OPTIMAL DESIGN OF BATCH-STORAGE NETWORK APPLICABLE TO SUPPLY CHAIN

Gyeongbeom Yi*, Euy Soo Lee** and In Beom Lee***

Department of Chemical Engineering , Pukyong National University, Busan, Korea
** Department of Chemical Engineering, Donggug University, Seoul, Korea
*** Department of Chemical Engineering , POSTECH, Pohang, Korea

Abstract: An effective methodology is reported for the optimal design of multisite batch production/transportation and storage networks under uncertain demand forecasting. We assume that any given storage unit can store one material type which can be purchased from suppliers, internally produced, internally consumed, transported to or from other plant sites and/or sold to customers. We further assume that a storage unit is connected to all processing and transportation stages that consume/produce or move the material to which that storage unit is dedicated. Each processing stage transforms a set of feedstock materials or intermediates into a set of products with constant conversion factors. A batch transportation process can transfer one material or multiple materials at once between plant sites. The objective for optimization is to minimize the probability averaged total cost composed of raw material procurement, processing setup, transportation setup and inventory holding costs as well as the capital costs of processing stages and storage units. A novel production and inventory analysis formulation, the PSW(Periodic Square Wave) model, provides useful expressions for the upper/lower bounds and average level of the storage inventory. The expressions for the Kuhn-Tucker conditions of the optimization problem can be reduced to two sub-problems. The first yields analytical solutions for determining lot sizes while the second is a separable concave minimization network flow subproblem whose solution yields the average material flow rates through the networks for the given demand forecast scenario. The result of this study will contribute to the optimal design and operation of large-scale supply chain system.

Keywords: Optimal, Lot-size, Multisite, Transportation, Distribution

1. INTRODUCTION

Recent major chemical industries are experiencing multiple difficulties such as customer demand reduction, lower margin and fierce competition. In order for the industries to stay profitable, they aggressively merge similar enterprises and take the advantage of the economies of scale not only from slimming the manpower of business management and/or process operation but also from consolidating production into bigger plants with higher efficiency. Rapid development of computer hardware and computer-aided optimization technology enabled such progress. As another strategy, many chemical companies are turning their focus to specialty chemicals or other high value added products produced in much smaller volume than common commodity chemicals. They are likely to be made in batch and using generic equipment. Their profit potential arises not so much from their efficient manufacture but timely delivering well-functional products to customers.

Reflecting such paradigm change of chemical industries, process system academia has already accumulated abundant volume of research articles about batch process design and operation. The researchers concern was not confined within resolving the technical puzzles in chemical plant boundary but enlarged to cover many sophisticated non-traditional issues tightly related with batch processing such as customer satisfaction under demand uncertainty (Gupta et. al. 2000), fair profit distribution between multienterprise members (Gjerdrum et. al. 2001), new product development and production (Papageorgiou et. al. 2001), multisite production and distribution (Mcdonald and Karimi 1997) and inventory control (Perea-Lopez et. al. 2001). Not only diversifying research subjects, the problem scale approached to a practical size owing to the increase of computational capacity. Tsiakis et. al. (2001) reported an optimal network design for the large-scale supply chain system that can cover European continental. In spite of remarkable advance of research scale and depth, human understanding for the supply chain management is quite limited. Most of the above researches stick to linear or linearized mixed integer model tracking the major tendency of business factors in a logbook style. Linear models have many good properties such as great flexibility, easy computation and easy implementation, etc. As a matter of fact, many of linear models are successful in real applications (Camm et. al. 1997, Zierer et. al. 1976). However, it is questionable if this logbook style linear model is the only way to represent the nature of commercial material flows through modern super-size enterprise network or in another word, if there is a new model that gives better insight or understanding as well as less computational burden. It is no doubt that better human understanding is the key success factor for computer-aided business optimization. Also, a good model is very useful for educational purpose.

One of the challenging problems in supply chain optimization is dealing with uncertainty. Major business uncertainties arise from demand forecasting, price change and resource supply in the future. Including computational mechanism to mitigate such uncertainty effects into the supply chain optimization model is a promising remedy (Gupta et. al. 2000, Tsiakis et. al. 2001). But adhering computational complexity to the uncertainty treatment as well as unknown probabilistic parameters prohibits such models from being used in real applications. Under such considerations, this study aims to develop a compact analytical solution to optimize the design and/or operation of large-scale supply chain including batch production and transportation processes among multiple sites under demand uncertainty.

. Yi and Reklaitis (2000) suggested a novel production and inventory analysis method called Periodic Square Wave (PSW) and applied it to the optimal design of parallel batch-storage system. They extended the plant structure to the sequential multistage batch-storage network in Yi and Reklaitis (2002). A non-sequential network structure that can deal with recycle material flows in a plant site was suggested in Yi and Reklaitis (2003). In this study, we will apply the PSW model to the multisite network involving batch transportation processes between sites. We address an arbitrary batch-storage circuit and exclude the details of operational or design constraints. For example, we would not concern about material movement system such as piping and pumping network between processes and storages within a plant site. Instead we focus on obtaining a compact set of analytical solutions. In order to obtain the analytical solution, we assume that all operations are periodical with unknown cycle times. Forecast demand uncertainty will be treated with scenario-based approach (Tsiakis et. al. 2001). Thus, the practical contribution of this study over our previous work exists in that we can deal with material transportation processes between plant sites and demand uncertainty. In spite of enlarged problem scope, the computational burden is minimal because of mostly analytical results.

2. OPTIMIZATION MODEL

Chemical plant, which converts the raw materials into final products through multiple physicochemical processing steps, is effectively represented by the batch-storage network, as shown at Fig.1. The chemical plant is composed of a set of storage units (J) and a set of batch processes (I) as shown in Fig. 1(a). The circle ($j \in J$) in the figure represents a storage unit, the square ($i \in I$) represents a batch process and the arrows represent the material flows. Each process requires multiple feedstock materials of variable flow rates (f_i^j) and produces multiple products with variable product yield (g_i^j) as shown in Fig. 1(b). Note that storage index j is superscript and process index i is subscript. If there is no material flow between a storage and a process, the corresponding feedstock composition or product yield value is zero. Each storage unit is dedicated to one material.

Each storage is involved with four types of material movement, purchasing from suppliers ($k \in K(j)$), shipping to consumers ($m \in M(j)$), feeding to processes and producing from processes as shown in Fig. 1(c). Note that the sets of suppliers K(j) or customers M(j) are storage dependent. The material flow from process to storage (or from storage to process) is represented by the Periodic Square Wave (PSW) model[1]. The material flow representation of PSW model is composed of four variables: the batch size B_i, the cycle time ω_i, the transportation time fraction x_i^{in} (or x_i^{out}), the start-up time t_i^{in} (or t_i^{out}). According to the same discussion in Reference [1], we assume that the feedstock feeding operations to the process (or the product discharging operations from the process) occur at the same time and their transportation time fractions are the same among feeding or discharging flows. That is, the superscript j is not necessary to discriminate the storage units in the x_i^{in} (or x_i^{out}) and t_i^{in} (or t_i^{out}). The material flow of raw material purchased is represented by order size B_k^j, cycle time ω_k^j, transportation time fraction x_k^j, and start-up time t_k^j t_i^{in} (or t_i^{out}). All transportation time fractions will be considered as parameters whereas the other will be the design variables as used in this study. The material flow of finished product sales is represented by $B_m^j, \omega_m^j, x_m^j, t_m^j$ in the same way. The arbitrary periodic function of the finished product demand forecast can be represented by the sum of periodic square wave functions with known values of $B_m^j, \omega_m^j, x_m^j, t_m^j$.

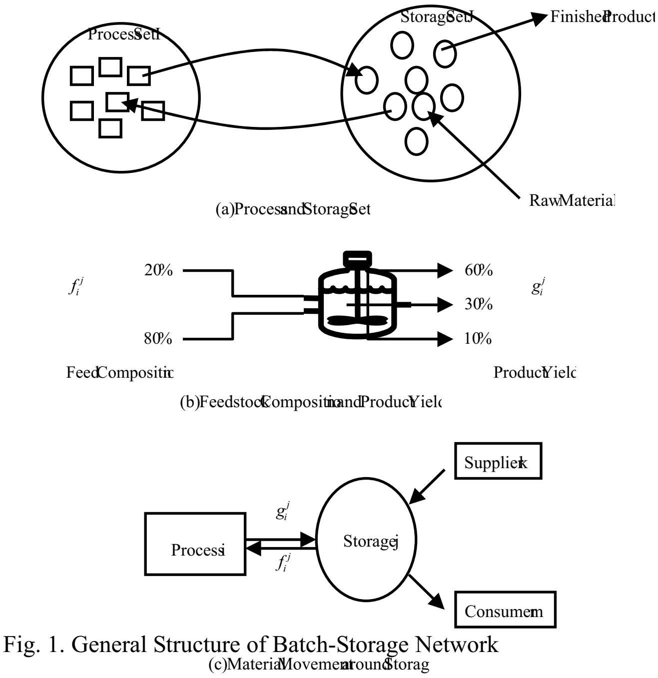

Fig. 1. General Structure of Batch-Storage Network

The feedstock flows from predecessor storages and the product flows to successor storages are of course not independent. From the fact that one production cycle in a process is composed of feedstock feeding, processing and product discharging, there exists the following timing relationship between the time delays of feedstock stage and the time delay of product stage.

$$t_i^{out} = t_i^{in} + \omega_i (1 - x_i^{out}) \tag{1}$$

Let $\sum_{j=1}^{|J|} f_i^j = \sum_{j=1}^{|J|} g_i^j$ be the average material flow rate through process i, which is B_i divided by ω_i. The average material flow through raw material storage and finished product storage are denoted by D_k^j, D_m^j respectively. The overall material balance around the storage results in the following relationships;

$$\sum_{i=1}^{|I|} g_i^j + \sum_{k=1}^{|K(j)|} D_k^j = \sum_{i=1}^{|I|} f_i^j + \sum_{m=1}^{|M(j)|} D_m^j \tag{2}$$

The size of storage j is denoted by V_s^j. The initial inventory of storage j is denoted by $V^j(0)$. The inventory hold-up of storage j at time t is denoted by $V^j(t)$. The inventory hold-up can be calculated by the difference between the incoming material flows from supply processes and the outgoing material flows into consumption processes. Special properties of the periodic square wave function are required to integrate the detail material balance equation. The resulting inventory hold-up functions for storages are;

$$
\begin{aligned}
V^j(t) &= V^j(0) \\
&+ \sum_{k=1}^{|K(j)|} B_k^j \left[\operatorname{int}\left[\frac{t-t_k^j}{\omega_k^j}\right] + \min\left\{1, \frac{1}{x_k^j} res\left[\frac{t-t_k^j}{\omega_k^j}\right]\right\}\right] \\
&+ \sum_{i=1}^{|I|} (G_i^j \omega_i) \left[\operatorname{int}\left[\frac{t-t_i^{out}}{\omega_i}\right] + \min\left\{1, \frac{1}{x_i^{out}} res\left[\frac{t-t_i^{out}}{\omega_i}\right]\right\}\right] \\
&- \sum_{m=1}^{|M(j)|} B_m^j \left[\operatorname{int}\left[\frac{t-t_m^j}{\omega_m^j}\right] + \min\left\{1, \frac{1}{x_m^j} res\left[\frac{t-t_m^j}{\omega_m^j}\right]\right\}\right] \\
&- \sum_{i=1}^{|I|} (F_i^j \omega_i) \left[\operatorname{int}\left[\frac{t-t_i^{in}}{\omega_i}\right] + \min\left\{1, \frac{1}{x_i^{in}} res\left[\frac{t-t_i^{in}}{\omega_i}\right]\right\}\right]
\end{aligned}
\tag{3}
$$

The upper bound of inventory hold-up, the lower bound of inventory hold-up and the average inventory hold-up of Eq. (3) are calculated by using the properties of flow accumulation function.

$$
\begin{aligned}
V_{ub}^j &= V^j(0) + \sum_{k=1}^{|K(j)|} (1-x_k^j) D_k^j \omega_k^j - \sum_{k=1}^{|K(j)|} D_k^j t_k^j \\
&+ \sum_{i=1}^{|I|} (1-x_i^{out}) G_i^j \omega_i - \sum_{i=1}^{|I|} G_i^j t_i^{out} \\
&+ \sum_{i=1}^{|I|} F_i^j t_i^{in} + \sum_{m=1}^{|M(j)|} D_m^j t_m^j
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
V_{lb}^j &= V^j(0) - \sum_{k=1}^{|K(j)|} D_k^j t_k^j - \sum_{i=1}^{|I|} G_i^j t_i^{out} \\
&- \sum_{i=1}^{|I|} (1-x_i^{in}) F_i^j \omega_i + \sum_{i=1}^{|I|} F_i^j t_i^{in} \\
&- \sum_{m=1}^{|M(j)|} (1-x_m^{in}) D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\overline{V^j} &= V^j(0) + \sum_{k=1}^{|K(j)|} \frac{(1-x_k^j)}{2} D_k^j \omega_k^j - \sum_{k=1}^{|K(j)|} D_k^j t_k^j \\
&+ \sum_{i=1}^{|I|} \frac{(1-x_i^{out})}{2} G_i^j \omega_i - \sum_{i=1}^{|I|} G_i^j t_i^{out} \\
&- \sum_{i=1}^{|I|} \frac{(1-x_i^{in})}{2} F_i^j \omega_i + \sum_{i=1}^{|I|} F_i^j t_i^{in} \\
&- \sum_{m=1}^{|M(j)|} \frac{(1-x_m^j)}{2} D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j
\end{aligned}
\tag{6}
$$

The purchasing setup cost of raw material j is denoted by A_k^j \$/order and the setup cost of process i is denoted by A_i \$/batch. The annual inventory holding cost of storage j is denoted by H^j \$/year/liter. The annual capital cost of process construction and licensing can not be ignored in the chemical process industries. In this article, we will assume that capital cost is proportional to process capacity in order to permit analytical solution. The objective function of designing batch-storage network is minimizing the total cost consisted of the setup cost of processes, inventory holding cost of storages and the capital cost of processes and storages.

$$
\begin{aligned}
TC &= \sum_{j=1}^{|J|} \sum_{k=1}^{|K(j)|} \left[\frac{A_k^j}{\omega_k^j} + a_k^j D_k^j \omega_k^j \right] \\
&+ \sum_{i=1}^{|I|} \left[\frac{A_i}{\omega_i} + a_i \omega_i \sum_{j=1}^{|J|} F_i^j \right] \\
&+ \sum_{j=1}^{|J|} \left[H^j \overline{V^j} + b^j V_{ub}^j \right]
\end{aligned}
\tag{7}
$$

where a_k^j is the annual capital cost of the purchasing facility of raw material j, a_i is the annual capital cost of process i and b^j is the capital cost of storage j. Without loss of generality, the storage size V_s^j will be determined by the upper bound of inventory holdup, V_{ub}^j. Therefore, Eq. (4) is the expression for storage capacities. The independent variables are selected as cycle times ω_k^j, ω_i, initial time delays t_k^j, t_i^{in} and average processing rates D_k^j, D_i. The inventory holdup $V^j(t)$ should be confined within the storage capacity. Sufficient conditions are $0 \le V_{lb}^j < V_{ub}^j \le V_s^j$. Since the storage size V_s^j should be determined through this analysis, only the conditions $0 \le V_{lb}^j$ are necessary. The problem is defined as minimizing total cost given by Eq. (7) subject to the constraints Eq. (5) ≥ 0 with respect to the non-negative search variables ω_k^j, ω_i, t_k^j, t_i^{in}, D_k^j, D_i. Kuhn-Tucker conditions result in analytical solution. Optimal cycle times are;

$$
B_k^j = \sqrt{\frac{A_k^j D_k^j}{\Psi_k^j}} \tag{8}
$$

Where

$$\Psi_k^j = \left(\frac{H^j}{2} + b^j\right)(1 - x_k^j) + a_k^j$$

$$B_i = \sqrt{\frac{A_i}{\Psi_i}} \tag{9}$$

where

$$\Psi_i = a_i \sum_{j=1}^{|J|} F_i^j + \left(1 - x_i^{in}\right)\sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)F_i^j$$
$$+ \left(1 - x_i^{out}\right)\sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)G_i^j$$

Optimal start-up times are;

$$\begin{aligned} &\sum_{k=1}^{|K(j)|} D_k^j t_k^j + \sum_{i=1}^{|I|}\left(G_i^j - F_i^j\right)t_i^{in} = V^j(0) \\ &- \sum_{m=1}^{|M(j)|}(1 - x_m^j)D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j \\ &- \sum_{i=1}^{|I|}\left[(1 - x_i^{in})F_i^j + (1 - x_i^{out})G_i^j\right]\omega_i \end{aligned} \tag{10}$$

Eq. (10) has $|K(j)| + |I|$ variables and $|J|$ equations. In most real cases, the variables outnumber the equations. We may need a secondary objective function to fix the additional freedom. Optimal storage sizes are;

$$\begin{aligned} V_{ub}^j &= \sum_{i=1}^{|I|}\left[(1 - x_i^{in})F_i^j + (1 - x_i^{out})G_i^j\right]\omega_i \\ &+ \sum_{k=1}^{|K(j)|}(1 - x_k^j)D_k^j \omega_k^j + \sum_{m=1}^{|M(j)|}(1 - x_m^j)D_m^j \omega_m^j \end{aligned} \tag{11}$$

The optimum value of objective function is;

$$\begin{aligned} &{}^*TC(D_k^j, F_i^j, G_i^j) = \\ &2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{A_k^j \Psi_k^j D_k^j} + 2\sum_{i=1}^{|I|}\sqrt{A_i \Psi_i} \\ &+ \sum_{j=1}^{|J|}\left(\frac{H^j}{2} + b^j\right)\sum_{m=1}^{|M(j)|} D_m^j \omega_m^j (1 - x_m^j) \end{aligned} \tag{12}$$

In order to determine the average flow rates D_k^j and D_i, a subsidiary optimization problem minimizing Eq. (12) subject to Eq. (2) should be solved. This is known as a separable concave minimization network flow problem. An analytic solution exists when Eq. (2) can be solved directly.

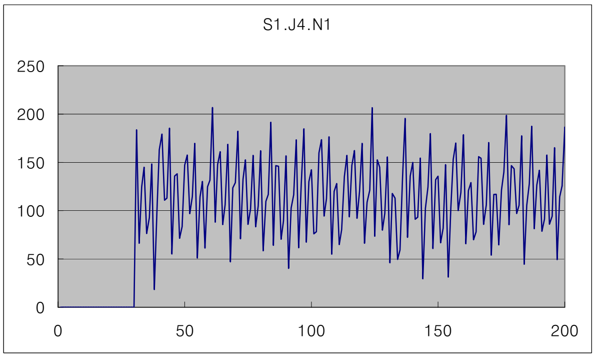

Fig. 2. Optimal Inventory Profiles of a Selected Material for Different Sites.

3. EXAMPLE SUPPLY CHAIN DESIGN

Fig. 3 is the layout of supply chain with one possible solution result. Bold lines represent composite delivery and thin lines represent separate delivery. Customer demands can occur on any materials stored in any site. We assume constant demands for convenience. There are three demand scenarios with equal probability of 1/3. The system purchases four raw materials and manufactures all the other materials at site 1 ~ 3. Then, they are transported to all the other sites. We assume that there are no transportation deliveries from site 4 ~ 18 to site 1 ~ 3 and no composite deliveries from site 1 ~ 9 to 10 ~ 18. Fig. 2 shows inventory profiles of a selected storage.

4. CONCLUSION

This study deals with the optimal lot size of production, inventory and distribution system composed of multiple sites with a set of storage units and a set of production batch processes in most general form. The material in a storage unit can be purchased, internally produced, internally consumed, sold out, transferred to and/or transferred from other site. A production process can consume multiple materials in storage units and produce multiple products into storage units. Two types of transportation processes that carry single material or multiple materials at once are considered. All operations are assumed to be periodical. Startup time and batch size of any production or transportation process operations are our major decision variables as well as average material flow rates. The economic factors considered in this study cover raw material purchase cost, setup cost, inventory holding cost, capital cost of facilities. Sequence dependent setup cost and backlogging cost are not considered in this study. Analytical optimal solution for the optimization problem indicates that the optimal design procedure can be decomposed into two phases; i) To determine the average material flow rates by solving separable concave minimization network flow problem, ii) To determine the batch sizes and cycle times by simple analytical equations. The average material flow rates can be calculated by any other methods such as ordinary linear programming. Because the network model is very general, the result can be applicable to the plant involving recycling flows which is known as very difficult to solve but also very popular in chemical industries. A scenario based approach has been applied to treat the demand forecast uncertainty.

The result of this study will contribute to the optimal design and operation of the large-scale supply chain, including diagnostic analysis. The optimality and simplicity of the analytical lot sizing solution open the possibility to design self-optimizing, real-time scheduling and inventory control systems of large-scale batch-storage networks under uncertainty

ACKNOWLEDGEMENT

This work was supported by a grant No.(R01-2002-000-00007-0) from Korea Science & Engineering Foundation.

REFERENCES

[1] Camm, J. D., T. E. Chorman, F. A. Dill, J. R. Evans, D. J. Sweeney and G. W. Wegryn, "Blending OR/MS, Judgement, and GIS: Restructuring P&G's Supply Chain," *INTERFACES*, **27**, 128(1997).

[2] Gjerdrum, J., N. Shah and L. G. Papageorgiou, "Transfer Prices for Multienterprise Supply Chain Optimization," *Ind. Eng. Chem*. Res., **40,** 1650(2001).

[3] Gupta, A., C. D. Maranas and C. M. McDonald, "Mid-term Supply Chain Planning under Demand Uncertainty: Customer Demand Satisfaction and Inventory Management," *Computers and Chemical Engineering*, **24**, 2613(2000).

[4] McDonald, C. M. and I. A. Karimi, "Planning and Scheduling of Parallel Semicontinuous Processes. 1. Production Planning," *Ind. Eng. Chem*. Res., **36,** 2691(1997).

[5] Papageorgiou, L. G., G. E. Rotstein and N. Shah, "Strategic Supply Chain Optimization for the Pharmaceutical Industries," *Ind. Eng. Chem*. Res., **40,** 275(2001).

[6] Perea-Lopez, E., I. E. Grossmann and B.E. Ydstie, "Dynamic Modeling and Decentralized Control of Supply Chains," *Ind. Eng. Chem*. Res., **40,** 2269(2001).

[7] Shectman, J. P. and N. V. Sahinidis, "A Finite Algorithm for Global Minimization of Separable Concave Programs," *Journal of Global Optimization*, **12**, 1(1998).

[8] Tsiakis, P., N. Shah and C. C. Pantelides, "Design of Multi-echelon Supply Chain Networks under Demand Uncertainty," *Ind. Eng. Chem*. Res., **40,** 3585(2001).

[9] Yi, G. and G. V. Reklaitis, "Optimal Design of Multiple Batch Units with Feedstock/Product Storages," *Chem. Eng. Comm.,* **181**, 79(2000).

[10] Yi, G. and G. V. Reklaitis, "Optimal Design of Batch-Storage Network Using Periodic Square Model," *AIChE J.,* **48**, 1737(2002).

[11] Yi, G. and G. V. Reklaitis, "Optimal Design of Batch-Storage Network with Recycle Streams," *AIChE J*. **49**, 3084(2003)

[12] Zierer, T. K., W. A. Mitchell and T. R. White, "Practical Applications of Linear programming to Shell's Distribution Problems," *INTERFACES*, **6**, 13(1976).

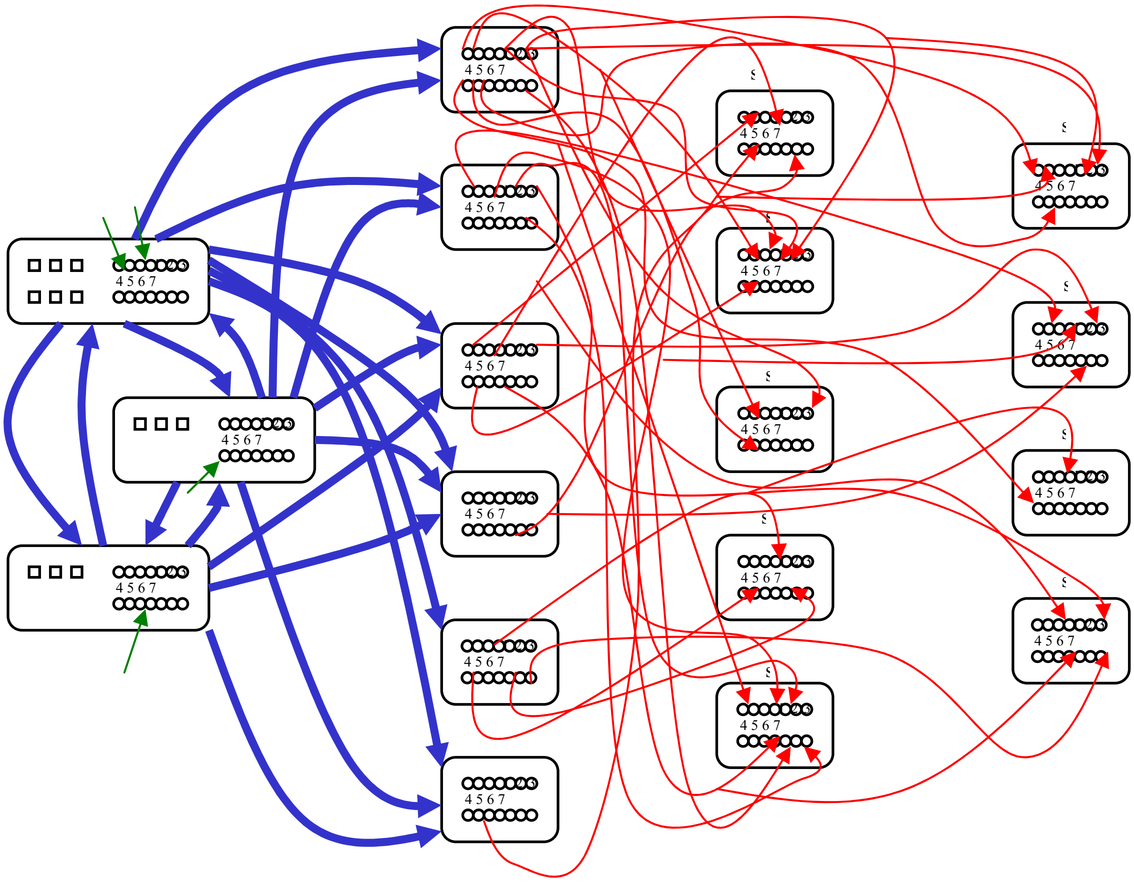

Fig. 3. A Motivating Example Process with Multisite Enterprise.