Relaxing of the Sampling Time Requirement in Prove of the EDMC Stability

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Abstract: Closed loop stability of Extended Dynamic Matrix Control (EDMC) is investigated for limited sampling time. Linear approximation of the sensitivity functions is employed in the derivation of the stability condition. It is shown that the closed loop system will be stable if the control moves suppression coefficient λ is taken arbitrarily large. Special cases such as $M = P = 1$ and $M = 1, P > 1$ are discussed in more details.

Keywords: EDMC, Stability, Sampling time

1. INTRODUCTION

EDMC is an extended version of Dynamic Matrix Control (DMC) that works on nonlinear processes. In this control algorithm, a linear approximation of the nonlinear model of the process is determined in each control interval applying a small perturbation on the control signal [1]. The main difference between ordinary and extended DMC is to how predict the unmodelled part of the process including the external disturbances. In contrary to ordinary DMC, this part of the process output is separated to predictable (difference between linear and nonlinear models outputs) and unpredictable (difference between the process and nonlinear model outputs) portions. The predictable portion is determined using iterative methods. Once the iterations converged the control moves are calculated using DMC formulas. The unpredictable portion is treated as in DMC. It has been shown in [1, 2] that the iterations are convergent and the closed loop system is stable if the following conditions are satisfied.

1) Steady state gain of the process does not change sign
2) Control move suppression coefficient is larger than zero
3) Sampling time is long enough ($T \to \infty$)
4) Control horizon (M) is 1
5) Set point is constant along the prediction horizon (P).

The main goal of the present work is to relax the third condition of the above list. The given prove in this paper is also applicable when the control horizon is longer than 1. Removing of these two conditions extends applicability of the existing stability theorem for wider rang of nonlinear processes.

The paper is organized as follows. In Section 2, a brief overview of EDMC is given. Deriving of the stability condition for limited sampling time is discussed in Section 3. Section 4 contains materials related to the special cases and finally paper is concluded in Section 5.

2. EDMC FORMULATION

In this section a brief description of DMC and EDMC formulations are presented.

2.1 DMC formulation

Output of a stable SISO system could be determined using the following discrete convolution model.

$$y(k) = \sum_{i=1}^{N} a_i \Delta u(k-i) + a_N u(k-N-1) \tag{1}$$

In which a_i's are the coefficients of the step response, u is the control signal, Δu is the control signal variation, and N is the number of samples that system reaches to the steady state. Any difference between the measured output and the one predicted by the model is represented as the external disturbance,

$$d(k) = y^{meas}(k) - \sum_{i=1}^{N} a_i \Delta u(k-i) - a_N u(k-N-1) \tag{2}$$

The future outputs of the system can be predicted based on the following matrix-vector relation.

$$\begin{bmatrix} y^p(k+1) \\ y^p(k+2) \\ \vdots \\ y^p(k+P) \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ \vdots & & & & \\ a_M & a_{M-1} & & \cdots & a_1 \\ \vdots & & & & \\ a_P & a_{P-1} & & \cdots & a_{P-M+1} \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix} + \begin{bmatrix} a_2 & a_3 & \cdots & a_N \\ a_3 & a_4 \cdots & a_N & 0 \\ \vdots & & & \\ a_{P+1} & \cdots a_N & \cdots & 0 \end{bmatrix} \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-N+1) \end{bmatrix} + \begin{bmatrix} a_N u(k-N) \\ a_N u(k-N+1) \\ \vdots \\ a_N u(k-N+P-1) \end{bmatrix} + \begin{bmatrix} d(k+1) \\ d(k+2) \\ \vdots \\ d(k+P) \end{bmatrix} \tag{3}$$

In which P and M are the prediction and the control horizons respectively. In compact form we have,

$$\boldsymbol{y}^p = \boldsymbol{A}\Delta\boldsymbol{u} + \boldsymbol{y}^{past} + \boldsymbol{d} \tag{4}$$

In DMC, the control moves, $\Delta\boldsymbol{u}$, are determined by the solution of the following optimization problem.

$$\underset{\Delta u}{Min} \sum_{i=1}^{P} \left[y^p(k+i) - y^p(k+i) \right]^2 + \sum_{j=1}^{M} \lambda \left[\Delta u(k+j-1) \right]^2 \tag{5}$$

The cost function includes difference between the desired ($\boldsymbol{y}^d$) and the predicted ($\boldsymbol{y}^p$) outputs and the weighted control moves as well (λ is the weighting factor). In the unconstraint case solution of the problem is given by:

$$\Delta \boldsymbol{u} = \left(\boldsymbol{A}^T \boldsymbol{A} + \lambda \boldsymbol{I}\right)^{-1} \boldsymbol{A}^T \left(\boldsymbol{y}^d - \boldsymbol{y}^{past} - \boldsymbol{d}\right) \tag{6}$$

$\boldsymbol{y}^{past}$ is the free response of the system and is determined using its model.

2.2 Extended DMC approach

In EDMC, $\boldsymbol{d}$ is not assumed to be completely unknown. It comprises from unpredictable disturbance ($\boldsymbol{d}^{ext}$) that represents difference between system and its nonlinear model outputs and the predictable disturbance ($\boldsymbol{d}^{nl}$), which is the difference between linear and nonlinear models outputs. This can be explained using the following equation.

$$\boldsymbol{d} = \boldsymbol{d}^{ext} + \boldsymbol{d}^{nl} \tag{7}$$

$\Delta\boldsymbol{u}$ is computed from the similar formula as in DMC. However, since $\boldsymbol{d}^{nl}$ (the known part of $\boldsymbol{d}$) depends nonlinearly on $\Delta\boldsymbol{u}$, the problem is solved using iterative methods. The criterion for the convergence of the iterations is to reach the following equality.

$$\boldsymbol{y}^{nl} = \boldsymbol{y}^{el} = \boldsymbol{A}\Delta\boldsymbol{u} + \boldsymbol{y}^{past} + \boldsymbol{d}^{ext} + \boldsymbol{d}^{nl} \tag{8}$$

$\boldsymbol{y}^{el}$ and $\boldsymbol{y}^{nl}$ are the predictions of the system output that are given by the linear and the nonlinear models.

Equation (8) contains P nonlinear equations for P unknowns $d^{nl}(k+1), \cdots, d^{nl}(k+P)$. Nonlinearity of the equations arises from the nonlinear relation that exists between $\boldsymbol{y}^{nl}$ and $\boldsymbol{u}$ (and therefore $\Delta\boldsymbol{u}$). Iterative methods such as fixed-point iteration or secant method are used to solve the P dimensional nonlinear vector equation.

3. NEW STABILITY CRITERION FOR EDMC

In this section we evaluate the stability criterion of SISO systems for $M > 1$ case. Regarding the stability, we assume that the iterative computation of $\boldsymbol{d}^{nl}$ in sample time k has been converged. Our goal is to derive a relation between the present input, u_k and the previous input, u_{k-1} .

$$\begin{aligned} u_k &= u_{k-1} + \Delta u_k \\ &= u_{k-1} + \boldsymbol{e}_1^T [(\boldsymbol{A}^T\boldsymbol{A} + \lambda^2 \boldsymbol{I})^{-1} \boldsymbol{A}^T (\boldsymbol{y}_{k+1}^{sp} - \boldsymbol{y}_{k+1}^{past} - \boldsymbol{d}_{k+1}^{nl})] \end{aligned} \tag{9}$$

In which $\boldsymbol{e}_1$ is a $M \times 1$ vector with all elements zero except the first element that is one. Note that for nominal system $\boldsymbol{d}^{ext} = \boldsymbol{0}$. To complete and simplify (9), the converged value of $\boldsymbol{d}_{k+1}^{nl}$ should be determined. When the convergence is occurred the following equality is satisfied.

$$\boldsymbol{y}^{nl} = \boldsymbol{y}^{el} \tag{10}$$

Output of the extended linear model $\boldsymbol{y}^{el}$ is obtained as follows.

$$\begin{aligned} \boldsymbol{y}^{el}(k+1) &= \boldsymbol{y}^{past}(k+1) + \boldsymbol{A}\Delta\boldsymbol{u}(k) + \boldsymbol{d}^{nl}(k+1) \\ &= \boldsymbol{y}^{past}(k+1) + \\ &\quad \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A} + \lambda^2\boldsymbol{I})^{-1}\boldsymbol{A}^T(\boldsymbol{y}^{sp}(k+1) - \boldsymbol{y}^{past}(k+1) \\ &\quad - \boldsymbol{d}^{nl}(k+1)) + \boldsymbol{d}^{nl}(k+1) \end{aligned} \tag{11}$$

Use of the following definition,

$$\boldsymbol{A}_0 = \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A} + \lambda^2\boldsymbol{I})^{-1}\boldsymbol{A}^T$$

and equations (10) and (11) one can derive the following relation for $\boldsymbol{d}_{k+1}^{nl}$.

$$(\boldsymbol{I} - \boldsymbol{A}_0)\boldsymbol{d}_{k+1}^{nl} = \boldsymbol{y}_{k+1}^{nl} - (\boldsymbol{I} - \boldsymbol{A}_0)\boldsymbol{y}_{k+1}^{past} - \boldsymbol{A}_0\boldsymbol{y}_{k+1}^{sp}$$

or

$$\boldsymbol{d}_{k+1}^{nl} = (\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{y}_{k+1}^{nl} - \boldsymbol{y}_{k+1}^{past} - (\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_0\boldsymbol{y}_{k+1}^{sp} \tag{12}$$

Substitute for $\boldsymbol{d}_{k+1}^{nl}$ from (12) in (9), we have,

$$\begin{aligned} u_k = u_{k-1} + \Delta u_k = u_{k-1} + \boldsymbol{e}_1^T [(\boldsymbol{A}^T\boldsymbol{A} + \lambda^2\boldsymbol{I})^{-1}\boldsymbol{A}^T \times \\ (\boldsymbol{y}_{k+1}^{sp} - (\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{y}_{k+1}^{nl} + (\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_0\boldsymbol{y}_{k+1}^{sp})] \end{aligned} . \tag{13}$$

Using the following two matrix relation for $\boldsymbol{A}_0$,

$$(\boldsymbol{I} - \boldsymbol{A}_0)^{-1} = \boldsymbol{I} + \frac{1}{\lambda^2}\boldsymbol{A}\boldsymbol{A}^T , \quad (\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_0 = \frac{1}{\lambda^2}\boldsymbol{A}\boldsymbol{A}^T \tag{14}$$

Equation (13) is simplified as,

$$u_k = u_{k-1} + \frac{1}{\lambda^2}\boldsymbol{e}_1^T\boldsymbol{A}^T(\boldsymbol{y}_{k+1}^{sp} - \boldsymbol{y}_{k+1}^{nl}) . \tag{15}$$

When sampling time is assumed to be large enough $(T \to \infty)$ [2], one can consider the following approximations.

$$\begin{aligned} &\boldsymbol{y}_{k+1}^{nl} = y^{nl}(k+1)\mathbf{1}_P, \quad \boldsymbol{y}_{k+1}^{sp} = y^{sp}(k+1)\mathbf{1}_P, \\ &\boldsymbol{A} = a\mathbf{1}_L, \quad e_1^T\boldsymbol{A}^T\mathbf{1}_P = aP \end{aligned} \tag{16}$$

Where

$$\mathbf{1}_P = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{P\times 1} , \quad \mathbf{1}_L = \begin{bmatrix} 1\,0\,0\cdots 0 \\ 1\,1\,0\cdots 0 \\ \vdots \\ 1\,1\,1\cdots 1 \\ \vdots \\ 1\,1\,1\cdots 1 \end{bmatrix}_{P\times M} \tag{17}$$

Therefore (15) is further simplified as,

$$u_k = u_{k-1} + \frac{aP}{\lambda^2}(y^{sp}(k+1) - y^{nl}(k+1)) \tag{18}$$

This equation is similar to the one derived in [2] but assumption $M = 1$ is not used in this case. Therefore results given in [2] for the stability of the closed loop system are also applicable for $M > 1$.

In the sequel, analysis of the stability is continued without infinite sampling time assumption. We start from (15) and expand the right hand side of the equation based on $\boldsymbol{e}_1$, $\boldsymbol{A}$, $\boldsymbol{y}_{k+1}^{sp}$, and $\boldsymbol{y}_{k+1}^{nl}$ elements.

$$u_k = u_{k-1} + \frac{1}{\lambda^2}\sum_{i=1}^{P} a_i (y^{sp}(k+i) - y^{nl}(k+i)) \qquad (19)$$

To examine the stability of a closed loop system that is characterized by (19), one can employ the Lyapunov linearization method. To this end, derivatives of the future outputs with respect to u_{k-1} are required.

$$\begin{gathered}\frac{\partial u_k}{\partial u_{k-1}} = 1 - \frac{1}{\lambda^2}\sum_{i=1}^{P} a_i \frac{\partial y^{nl}(k+i)}{\partial u_{k-1}} = 1 - \frac{1}{\lambda^2} \\ \left(a_1 \frac{\partial y^{nl}(k+1)}{\partial u_{k-1}} + a_2 \frac{\partial y^{nl}(k+2)}{\partial u_{k-1}} + \cdots + a_p \frac{\partial y^{nl}(k+P)}{\partial u_{k-1}} \right)\end{gathered} \qquad (20)$$

To solve the problem, the linear approximations of $y^{nl}(k+i)$ are used in the computation.

$$\begin{aligned}
y(k+1) &= a_1 u(k) + (a_2 - a_1)u(k-1) + (a_3 - a_2)u(k-2) + \cdots \\
&\quad + (a_N - a_{N-1})u(k-N+1) \\
y(k+2) &= a_1 u(k+1) + (a_2 - a_1)u(k) + (a_3 - a_2)u(k-1) + \cdots \\
&\quad + (a_N - a_{N-1})u(k-N+2) \\
&\vdots \\
y(k+M) &= a_1 u(k+M-1) + (a_2 - a_1)u(k+M-2) + \cdots \\
&\quad + (a_{M+1} - a_M)u(k-1) + \cdots + (a_N - a_{N-1})u(k-N+M) \\
y(k+M+1) &= a_2 u(k+M-1) + (a_3 - a_2)u(k+M-2) + \cdots \\
&\quad + (a_{M+2} - a_{M+1})u(k-1) + \cdots \\
&\quad + (a_N - a_{N-1})u(k-N+M+1) \\
&\vdots \\
y(k+P+1) &= a_{P-M+1} u(k+M-1) + (a_{P-M+2} - a_{P-M+1})u(k+M-2) \\
&\quad + \cdots + (a_{P+1} - a_P)u(k-1) + \cdots \\
&\quad + (a_N - a_{N-1})u(k-N+P)
\end{aligned} \qquad (21)$$

Let us use the following definitions.

$$\begin{gathered}\boldsymbol{A} = \left[\underline{\boldsymbol{a}}_1 \ \underline{\boldsymbol{a}}_2 \ \cdots \ \underline{\boldsymbol{a}}_M\right] \\ z_1 = \frac{\partial u_k}{\partial u_{k-1}},\ z_2 = \frac{\partial u_{k+1}}{\partial u_{k-1}},\ \cdots,\ z_M = \frac{\partial u_{k+M-1}}{\partial u_{k-1}}\end{gathered} \qquad (22)$$

Based on (20) and (21), z_i are determined as follows.

$$\begin{aligned}
z_1 &= 1 - \frac{1}{\lambda^2}\underline{\boldsymbol{a}}_1^T \boldsymbol{l}, \\
z_2 &= z_1 - \frac{1}{\lambda^2}\underline{\boldsymbol{a}}_2^T \boldsymbol{l}, \\
&\vdots \\
z_M &= z_{M-1} - \frac{1}{\lambda^2}\underline{\boldsymbol{a}}_M^T \boldsymbol{l}
\end{aligned} \qquad (23)$$

In which $\boldsymbol{l}$ is a column vector and is defined as follows,

$$\boldsymbol{l} = \begin{bmatrix}
a_1 z_1 + a_2 - a_1 \\
a_1 z_2 + (a_2 - a_1)z_1 + a_3 - a_2 \\
\vdots \\
a_1 z_M + (a_2 - a_1)z_{M-1} + \cdots + (a_M - a_{M-1})z_1 + \\
a_{M+1} - a_M \\
a_2 z_M + (a_3 - a_2)z_{M-1} + \cdots + (a_{M+1} - a_M)z_1 + \\
a_{M+2} - a_{M+1} \\
\vdots \\
a_{P-M+1} z_M + (a_{P-M+2} - a_{P-M+1})z_{M-1} + \cdots + \\
(a_P - a_{P-1})z_1 + a_{P+1} - a_P
\end{bmatrix} \qquad (24)$$

Substitute for z_i in (24) from (23) and with some manipulation, $\boldsymbol{l}$ is reformulated as follows.

$$\boldsymbol{l} = \begin{bmatrix}
a_2 - \frac{1}{\lambda^2} a_1 \underline{\boldsymbol{a}}_1^T \boldsymbol{l} \\
a_3 - \frac{1}{\lambda^2}(a_2 \underline{\boldsymbol{a}}_1^T + a_1 \underline{\boldsymbol{a}}_2^T)\boldsymbol{l} \\
\vdots \\
a_{P+1} - \frac{1}{\lambda^2}(a_P \underline{\boldsymbol{a}}_1^T + \ldots + a_{P+M-1}\underline{\boldsymbol{a}}_M^T)\boldsymbol{l}
\end{bmatrix} \qquad (25)$$

or in simple form it can be written as,

$$\boldsymbol{l} = \widehat{\boldsymbol{b}} - \frac{1}{\lambda^2}\widehat{\boldsymbol{B}}\boldsymbol{l} \qquad (26)$$

Where

$$\widehat{\boldsymbol{b}} = \begin{bmatrix} a_2 \\ a_3 \\ \vdots \\ a_{P+1} \end{bmatrix},\quad \widehat{\boldsymbol{B}} = \begin{bmatrix}
a_1 \underline{\boldsymbol{a}}_1^T \\
a_2 \underline{\boldsymbol{a}}_1^T + a_1 \underline{\boldsymbol{a}}_2^T \\
a_M \underline{\boldsymbol{a}}_1^T + \ldots + a_1 \underline{\boldsymbol{a}}_M^T \\
a_{M+1} \underline{\boldsymbol{a}}_1^T + \ldots + a_2 \underline{\boldsymbol{a}}_M^T \\
\vdots \\
a_P \underline{\boldsymbol{a}}_1^T + \ldots + a_{P+M-1} \underline{\boldsymbol{a}}_M^T
\end{bmatrix} = \boldsymbol{A}\boldsymbol{A}^T . \qquad (27)$$

Equation (26) can be solved for $\boldsymbol{l}$.

$$\boldsymbol{l} = (\boldsymbol{I} + \frac{1}{\lambda^2}\boldsymbol{A}\boldsymbol{A}^T)^{-1}\widehat{\boldsymbol{b}} \qquad (28)$$

Using definition of $\boldsymbol{A}_0$, and the following relation,

$$(\boldsymbol{I} + \frac{1}{\lambda^2}\boldsymbol{A}\boldsymbol{A}^T)^{-1} = \boldsymbol{I} - \boldsymbol{A}_0,$$

relation for $\boldsymbol{l}$ becomes,

$$\boldsymbol{l} = (\boldsymbol{I} - \boldsymbol{A}_0)\widehat{\boldsymbol{b}} = \boldsymbol{b} - \boldsymbol{A}_0\widehat{\boldsymbol{b}} . \qquad (29)$$

From the first row of (24), z_1 is determined as follows.

$$z_1 = \frac{1}{a_1}(l_1 + a_1 - a_2) = \frac{1}{a_1}(\boldsymbol{e}_1^T \boldsymbol{l} + a_1 - a_2)$$

$$= \frac{1}{a_1}(\boldsymbol{e}_1^T \hat{\boldsymbol{b}} - \boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}} + a_1 - a_2) = \frac{1}{a_1}(a_1 - \boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}}) \qquad (30)$$

According to the Lyapunov linearization method (or using the contraction mapping theorem), the closed loop system is stable if $z_1 < 1$. This is equivalent to have $\boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}} / a_1 > \boldsymbol{0}$.

To derive the above criterion, the sampling time is not considered long enough and also control horizon is higher than 1 ($M > 1$). The same line of computations can be followed to determine the similar condition in which a MIMO system is stable.

4. SPECIAL CASES

In this section in order to investigate mentioned stability criterion in detail, we consider some special cases.

1) $P = M = 1$

$$\begin{aligned}
&\boldsymbol{A}_0 = a_1 (a_1^2 + \lambda^2)^{-1} a_1, \\
&\hat{\boldsymbol{b}} = a_2, \\
&\frac{1}{a_1} \boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}} = \frac{1}{a_1} \frac{a_1^2}{a_1^2 + \lambda^2} a_2 \\
&\qquad = \frac{a_1 a_2}{a_1^2 + \lambda^2} > 0
\end{aligned} \qquad (31)$$

or $a_1 a_2 > 0$. This is compatible with the Peterson's result [1] in which it is assumed that the steady state gain of the system does not have sign changes.

2) $M = 1, P > 1$

$$\begin{aligned}
&\boldsymbol{A} = [a_1 \; a_2 \; ... \; a_P]^T \\
&\frac{1}{a_1} \boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}} = \frac{\boldsymbol{A}^T \hat{\boldsymbol{b}}}{\boldsymbol{A}\boldsymbol{A}^T + \lambda^2} > 0 \\
&\boldsymbol{A}^T \hat{\boldsymbol{b}} > 0
\end{aligned} \qquad (32)$$

or equivalently,

$$a_1 a_2 + a_2 a_3 + ... + a_P a_{P+1} = \sum_{i=1}^{P} a_i a_{i+1} > 0 \qquad (33)$$

3) $M = 2, P > 2$

$$\boldsymbol{A}^T \boldsymbol{A} + \lambda^2 \boldsymbol{I} = \begin{bmatrix} \sum_{i=1}^{P} a_i^2 + \lambda^2 & \sum_{i=1}^{P-1} a_i a_{i+1} \\ \sum_{i=1}^{P-1} a_i a_{i+1} & \sum_{i=1}^{P-1} a_i^2 + \lambda^2 \end{bmatrix} \qquad (34)$$

and

$$\frac{1}{a_1} \boldsymbol{e}_1^T \boldsymbol{A}_0 \hat{\boldsymbol{b}} = \frac{\begin{bmatrix} \sum_{i=1}^{P-1} a_i^2 + \lambda^2 & -\sum_{i=1}^{P-1} a_i a_{i+1} \end{bmatrix}}{\det(\boldsymbol{A}^T \boldsymbol{A} + \lambda^2 \boldsymbol{I})} \begin{bmatrix} \sum_{i=1}^{P} a_i a_{i+1} \\ \sum_{i=1}^{P-1} a_i a_{i+2} \end{bmatrix} \qquad (35)$$

or equivalently,

$$= \frac{(\sum_{i=1}^{P-1} a_i^2 + \lambda^2)(\sum_{i=1}^{P} a_i a_{i+1}) - (\sum_{i=1}^{P-1} a_i a_{i+1})(\sum_{i=1}^{P-1} a_i a_{i+2})}{(\sum_{i=1}^{P} a_i^2 + \lambda^2)(\sum_{i=1}^{P-1} a_i^2 + \lambda^2) - (\sum_{i=1}^{P-1} a_i a_{i+1})^2} > 0 \qquad (36)$$

5. CONCLUSIONS

A new closed loop stability criterion is proposed by which the Extended DMC controller can be employed for finite sampling time as well as higher control horizon. This improves existing result, which was based on the infinite sampling time and $M = 1$ assumption.

As a future work, the stability analysis can be extended for MIMO systems. Also the linear approximation that was used in the calculation of the Jacobean matrix (equations (21) and (22)) can be eliminated. Works in these two areas are in progress and results will be submitted latter.

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