

## Robust $H_\infty$ State Feedback Congestion Control of ATM for linear discrete-time systems with Uncertain Time-Variant Delay

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**Abstract:** This paper focuses on congestion control for ATM network with uncertain time-variant delays. The time-variant delays can be distinguished into two distinct components. The first one is represented by time-variant queueing delays in the intermediate switches that are occurred in the return paths of RM cells. The next one is a forward path delay. It is solved by the VBR model which quantifies the data propagation from the sources to the switch. Robust  $H_\infty$  control is studied for solving congestion problem with norm-bounded time-varying uncertain parameters. The suitable robust  $H_\infty$  controller is obtained from the solution of a convex optimization problem through LMI technique.

**Keywords:** ATM,  $H_\infty$  control, uncertain time-variant delays, discrete-time system, LMI

### 1. INTRODUCTION

Congestion control is a process by which networks use feedback to adjust the influx of data such that the customer's QoS (quality of service) requirements are met while simultaneously attempting to maximize the utilization of the network's resources. To solve the congestion control, explicit rate control is used. This mechanism allows the switches to explicitly designate the cell transmission rate by modifying the explicit rate (ER) value of the RM cell. The delays in ATM networks have four components: packet delays, transmission delays, processing delays, and queueing delays. The first three categories are fairly constant, but the fourth one is the major source of time variance.

For some representative prior works on this general topic, robust congestion control in high speed network is studied [9]. It illustrates models that include simple delays (i.e. constant delay). These models are easy to understand and organize, but they are not sufficient to apply to practical systems. There are some papers that study the effect of uncertain time-variant delay in ATM network, but they use queueing model to apply delay and show some simple congestion controller. We can distinguish two distinct components of the time-variant delay. The one is the return path of RM cells. They experience time-variant queueing delays in the intermediate switches. This mechanism is illustrated by Hold Freshest Sample Model ("holds" the same rate until it receives "fresh" information.) [2]. The another is forward path delay. It is solved by VBR model, which quantifies the propagation of data volume from the sources to the switch. In case of multi queueing system, we may consider max-min fairness. The introduced system is capable of modeling time-variant communication delays between a single congested node and several sources, rate and buffer nonlinearities, RM cells loss caused by mismatches between time-variant RA cells period and the cycle of controller. These systems have time-variant delay that we can't know accurate numerical value. Therefore this paper will apply time-variant delay to robust  $H_\infty$  state feedback control of linear discrete-time uncertain dynamical systems in ATM networks.

This paper studies, via a linear matrix inequality approach, the problem of robust  $H_\infty$  state feedback control for discrete system with time-variant delay of parametric uncertainty in ATM network. As a result, the system under consideration is subjected to time-varying norm-bounded parameter uncertainties. This paper includes several parts as follows. First, introduction of congestion control of ATM network is shown in brevity. Second, the state equation of time-variant

delay model is studied. Third, we address the problem of robust  $H_\infty$  state feedback control in which both robust stability and a prescribed  $H_\infty$  performance are achieved irrespective of the uncertainty. Finally, we demonstrate the result through several examples.

### 2. ATM NETWORK WITH TIME-VARIANT DELAY MODEL

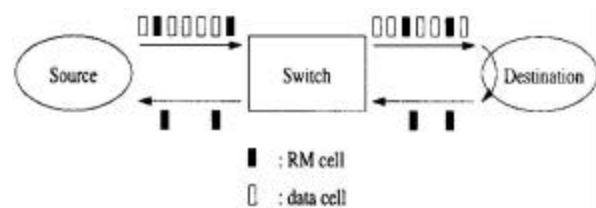


Fig. 1 ATM Service Model

Fig.1 shows a simple network with a single congested node and end to end RM cell routing. Bandwidth for the ABR traffic on the congested link is  $b_0$ . The congested switch uses the RM cells on the return path to inform the sources about the rate at which they should transmit. The delay these RM cells undergo from the congested node to the source will be time-variant in nature.

#### 2.1 Single source model

Fig. 2 illustrates two paths of one single communication link. One is the return path RM cells travel from the switch to the source. Another is the forward path the user data travels from the source through the congested switch. Thus we need two different models corresponding to the two different quantities. Both modes will be formulated in discrete time with sampling period  $T$ , since this simplifies the analysis of the arising system.  $b(n)$  denotes the rate computed at the congested node at time instant  $n$ , and  $r(n)$  is the rate at which the source transmits at time instant  $n$ .  $z(n)$  is the number of cells that arrive at the congested switch between time instant  $n-1$  and time instant  $n$ .

The generation of RM cells in general does not follow a fixed period, thus creating a rate mismatch between the controller rate and the RM cells rate. This rate mismatch may introduce a time-variant delay since an RM cell will not always be available at the time instant the controller computes

its output.

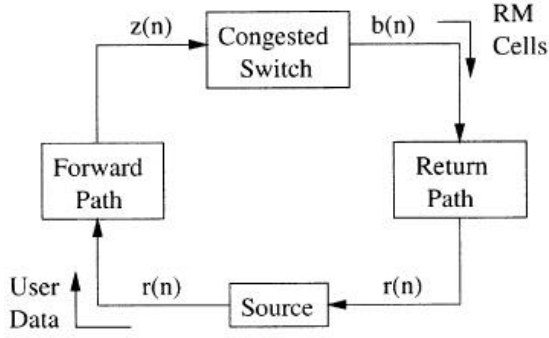


Fig. 2 Single source network model

The return paths of RM cells experience time-variant queuing delays in the intermediate switches. We introduce HFS (Hold Freshest Sample)[2] interface for explaining the return paths delays. This interface holds the most recently received sample both at the plant and controller input as long as necessary. If an old sample arrives after a newer one was received it is simply discarded. At any time instant  $n$ , we can have one of the following three situations. First case is no signal coming at the output of the time-variant delay block. In this case we simply hold the previous sample, thus we have:

$$r(n) = r(n-1) \quad (1)$$

Second case is all the signals that arrived at time  $n$  are older than the freshest sample at time  $n-1$ . Also in this situation we discard the newly arrived signals and we hold the previous sample:

$$r(n) = r(n-1) \quad (2)$$

Final case is at least one more recent sample arrived at time instant  $n$  and we choose the most recent one:

$$r(n) = b(n - \tau(n)) \quad (3)$$

where  $t(n)$  is the delay encountered by an RM cell on the return path. The delay  $t(n)$  is satisfying the inequality:

$$0 \leq \underline{\tau} \leq \tau(n) \leq \bar{\tau} < \infty \quad (4)$$

where  $\underline{\tau}$ , and  $\bar{\tau}$  are minimum and maximum delays. Eq. (3) is written as:

$$r(n) = \alpha_{\tau_{\min}}(n) b(n - \tau_{\min}) + \dots + \alpha_{\tau_{\max}}(n) b(n - \tau_{\max}) \quad (5)$$

$$\alpha_j = \begin{cases} 1, & \text{if } j = \tau(n) \\ 0, & \text{otherwise} \end{cases}, \quad \sum_{j=\tau_{\min}}^{\tau_{\max}} \alpha_j = 1 \quad (6)$$

For the queuing delays, the maximum and minimum delays are simple to compute. The minimum delay is zero corresponding to an empty tandem queue, while the maximum delay occurs when all the queues from the switch to the source are at maximum occupancy.

The forward path model quantifies the propagation of data volume from the sources to the switch at any given time  $n$ . We assume that there are no packet/cell losses on the

communication channel, and we do not require the FIFO property. At any time instant  $n$ , there is one of the following two situations. One is no signal at the output of the time-variant delay. In this case:

$$z(n) = 0 \quad (7)$$

Another is at least one signal at the output of the time-variant delay. In this case all the values that arrived during this time period are added, since the communication link cannot lose data:

$$z(n) = \sum_{i=\tau_{\min}}^{\tau_{\max}} \beta_i(n-i) b(n-i) \quad (8)$$

$$\beta_i(n) = \begin{cases} 1, & \text{if } i = \tau(n) \\ 0, & \text{otherwise} \end{cases}, \quad \sum_{i=\tau_{\min}}^{\tau_{\max}} \beta_i = 1 \quad (9)$$

The delays  $t_{1,i}(n)$ ,  $t_{2,i}(n)$  correspond to the HFS/VBR models where the index  $i$  denotes the source with  $i = 1, \dots, M$ . We assume that the delays are bounded:

$$0 \leq \tau_{1,i}(n) \leq \bar{\tau}_{1,i}, \quad 0 \leq \tau_{2,i}(n) \leq \bar{\tau}_{2,i}, \quad (10)$$

The state-space equation includes the delays generated by  $a$  and  $\beta$  in Eqs. (6), (9) and data transfer rates are shown as follows:

$$x_i(n+1) = A_i(n) x_i(n) + B_i(n) b(n) \quad (11)$$

$$z_i(n) = C_i(n) x_i(n) + D_i(n) b(n) \quad (12)$$

where

$$B_i(n) = \begin{bmatrix} w_i \\ 0 \\ \vdots \\ 0 \\ w_i R_i \beta[\bar{\tau}_{2,i}, i] \alpha[0, i] \\ w_i R_i \beta[\bar{\tau}_{2,i} - 1, i] \alpha[0, i] \\ \vdots \\ w_i R_i \beta[1, i] \alpha[0, i] \end{bmatrix} \quad (14)$$

$$D_i(n) = R_i w_i \beta[0, i] \alpha[0, i] \quad (16)$$

$x_i(n)$  corresponds to the state of the  $i$ th delay line corresponding to the  $i$ th source, including the RM cell path from the congested switch to the source and the data cell path from the source to the congested switch. We denote with  $a_j$ ,  $j = 1, \dots, \bar{\tau}_{1,i}$ ,  $i = 1, \dots, M$ ,  $j$ th time-variant coefficient  $a_j$  of the HFS model for the  $i$ th return path.  $\beta$  is similar to  $a$ . The weights  $w_i(n)$  represent the "fair" share of the bandwidth allocated to source  $i$  and can be computed using a max-min fairness algorithm[3]. And rate saturation  $R_i$  is defined for the  $i$ th source as follows:

$$\text{sat}_{R_i}(r_i) = R_i(n) (r - r_0) + r_0, \quad (17)$$

where  $R_i(n) \in [R_{i,\min}, 1]$

$$A_i(n) = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ R_i\beta_i[\bar{\tau}_{2,i}, i]\alpha[1, i] & R_i\beta_i[\bar{\tau}_{2,i}, i]\alpha[2, i] & \dots & \dots & R_i\beta_i[\bar{\tau}_{2,i}, i]\alpha[\bar{\tau}_{1,i}, i] & 0 & \dots & 0 & 0 \\ R_i\beta_i[\bar{\tau}_{2,i}-1, i]\alpha[1, i] & R_i\beta_i[\bar{\tau}_{2,i}-1, i]\alpha[2, i] & \dots & \dots & R_i\beta_i[\bar{\tau}_{2,i}-1, i]\alpha[\bar{\tau}_{1,i}, i] & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ R_i\beta_i[1, i]\alpha[1, i] & R_i\beta_i[1, i]\alpha[2, i] & \dots & \dots & R_i\beta_i[1, i]\alpha[\bar{\tau}_{1,i}, i] & 0 & \dots & 1 & 0 \end{bmatrix} \quad (13)$$

$$C_i(n) = [R_i\beta[0, i]\alpha[1, i]; R_i\beta[0, i]\alpha[2, i]; \dots; R_i\beta[0, i]\alpha[\bar{\tau}_{1,i}, i]; 0; \dots; 0; 1] \quad (15)$$

## 2.2 Multi source model

The congested switch model consists of a finite buffer, a queue, and a rate control component. The buffer receives incoming data from all sources. The data rates are converted to data by multiplication with the sampling period  $T$ . At each time instant  $n$  the buffer level  $y_s(n)$  is shown as follows:

$$\begin{aligned} y_s(n+1) &= y_s(n) + \left( \sum_{i=1}^M z_i T + \omega(n) \right) - b_0 T \\ &= y_s(n) + T \sum_{i=1}^M [C_i x_i + D_i b(n)] + \omega(n) - b_0 T \\ &= y_s(n) + T \sum_{i=0}^M [C_i x_i + D_i (\Delta b + u(n) + b_0)] + \omega(n) - b_0 T \\ &= y_s(n) + T \sum_{i=0}^M C_i x_i + \omega(n) + T \left( \sum_{i=1}^M D_i - 1 \right) b_0 + T \sum_{i=1}^M D_i (\Delta b + u) \end{aligned} \quad (18)$$

where  $\omega(n)$  is the exogenous disturbance,  $b_0$  denotes available bandwidth and  $u(n)$  is the control input.  $\Delta b$  represents the queue control component of the bandwidth.

$$\Delta b = G(y_0 - y_s) \quad (19)$$

where  $G$  denotes proper gain, and  $y_0$  is queue set point. We can express the entire closed-loop system in state space form as follows:

$$x(n+1) = A(n)x(n) + B(n)u(n) + H(n)\omega(n) \quad (20)$$

$$y(n) = C(n)x(n) + D(n)u(n) \quad (21)$$

$$y_s(n) = Ex(n) \quad (22)$$

where

$$A = \begin{bmatrix} 1 & 0 & T \sum_{i=1}^M D_i & T \left( \sum_{i=1}^M D_i - 1 \right) & TC_1 & TC_2 & \dots & TC_M \\ 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ -G & G & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & B_1 & B_1 & A_1 & 0 & \dots & 0 \\ 0 & 0 & B_2 & B_2 & 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & B_M & B_M & 0 & 0 & \dots & A_M \end{bmatrix} \quad (23)$$

$$B = \begin{bmatrix} T \sum_{i=1}^M D_i \\ 0 \\ 1 \\ 0 \\ B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix}, \quad H = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (24)$$

$$C = \begin{bmatrix} 0 & 0 & D_1 & D_1 & C_1 & 0 & \dots & 0 \\ 0 & 0 & D_2 & D_2 & 0 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & D_M & D_M & 0 & 0 & \dots & C_M \end{bmatrix} \quad (25)$$

$$D = [D_1^T \ D_2^T \ \dots \ D_M^T]^T \quad (26)$$

$$E = [1 \ 0 \ 0 \ \dots \ 0]^T \quad (27)$$

The state vector  $x(n)$  is composed of the queue length, queue set point, data transfer rates, and the state of the delay lines:

$$x(n) = [y_s \ y_0 \ \Delta b \ b_0 \ x_1(n)^T \ x_2(n)^T \ \dots \ x_M(n)^T]^T \quad (28)$$

## 3. ROBUST CONTROLLER DESIGN

In this section, a robust controller will be designed such that the closed-loop system (20), (21), (22) with this control law is robustly stable and has a robust  $H_s$  performance  $\gamma$  for all admissible parameter uncertainty.

### 3.1 $H_s$ control with norm-bounded time-varying uncertain parameter.

We consider the equations is including uncertain parameter.

$$x(n+1) = [A + \Delta A(n)]x(n) + [B + \Delta B(n)]u(n) + H(n)\omega(n) \quad (29)$$

$$y(n) = [C + \Delta C(n)]x(n) + [D + \Delta D(n)]u(n) \quad (30)$$

$$y_s(n) = Ex(n) \quad (31)$$

where  $A, B, H, C$  and  $D$  are known real constant matrices of appropriate dimensions which describe the “nominal” systems, and  $?A(n), ?B(n), ?C(n), ?D(n)$  represent the time-varying parameter uncertainty. The parameter uncertainty is assumed to be of the following structure:

$$\begin{bmatrix} \Delta A(n) & \Delta B(n) \\ \Delta C(n) & \Delta D(n) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(n) \begin{bmatrix} N_1 & N_2 \end{bmatrix} \quad (32)$$

where  $E_1, E_2$  and  $N_1, N_2$  are constant matrices of appropriate dimensions defining the structure of the uncertainty.  $F(n)$  is the model of the time varying parameter uncertainty.  $F(n)$  satisfies the following equation:

$$F(n)F(n)^T \leq I \quad (33)$$

In this paper, we are concerned with the problem of robust state feedback control for the uncertain time delay system for all admissible uncertainties. Our attention is to design a state feedback controller:

$$u(n) = Kx(n) \quad (34)$$

such that for a given scalar  $? > 0$ , all non-zero  $?_k \in l_2[0, \infty)$  and all parameter uncertainties satisfying Eqs. (32)-(33),

$$\|z\|_{[0, \infty)} < \gamma \|w\|_{[0, \infty)} \quad (35)$$

In this situation, the system (29)-(31) with the controller (34) is said to have a robust  $H_\infty$  performance (35).

Assume 1.

$$\begin{bmatrix} D \\ E_2 \end{bmatrix} \text{ is of full column rank.}$$

It means that the  $H_\infty$  control problem for the system is ‘non-singular’. Observe that, if the parameter uncertainty in the control matrix disappears, that is  $E_2 = 0$ , the assumption reduces to  $D^T D > 0$ , which is a standard assumption in the non-singular  $H_\infty$  control problem for the nominal system. If there exists a matrix  $P > 0$  in Eq. (36) such that for any admissible uncertainty  $?A(k)$ , the unforced system is said to be quadratically stable with an  $H_\infty$ -norm bounded  $?$ .

$$\tilde{A}^T P \tilde{A} + \tilde{A}^T P H (I - \gamma^{-2} H^T P H)^{-1} H^T P \tilde{A} - P + E^T E < 0 \quad (36)$$

with

$$I - \gamma^{-2} H^T P H > 0 \quad (37)$$

or equivalently, there exists a matrix  $Q > 0$  such that for any admissible uncertainty  $?A(k)$

$$\tilde{A} Q \tilde{A}^T + \tilde{A} Q E^T (I - \gamma^{-2} E Q E^T)^{-1} E Q \tilde{A}^T - Q + H H^T < 0 \quad (38)$$

with

$$I - \gamma^{-2} E Q E^T > 0 \quad (39)$$

where

$$\tilde{A} = A + \Delta A(k) \quad (40)$$

### 3.2 Transformation of Riccati equation.

These equations in Eg. (36) and Eg.(38) are difficult to apply to the uncertain parameter systems. Thus these problems can be reformulated as the scaled  $H_\infty$  analysis and syntheses problems of an auxiliary system which is independent of the uncertainty in the system. For a parameter  $e > 0$ , introduce an auxiliary system:

$$x_a(n+1) = A x_a(n) + B u(n) + H_a w_a(n) \quad (41)$$

$$y(n) = C x_a(n) + D u(n) \quad (42)$$

$$y_s(n) = E_{1a} x_a(n) + E_{2a} \quad (43)$$

where

$$H_a = [\sqrt{e} M_1 \quad \gamma^{-1} H] \quad (44)$$

$$E_{1a} = \begin{bmatrix} N_1 \\ \sqrt{e} \end{bmatrix}, \quad E_{2a} = \begin{bmatrix} N_2 \\ \sqrt{e} \\ 0 \quad 0 \quad \dots \quad 0 \end{bmatrix} \quad (45)$$

Uncertain system of (20)-(22) (setting  $u(k) = 0$ ) is quadratically stable with a scaled  $H_\infty$ -norm bound  $? > 0$ , if and only if the unforced auxiliary system of (41)-(43) (setting  $u(k) = 0$ ) is stable with unitary  $H_\infty$ -norm bound[5]. Furthermore, the uncertain system (20)-(22) is quadratically stabilizable with an  $H_\infty$ -norm bound  $? > 0$ , if and only if the auxiliary system (41)-(43) is stabilizable with unitary  $H_\infty$ -norm bound. An appropriate state feedback gain is

$$K = -(E_{2a}^T E_{2a} + B^T P B)^{-1} (B^T P A + E_{2a}^T E_{1a})^T \quad (46)$$

where  $P > 0$  is the stabilizing solution of the following Riccati equation:

$$A^T P A - P - \begin{bmatrix} B^T P A + E_{2a}^T E_{1a} \\ H^T P A \end{bmatrix} \Xi(P)^{-1} \begin{bmatrix} B^T P A + E_{2a}^T E_{1a} \\ H^T P A \end{bmatrix} + E_{1a}^T E_{1a} < 0 \quad (47)$$

with

$$I - H^T P H > 0 \quad (48)$$

where

$$\Xi(P) = \begin{bmatrix} E_{2a}^T E_{2a} + B^T P B & B^T P H \\ H^T P B & H^T P H - I \end{bmatrix} \quad (49)$$

An algorithm for finding the stabilization solution of (47) is represented by following equation [6]. Assume that  $(A, B)$  is stabilizable and that  $(A, B, E_{1a}, E_{2a})$  is left-invertible and has no invariant zeros on the unit circle. Then there exists a matrix  $L = 0$  such that

$$B^T L B + E_{2a}^T E_{2a} > 0 \quad (50)$$

$$A^T L A - L - (A^T L B + E_{1a}^T E_{2a}) (B^T L B + E_{2a}^T E_{2a})^{-1} (B^T L A + E_{2a}^T E_{1a}) + E_{1a}^T E_{1a} < 0 \quad (51)$$

and

$$A - B (B^T L B + E_{2a}^T E_{2a})^{-1} (B^T L A + E_{2a}^T E_{1a}) \quad (52)$$

is asymptotically stable. The input is

$$u(n) = (B^T L B + E_{2a}^T E_{2a})^{-1} (B^T L A + E_{2a}^T E_{1a}) x(n) \quad (53)$$

This Riccati equation (51) is in an inequality. Thus this equation can be solved easily by using LMI algorithm. It is as follows that definition of the LMI algorithm [4]. Given constant matrices M, L and Q of appropriate dimensions where M and Q are symmetric, then  $Q > 0$  and  $M + L^T Q^{-1} L < 0$  if and only if

$$\begin{bmatrix} M & L^T \\ L & -Q \end{bmatrix} < 0 \quad (54)$$

or equivalently

$$\begin{bmatrix} -Q & L \\ L^T & M \end{bmatrix} < 0 \quad (55)$$

Now, we apply (51) to the LMI.

$$L \leq 0, \quad B^T L B + E_{2a}^T E_{2a} > 0 \quad (56)$$

$$\begin{bmatrix} A^T L A - L + E_{1a}^T E_{1a} & A^T L B + E_{1a}^T E_{2a} \\ B^T L A + E_{2a}^T E_{1a} & B^T L B + E_{2a}^T E_{2a} \end{bmatrix} < 0 \quad (57)$$

Using the solution of (56-57), we can obtain a closed-loop  $H_\infty$  controller.

#### 4. EXAMPLES

We consider the examples to illustrate our results. This example uses the following parameters for our systems. Because the practical multi system is too complex to solve, the proposed system has only 3 sources.

- ? Bandwidth available for ABR traffic  $b_0 = 3000$  cells/s.
- ? Maximum rate  $2b_0 = 6000$  cells/s.
- ? Buffer set point  $y_0 = 5000$  cells.
- ? Controller cycle time  $T = 10$  ms.
- ? Maximum delay on the return path  $\bar{\tau}_1 = 3T$ .
- ? Maximum delay on the forward path  $\bar{\tau}_2 = 2T$ .
- ? Queue control gain  $G = 5$ .
- ? Fair share bandwidth  $w_1 = 0.3, w_2 = 0.3, w_3 = 0.4$ .
- ? Minimum saturation  $R_{i,min} = 0.5$ .
- ? Disturbance  $? \in (-30 \text{ cells}, 30 \text{ cells})$

The system starts at '0' queue size. Each source joins sequentially at 10ms intervals. The results of the simulation are shown in Fig. 3 and Fig. 4.

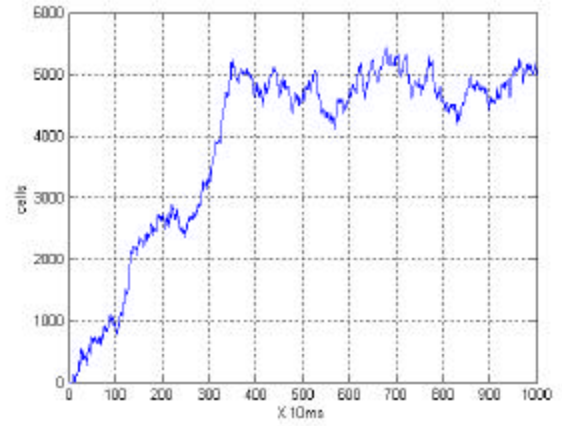


Fig. 3 Queue length of uncertain systems

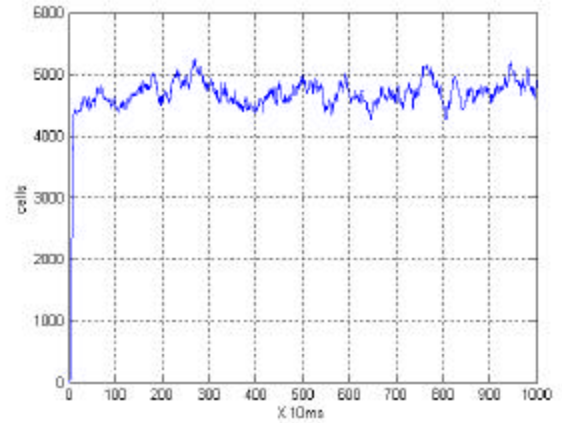


Fig. 4 Queue length of robust  $H_8$  control system

Fig. 3 shows the queue length in the case of the uncertain time-delay system with disturbance. Since the prior researches of [1] is not considered exogenous disturbance, control system is not guarantee the robustness as shown in Fig. 3. On the contrary, our proposed controller is robustly stabilizing against an exogenous disturbance as shown in Fig. 4, and it should be noted that our proposed controller is able to improve the performance. Both plots are saturated to queue set point in 5000 cells.

#### 5. CONCLUSION

In this paper, we have proposed the robust  $H_8$  state feedback congestion controller for linear discrete-time systems with uncertain time-variant delay in ATM network. The LMI algorithm is used for solving the Riccati inequalities. The examples show that the proposed controller has an effect on congestion control of ATM network. Since its effectiveness, the proposed controller can be applied to congestion control in other communication networks using packet and feedback.

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