in this paper, the friction compensation using Coulomb friction with stiction is also added to the controller. The oscillation amplitude of the arm angle that can be reduced remarkably is also shown in the experimental results.

Keywords: Rotational inverted pendulum, coefficient diagram method, state feedback, friction compensation

1. INTRODUCTION

Inverted pendulum is a famous tool for testing the effectiveness of many control schemes. Owing to their nonlinearity and unstable characteristic, the controller development had been a great interest of many researchers [1-6]. So far, many controllers had been implemented either linear or nonlinear controllers. The nonlinear controllers guarantee a wide range operation and overcome the hard nonlinearity [2-3]. In spite of having some drawbacks, a linear controller, however, is easier to be designed and implemented [4-6]. As proposed in [4], a linear controller based on linear quadratic regulator (LQR) with an integrator augmented to the rotating arm angle can satisfy the required specification. The integrator was needed to reject the steady-state error in controlling the inverted pendulum system due to the noise generated by the hardware. Unfortunately the choice in selecting the proper weighting matrix was still trial and error. Furthermore, when the inverted pendulum is linearized by neglecting the friction simply, it will lead to limit cycles, which implies to somewhat an oscillatory result [7-8]. As reported in [9], coefficient diagram method (CDM) can satisfy time domain specification and the design is simple. In CDM the stability and speed of the closed-loop system are related to the stability index and the equivalent time constant respectively. Then, the desired characteristic polynomial based on these parameters can be composed.

In this paper, a design of a controller to stabilize the inverted pendulum in upright position while maintaining the arm position angle in certain position using CDM will be presented. As the responses exhibit significant oscillation, friction compensation using Coulomb friction with stiction will also be introduced.

The rotational inverted pendulum shown in Fig. 2 is a SIMO system with motor torque input and two outputs i.e. the pendulum angle θ and the arm angle β. By employing the Newton-Euler formulation, a nonlinear model of the inverted pendulum system can be obtained. As a linear controller will be designed, the model must be linearized about upright position. After representing the linear model including one augmented integrator in state space form, it will be transformed into controllable canonical form utilizing a transformation matrix [10] so that the CDM concept can be applied. Then, each element of the state feedback gain matrix and the integral gain can be designed by matching the closed-loop characteristic polynomial of the system to those obtained from the CDM concept.

The experimental results of the proposed control system with and without friction compensation are also shown.

2. PLANT AND CDM CONCEPT

2.1 Overview of the plant

After applying Newton-Euler formulation, the inverted pendulum model is derived as the nonlinear equation as follows [4]

$$\dot{x} = f\left(x(t)\right) + g\left(x(t)\right)u\,, \tag{1}$$

where

$$f(x) = \begin{bmatrix} x_2 \\ -\left(\dfrac{R\cos(x_1)}{mlR^2\sin^2 x_1 + Jl}\right)\left(mlRx_2^2\sin x_1 - bx_4 - \dfrac{\left(J+mR^2\right)g\sin x_1}{R\cos x_1}\right) \\ x_4 \\ \dfrac{1}{J+mR^2\sin^2 x_1}\left(-mRg\cos x_1\sin x_1 + mlRx_2^2\sin x_1 - bx_4\right) \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 \\ -\left(\dfrac{R\cos(x_1)}{mlR^2\sin^2 x_1 + Jl}\right) \\ 0 \\ \dfrac{1}{J+mR^2\sin^2 x_1} \end{bmatrix},$$

and where $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \theta & \dot{\theta} & \beta & \dot{\beta} \end{bmatrix}^T$, $u = \tau_m$, τ_m is the torque applied to the pivot, θ is the pendulum angle, β is the arm angle, m is the mass of the pendulum, l is the distance from the pivot point to the center of mass of the pendulum, R is the length of the rotating arm, J and b are the moment of inertia of the rotating arm and the pivot's friction coefficient respectively.

The equilibrium points which satisfy the following equation

$$0 = f(x(t)) + g(x(t))u \tag{2}$$

are $x_{ss} = [0 \quad 0 \quad c \quad 0]^T$ or $x_{ss} = [\pi \quad 0 \quad c \quad 0]^T$ and $u_{ss} = 0$ where c is any constant. First equilibrium point corresponds to the upright position which is unstable, while the second is the hanging position. Linearizing about its upright position we have

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{Jg + mgR^2}{Jl} & 0 & 0 & \dfrac{bR}{Jl} \\ 0 & 0 & 0 & 1 \\ \dfrac{-mgR}{J} & 0 & 0 & \dfrac{-b}{J} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ \dfrac{-R}{Jl} \\ 0 \\ \dfrac{1}{J} \end{bmatrix}.$$

As our main interests are the arm angle β and the pendulum angle θ, the output equation is

$$y(t) = Cx(t) , \tag{4}$$

where $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

2.2 Concept of CDM

In CDM, the characteristic polynomial is given in the following form

$$P(s) = a_n s^n + \cdots + a_1 s^1 + a_0 = \sum_{i=0}^{n} a_i s^i . \tag{5}$$

Based on Eq. (5) the performance specification known as stability index γ_i, equivalent time constant τ and stability limit γ_i^* can be synthesized as these equations

$$\gamma_i = \frac{a_i^2}{a_{i-1}a_{i+1}}, \qquad (i = 1, 2, \cdots, n-1) \tag{6}$$

$$\tau = \frac{a_1}{a_0} , \tag{7}$$

$$\gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}} , \tag{8}$$

where $i = 1 \sim n-1, \quad \gamma_1 = \gamma_n = \infty$.

Then the characteristic polynomial in term of γ_i, τ and a_0 can be expressed back as follows

$$P_m(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] . \tag{9}$$

The choice of stability index γ_i due to the control design specifications must satisfy the following inequality

$$\gamma_i > 1.5\gamma_i^* , \tag{10}$$

and τ normally can be chosen from settling time specification as

$$\tau = t_s / (2.5 \sim 3) . \tag{11}$$

However, in general the stability index known as standard stability index is recommended as

$$\gamma_{n-1} = ... = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5 . \tag{12}$$

3. CONTROL SYSTEM STRUCTURE

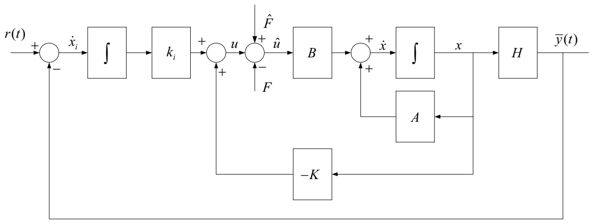


Fig. 1 Control system structure.

In this section, the CDM controller design procedure and friction compensation will be described respectively. Since an integrator is added to the arm of rotational inverted pendulum system for rejecting the steady-state error, the augmented system can be constructed as shown in Fig.1, and

$$\dot{x}_a(t) = A_a x_a(t) + B_a u(t) + Gr(t) \tag{13}$$

$$y(t) = C_a x_a(t) , \tag{14}$$

$$\dot{x}_i(t) = r(t) - Hx(t) \tag{15}$$

can also be obtained, where

$$x_a = \begin{bmatrix} x \\ x_i \end{bmatrix}, A_a = \begin{bmatrix} A & 0 \\ -H & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } C_a = \begin{bmatrix} C \\ 0 \end{bmatrix}^T$$

and where $r(t)$ is the reference signal to the arm angle, $H = [0 \quad 0 \quad 1 \quad 0]$ is derived from second row of C matrix and $x_i(t)$ is the state variable obtained by augmenting an integrator to the arm angle.

If the pair of A and B in Eq. (3) is controllable and $A_a = \begin{bmatrix} A & B \\ -H & 0 \end{bmatrix}$ is full rank, then the augmented system is completely state controllable. Therefore, the control law $u(t)$ can be assigned as

$$u(t) = -K_a x_a(t) \tag{16}$$

where $K_a = [K \quad -k_i]$, and where $K = [k_1 \quad k_2 \quad ... \quad k_{n-2} \quad k_{n-1}]$ is the state feedback gains matrix and $k_i = -k_n$ is the integral gain. Then the following relation can be derived as

$$\dot{x}_a(t) = (A_a - B_a K_a) x_a(t) + Gr(t) = A_{cl} x_a(t) + Gr(t) . \tag{17}$$

3.1 CDM controller

In this sub-section, the design procedure for assigning the feedback gain matrix K and integral gain k_i of the system shown in Fig. 1 by CDM is proposed. It can be done by matching the closed-loop characteristic polynomial of Eq. (17) to the characteristic polynomial obtained from CDM as the following procedure:

1. Transform the closed-loop system (17) into controllable canonical form as

$$\dot{z}(t) = T^{-1}A_{cl}Tz(t) + Gr(t) \quad (18)$$

by introducing a new state $z(t) = T^{-1}x_a(t)$. The transformation matrix T is defined as $T = MW$, where M and W are given by [10]

$$M = \begin{bmatrix} B_a & A_aB_a & A_a^2B_a & \cdots & A_a^{n-1}B_a \end{bmatrix}$$

$$W = \begin{bmatrix} \delta_1 & \delta_2 & \cdots & \delta_{n-1} & 1 \\ \delta_2 & \delta_3 & \cdots & 1 & 0 \\ \vdots & \vdots & & 0 & 0 \\ \delta_{n-1} & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

and where $\delta_{n-1}, \delta_{n-2}, ..., \delta_1$ are the coefficient of the open-loop characteristic polynomial

$$P_o(s) = |sI - A_a| = s^n + \delta_{n-1}s^{n-1} + \ldots + \delta_1 s + \delta_0 \ .$$

2. Find the closed-loop characteristic polynomial of system (18) as

$$P_o(s) = |sI - T^{-1}A_{cl}T|$$
$$= s^n + (\delta_{n-1} + \hat{k}_n)s^{n-1} + (\delta_{n-2} + \hat{k}_{n-1})s^{n-2} + \ldots + (\delta_0 + \hat{k}_1) \ , \quad (19)$$

where

$$K_aT = \begin{bmatrix} \hat{k}_1 & \hat{k}_2 & \cdots & \hat{k}_{n-1} & | \hat{k}_n \end{bmatrix}. \quad (20)$$

3. Choose the equivalent time constant τ and the stability index γ_i and derive the desired characteristic polynomial

$$P(s) = s^n + \sum_{i=2}^{n-1}\left(\prod_{j=i}^{n-1}\gamma_j^{n-j}\prod_{k=1}^{i-1}\gamma_k^{n-i}\frac{1}{\tau^{n-i}}s^i\right) + \sum_{i=0}^{1}\left(\prod_{j=1}^{n-1}\gamma_{n-j}^{j}\frac{1}{\tau^{n-i}}s^i\right)$$
$$= s^n + a_{n-1}s^{n-1} + \cdots + a_1s^1 + a_0 \quad (21)$$

from the characteristic polynomial (9) which is assumed to be monic (i.e. $a_n = 1$) so that $a_0 = (\prod_{j=1}^{n-1}\gamma_{n-j}^{j})/\tau^n$.

4. Equate the closed-loop characteristic polynomial (19) with the desired characteristic polynomial (21) to obtain

$$K_a = \begin{bmatrix} a_0 - \delta_0 & a_1 - \delta_1 & \ldots & a_{n-2} - \delta_{n-2} & | \ a_{n-1} - \delta_{n-1} \end{bmatrix} T^{-1}. \quad (22)$$

3.2 Friction compensation

The friction compensation is introduced because of the ineligible limit cycles generated mainly by motor driving the arm. Some methods for friction compensation have been described in [8]. However, a simple method of Coulomb friction with stiction $\hat{F}$ which can effectively reduce the oscillation amplitude is employed and expressed as

$$\hat{F} = \begin{cases} F_c \operatorname{sgn}(\dot{\beta}) & \text{if } \dot{\beta} \neq 0, \\ \overline{F} & \text{if } \dot{\beta} = 0 \text{ and } |\overline{F}| < F_s, \\ F_s \operatorname{sgn}(\overline{F}) & \text{otherwise} \end{cases} \quad (23)$$

where F_c is the coulomb friction constant, F_s is the stiction constant and $\overline{F}$ is the resultant forces acting on the slip ring. In this case $\overline{F} = \frac{-mgR}{J}x_1(t) + \frac{-b}{J}x_4(t) + \frac{1}{J}u(t)$.

The friction compensation added to the system is shown in Fig. 1 and then is applied to the control law as

$$\hat{u} = K_a x_a + \hat{F} \ . \quad (24)$$

4. EXPERIMENTS

4.1 Experimental setup

In order to verify the effectiveness of the controller the experiments in controlling the pendulum angle θ and the arm angle β has been done. The physical parameters of the rotational inverted pendulum used in the experiments are shown in Table 1.

Table 1 Parameters of the inverted pendulum.

Pendulum mass (m)	0.05 kg
Pendulum length (l)	0.48 m
Arm length (R)	0.47 m
Moment of inertia (J)	0.03264 $kg \cdot m^2$
Viscous coefficient (b)	0.00351 $kg \cdot m^2 / s$

As shown in Fig 2, the experimental apparatus consists of three main parts: the inverted pendulum system, the interfaces and the digital controller. The pendulum system composes of pendulum, rotating-arm, a high torque permanent magnet DC motor and two angular positions sensors to detect the pendulum angle θ and the arm position angle β. The interface devices are two microcontrollers PIC16C55 to filter the quadrature signal from each encoder, one microcontroller 89C1051 as a sampling clock generator, one eight-bit D/A converter and servo amplifier. A personal computer with Intel Pentium II 350 MHz processor is used as the digital controller. The control program is written in C language and the sampling period is set at 25 milliseconds.

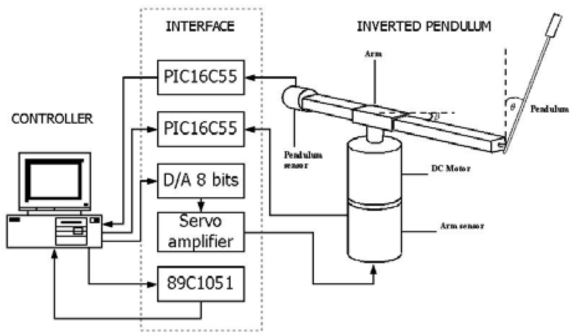
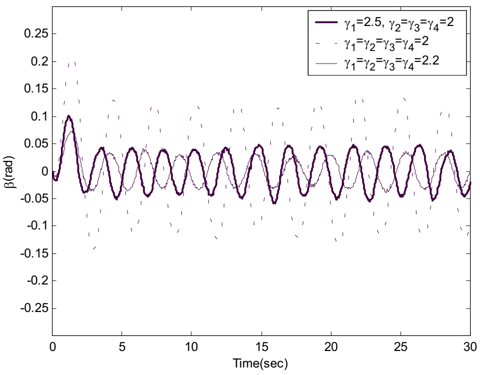


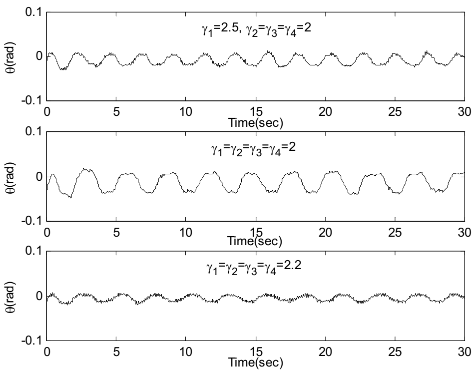
Fig. 2 Experimental apparatus.

4.2 Effect of stability index γ_i

First, the system responses corresponding to the variation of stability index will be observed. By varying the stability index from the standard stability index $\gamma_1 = 2.5, \gamma_2 = \gamma_3 = \gamma_4 = 2$ to $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 2$ and to $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 2.2$ then their corresponding gain K_a of the augmented system for τ =1.2 seconds can be obtained. As depicted in Fig. 3 the dark line, the light line and the dashed line respectively show their experimental results. It is shown that the responses of the system with different values of stability index oscillate around the zero radian line, which means that the integrator added to the arm angle of the rotational inverted pendulum can reject the steady-state error.



(a) Arm angle



(b) Pendulum angle

Fig. 3 System responses when the stability index is varied.

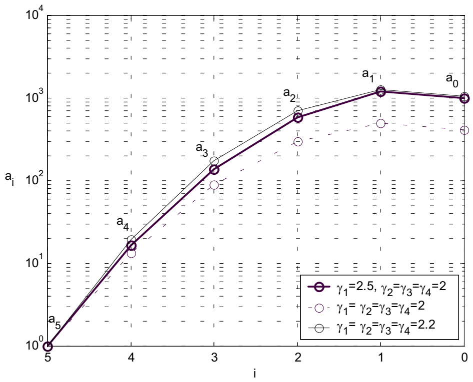


Fig. 4 Coefficient diagrams.

As reported in [9] the stability in CDM can be qualitatively observed visually using graphical interpretation known as coefficient diagram. The coefficient diagram of the proposed control system with varying stability index is shown in Fig. 4. By relating the responses in Fig. 3 to its coefficient diagram in Fig. 4, one can infer that the greater curvature of the coefficient diagrams implies to the smaller oscillation of its corresponding system responses. On the other words, the more stable system will lead to the smaller oscillation.

4.3 Effect of friction compensation

Indeed increasing the stability index can reduce the amplitude of oscillation. However, the control signal will be high which is undesirable. Therefore, simple model based friction compensation using Coulomb friction with stiction is developed and then is applied to the system at t =10 seconds. Fig. 5 shows the responses of the inverted pendulum when the standard stability index γ_i and equivalent time constant τ =1.2 seconds are used. It is seen that the oscillation amplitude of the arm angle β can be reduced significantly.

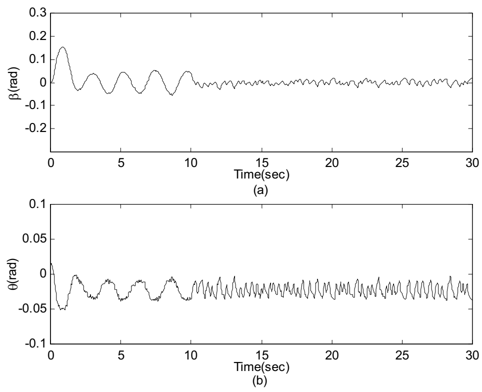


Fig.5 System responses when friction compensation is applied.

4.4 Tracking capability

In order to show the tracking capability, the reference input of the arm angle is changed from zero radian to one radian at 10 seconds for equivalent time constant τ =1.2 seconds. The result depicted in Fig. 6, shows that the output arm angle can track the constant reference input and oscillates around the one radian line, while the effect of the step change in arm angle does not affect the oscillatory behavior of the steady state response. It can also be observed that the rotational inverted pendulum angle is still almost unaffected at the steady state.

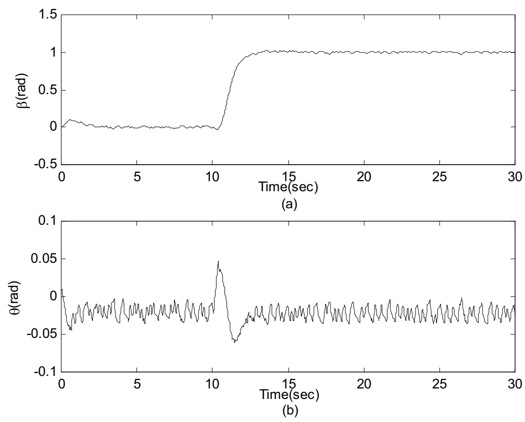


Fig. 6 Tracking capability response.

5. CONCLUSION

In conclusion, the controller designed by CDM incorporating simple friction compensation for a servo type system which is a rotational inverted pendulum with an integrator added to its arm have been proposed. The controller is implemented to control the system and the satisfied performances have been achieved. The good capabilities of angle position error rejection and tracking can be obtained as shown in the experiments. Furthermore, the coefficient diagram used for investigating system stability due to variation of stability index γ_i has also been shown.

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