Design of Reduced Order H2 Controller -Application to Anti-Sway-Control of a Traveling Crane-

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Abstract: For the anti-sway control of traveling cranes, there are several solutions, i.e., by fuzzy control, by optimal control theory, etc. Each of them is reported to be effective. And, H infinity control and H_2 control can be also used. However, the full order observer which estimates all states in the controlled object is used in these methods. Therefore, the orders of these controllers are apt to be higher than that of the optimal controller, etc. Because the conventional H_2 controller which minimizes H_2 norm consists of two parts, that is: feedback gains which make the controlled object stable and the full order observer which estimate those states. If the minimal order observer is used instead of the full order one, the order of the controller can be reduced.

In this paper, we propose a new method based on the minimalization of H_2 norm using the minimal order observer. And, we confirm the effect of a new H_2 controller in the experiments of the anti-sway control of a traveling crane.

Keywords: reduced controller, H2 control, minimal order observer, anti-sway, crane

1. INTRODUCTION

There are several methods of controlling the sway of traveling cranes, such as fuzzy control[1] and optimal control[2], each of which is reported to be effective[3]. The development of a simple method for designing control systems is important for field engineers. In a control system, the controller is generally designed by using a mathematical model of the controlled object. However, it is impossible to make a model which accurately expresses the characteristics of the controlled object. Therefore, the controller obtained by this method is not always appropriate for actual systems, even if it is valid theoretically. That is: the models for actual systems have always uncertainties. H infinity control or H2 control is well known as the method which overcome such uncertainties. However, the conventional H infinity control or H2 control are made on the basis of the mathematical models and the full order observers. As a result, the obtained control system has a redundancy. It is considered that we use the minimal order observer for the control system design instead of the full order one.

In this paper, the control system is designed by using an H_2 controller reduced by the minimal order observer, which is a new approach that we propose. The results of simulations and experiments show that this system has good performance.

2. REDUCED ORDER H2 CONTROL

2.1 Minimal order observer

Let us consider a system such as

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ y = C_2 x + D_{21} w \\ z = C_1 x + D_{12} u \end{cases} \tag{1}$$

For the controlled object shown in the above equation, we apply a transfer matrix as equation (1) based on the design method of Gopinath

$$S = [C_2 \quad C^{\#}]^T \tag{2}$$

where $C^{\#}$ is chosen so that S is full rank.

Putting $\bar{x} := Sx$, we obtain

$$\begin{cases} \dot{\bar{x}} = \bar{A}x + \bar{B}_1 w + \bar{B}_2 u \\ y = \bar{C}_2 x + D_{21} w \\ z = \bar{C}_1 x + D_{12} u \end{cases} \tag{3}$$

where

$$\begin{aligned} &\bar{A} := SAS^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ &\bar{B}_1 := SB_1 = \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix}, \bar{B}_2 := SB_2 = \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix} \\ &\bar{C}_2 := C_2 S^{-1} = \begin{bmatrix} I_p & 0 \end{bmatrix}, \bar{C}_1 := C_1 S^{-1}. \end{aligned} \tag{4}$$

In equation (1), we consider estimating the unknown state variables except y, which is observable, by using

$$\omega = U\bar{x} \quad U := \begin{bmatrix} -L & I_{n-P} \end{bmatrix}. \tag{5}$$

Expressed the minimal order observer which estimates ω as

$$\dot{\omega} = \hat{A}\omega + \hat{B}y + \hat{J}u , \tag{6}$$

we put

$$\begin{aligned} &\hat{A}U + \hat{B}\bar{C}_2 = U\bar{A} \\ &\hat{J} = U\bar{B}_2 . \end{aligned} \tag{7}$$

Assuming disturbance w as step one, we can obtain $\omega = U\bar{x}$ as $t \to \infty$.

Next, we put the estimate of $\bar{x}$ as

$$\hat{\bar{x}} = \hat{D}y + \hat{C}\omega . \tag{8}$$

Then, we also obtain $\hat{\bar{x}} = \bar{x}$ by putting

$$\begin{aligned} &\hat{D}\bar{C}_2 + \hat{C}U = I_n \\ &\hat{D}D_{21} = 0 . \end{aligned} \tag{9}$$

From equations (6) and (8), the minimal order observer is expressed as

$$\begin{aligned} &\dot{\omega} = \hat{A}\omega + \hat{B}y + \hat{J}u \\ &\hat{\bar{x}} = \hat{D}y + \hat{C}\omega , \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\hat{A} = -LA_{12} + A_{22} = U\bar{A}\hat{C} \\ &\hat{B} = -LA_{11} + A_{21} + \hat{A}L = U\bar{A}\hat{D} \\ &\hat{C} = \begin{bmatrix} 0 \\ I_{n-P} \end{bmatrix}, \hat{D} = \begin{bmatrix} I_P \\ L \end{bmatrix} \\ &\hat{J} = U\bar{B}_2 \ , \hat{D}D_{21} = 0 . \end{aligned} \tag{11}$$

2.2 Reduced order H2 controller

When the transfer function $\Phi(s)$ from $w(s)$ to $z(s)$ is defined, our purpose is to obtain an H_2 controller which satisfies the following conditions:

1) $\Phi(s)$ is internally stable, and

2) $\|\Phi(s)\|_2 \to \min$,

on the assumptions that

a1) $\begin{pmatrix} A & B_2 \end{pmatrix}$ can be stabilized and $\begin{pmatrix} A_{12} & A_{22} \end{pmatrix}$ is detectable,

a2) D_{12} is column full rank and B_{11} is row full rank, and

a3) $\begin{bmatrix} \overline{A}-j\omega I & \overline{B}_2 \\ \overline{C}_1 & D_{12} \end{bmatrix} \forall\omega$ and $\begin{bmatrix} A_{22}-j\omega I & B_{12} \\ A_{12} & B_{11} \end{bmatrix} \forall\omega$

are column full rank and row full rank respectively.

Here, assume the control law $u=-F\hat{\overline{x}}$ for equation (14). Then, we obtain the following equation substituting equation (18) for the above control law:

$$u=-F\hat{D}\overline{C}_2\overline{x}-F\hat{C}\omega \ (\because \hat{D}D_{21}=0\ ). \tag{11}$$

The following equation is obtained by substituting equation (11) into equation (3):

$$\begin{cases} \begin{bmatrix} \dot{\overline{x}} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_F & -\overline{B}_2F\hat{C} \\ 0 & A_K \end{bmatrix}\begin{bmatrix} \overline{x} \\ e \end{bmatrix} + \begin{bmatrix} \overline{B}_1 \\ B_K \end{bmatrix} w \\ z=\begin{bmatrix} C_F & -D_{12}F\hat{C} \end{bmatrix}\begin{bmatrix} \overline{x} \\ e \end{bmatrix} \\ y=\begin{bmatrix} \overline{C}_2 & D_{21} \end{bmatrix}\begin{bmatrix} \overline{x} \\ w \end{bmatrix} \end{cases}$$

From the above equation, the transfer function $\Phi(s)$ is expressed by

$$z(s)=\Phi(s)\,w(s)$$
$$\Phi(s):=G_c\overline{B}_1-U_cF\hat{C}G_f\ , \tag{12}$$

where

$$G_c=\left[\begin{array}{c|c} A_F & I \\ \hline C_F & 0 \end{array}\right],\ G_f=\left[\begin{array}{c|c} A_K & B_K \\ \hline I & 0 \end{array}\right]$$
$$U_c=\left[\begin{array}{c|c} A_F & \overline{B}_2 \\ \hline C_F & D_{12} \end{array}\right], \tag{13}$$
$$A_F:=\overline{A}-\overline{B}_2F\ ,\ C_F:=\overline{C}_1-D_{12}F\ ,$$
$$A_K:=\hat{A}\ ,\ B_K:=-U\overline{B}_1+\hat{B}D_{21}\ .$$

When A_F , A_K are stable, the 2 norm of the transfer function $\Phi(s)$ above is calculated as

$$\|\Phi(s)\|_2^2=trace\left(\overline{B}_1^TX\overline{B}_1\right)+R_F^{-1}trace\left(F\hat{C}Y\hat{C}^TF^T\right), \tag{14}$$

where

$$XA_F+A_F^TX+C_F^TC_F=0$$
$$D_{12}^TC_F+\overline{B}_2^TX=0 \tag{15}$$
$$D_{12}^TD_{12}=\delta I\ ,$$

and

$$YA_K^T+A_KY+B_KB_K^T=0\ . \tag{16}$$

From equation (15), we obtain

$$X\overline{A}+\overline{A}^TX-\left(D_{12}{}^T\overline{C}_1+\overline{B}_2{}^TX\right)^TR_F{}^{-1}\left(D_{12}{}^T\overline{C}_1+\overline{B}_2{}^TX\right)+\overline{C}_1{}^T\overline{C}_1=0$$
$$F=R_F{}^{-1}\left(D_{12}{}^T\overline{C}_1+\overline{B}_2{}^TX\right) \tag{17}$$
$$R_F:=D_{12}{}^TD_{12}>0\ .$$

and from equation (16), we also obtain

$$YA_{22}{}^T+A_{22}Y-\left(YA_{12}{}^T+B_{12}B_{11}{}^T\right)R_K{}^{-1}\left(A_{12}Y+B_{11}B_{12}{}^T\right)+B_{12}B_{12}{}^T=0$$
$$L=\left(YA_{12}{}^T+B_{12}B_{11}{}^T\right)R_K{}^{-1}, \tag{18}$$
$$R_K:=B_{11}B_{11}{}^T>0\ .$$

From equation (11), the H_2 controller is given by

$$u(s)=-\left[\begin{array}{c|c} UA_F\hat{C} & UA_F\hat{D} \\ \hline F\hat{C} & F\hat{D} \end{array}\right]y(s)\ , \tag{19}$$

where

$$U:=\begin{bmatrix} -L & I_{n-p} \end{bmatrix},\ A_F:=\overline{A}-\overline{B}_2F$$
$$\hat{C}=\begin{bmatrix} 0 \\ I_{n-p} \end{bmatrix},\ \hat{D}=\begin{bmatrix} I_P \\ L \end{bmatrix}. \tag{20}$$

Here, feedback gain F and observer gain L are given by equation (17) and equation (18) respectively.

Further, equation (19) is transformed as the following equation:

$$u=-F_py(s)-F_{n-p}\left\{\left(sI-UA_F\hat{C}\right)^{-1}UA_F\hat{D}+L\right\}y(s)$$
$$F:=[F_p\quad F_{n-p}]\ .$$

From the result above, we can see that this control law consists of the measured outputs and the estimated outputs.

3. SIMULATION AND EXPERIMENT

3.1 Controlled object

We confirm the effectiveness of a reduced order H2 controller using a traveling crane in this chapter. Figure 1 shows a model of the controlled object, where x is the position of the trolley, l the lope length, m the mass of a load, M the mass of the trolley, θ the sway angle of a load and u the external force. For fig. 1, the motion of a traveling crane is expressed by

$$\begin{cases} (M+m)\ddot{x}=ml\dot{\theta}^2\sin\theta-ml\ddot{\theta}\cos\theta+u \\ l\ddot{\theta}=-\ddot{x}\cos\theta-g\sin\theta \end{cases}. \tag{21}$$

Upon approximating these equations around $\theta=0$ and assuming $u=K_v(v-\dot{x})$, equation (21) is transformed into the following equation:

$$\begin{cases} M\ddot{x}=-K_v\dot{x}+mg\theta+K_vv \\ Ml\ddot{\theta}=K_v\dot{x}-(M+m)g\theta-K_vv \end{cases} \tag{22}$$

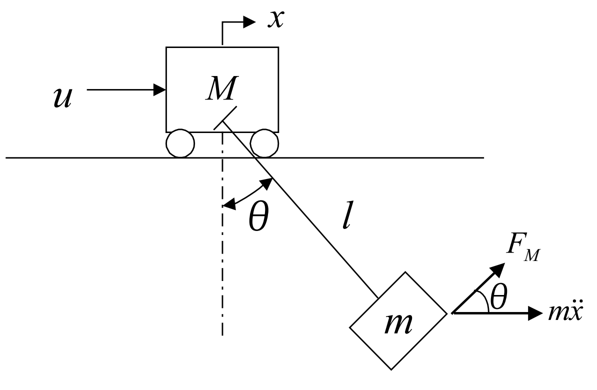


Fig. 1 Crane model

Considering controlled outputs and disturbances for above equations, the state equation is expressed as

$$\begin{cases} \dot{x} = A_P x + B_d d + B_P v \\ y_z = C_z x \\ y = C_P x + D_{rp} r \end{cases} \tag{23}$$

where,

$x_1 := x\,, x_2 := \dot{x}\,, \theta_1 := \theta\,, \theta_2 := \dot{\theta}\,,$

$$x := \begin{bmatrix} x_1 \\ \theta_1 \\ x_2 \\ \theta_2 \end{bmatrix}, A_P := \begin{bmatrix} 0 & \mathrm{I} \\ A_{P2} & A_{P3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mg}{M} & -\frac{K_v}{M} & 0 \\ 0 & -\frac{(M+m)g}{Ml} & -\frac{K_v}{Ml} & 0 \end{bmatrix},$$

$$B_P := \begin{bmatrix} 0 \\ 0 \\ \frac{K_v}{M} \\ -\frac{K_v}{Ml} \end{bmatrix}, B_d := \begin{bmatrix} B_{d1} \\ B_{d2} \end{bmatrix} = \begin{bmatrix} -b_{d1} & 0 & 0 & 0 \\ 0 & -b_{d2} & 0 & 0 \\ 0 & 0 & -b_{d3} & 0 \\ 0 & 0 & 0 & -b_{d4} \end{bmatrix},$$

$$C_P := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, C_z := I_{4\times4}, D_{rp} := \begin{bmatrix} d_{rp1} & 0 & 0 & 0 \\ 0 & d_{rp2} & 0 & 0 \end{bmatrix},$$

$m = 0.415[kg]$, $M = 18[kg]$, $l = 1[m]$, $g = 9.8[m/s^2]$.

From equation (23), a transfer function from input $w(s)$ to output $z(s)$ is expressed by

$$\begin{aligned} z(s) &= \begin{bmatrix} z_1(s) \\ z_2(s) \end{bmatrix} = \begin{bmatrix} W_1(s) y_z(s) \\ W_2(s) u(s) \end{bmatrix} \\ &= \begin{bmatrix} W_1(s) P_{wz}(s) & W_1(s) P_z(s) \\ P_w(s) & P(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix}, \end{aligned} \tag{24}$$

where

$$w(s) := \begin{bmatrix} d(s) \\ r(s) \end{bmatrix}, P(s) := \left[\begin{array}{c|c} A_P & B_P \\ \hline C_P & 0 \end{array}\right], P_z(s) := \left[\begin{array}{c|c} A_P & B_P \\ \hline C_z & 0 \end{array}\right],$$

$$P_w(s) := \left[\begin{array}{c|cc} A_P & B_d & 0 \\ \hline C_P & 0 & D_{rp} \end{array}\right], P_{wz}(s) := \left[\begin{array}{c|cc} A_P & B_d & 0 \\ \hline C_z & 0 & 0 \end{array}\right],$$

$$W_1(s) = \left[\begin{array}{c|c} A_{w1} & B_{w1} \\ \hline C_{w1} & D_{w1} \end{array}\right], W_2(s) = \left[\begin{array}{c|c} A_{w2} & B_{w2} \\ \hline C_{w2} & D_{w2} \end{array}\right]$$

Here, we apply a transfer matrix based on the design method of Gopinath for the equation (24), and we can obtain the following equation.

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_1 w + \bar{B}_2 u \\ z = \bar{C}_1 x + \qquad D_{12} u \\ y = \bar{C}_2 \bar{x} + D_{21} w \end{cases} \tag{25}$$

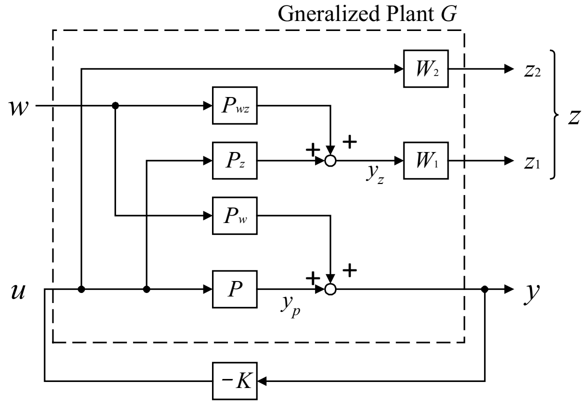
where,

$$\bar{A} := \begin{bmatrix} A_p & 0 & 0 \\ B_{w1} C_z & A_{w1} & 0 \\ 0 & 0 & A_{w2} \end{bmatrix}, \bar{B}_1 := \begin{bmatrix} [B_d \quad 0] \\ 0 \\ 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} B_p \\ 0 \\ B_{w2} \end{bmatrix}$$

$$\bar{C}_1 := \begin{bmatrix} D_{w1} C_z & C_{w1} & 0 \\ 0 & 0 & C_{w2} \end{bmatrix}, \bar{C}_2 := \begin{bmatrix} C_p & 0 & 0 \end{bmatrix}$$

$$D_{12} := \begin{bmatrix} 0 \\ D_{w2} \end{bmatrix}, D_{21} = \begin{bmatrix} 0 & D_{rp} \end{bmatrix}.$$

The block diagram in Fig.2 is H_2 control system.



P :Plant for Control,:Plant for Output-evaluation
P_w:Plant for Disturbance,
P_{wz}:Plant for Disturbance-evaluation,
K:Controller,u:Control Input, w:Disturbance,
y_p: Measured Output, z:Controlled Outputs,
W_1, W_2:Weighting Functions

Fig.2 H_2 control system

3.2 Selection of weights

For the weighting function, we chose the following weights accordingly:

- Selection of weight W_1

W_{11}, W_{22}, W_{33} and W_{44} are respectively weights for the position of a trolley, the angle of a load, the velocity of a trolley and the angular velocity of a load. Particularly for W_{11}, pseudo integrations are used so that the trolley position x can follow the reference value. And the gain of W_{44} has a large value so that it can reduce the quickly motion and sway of a load.

$$W_1 = diag\left(\frac{20}{s+0.9}\cdot\frac{50}{s+1}, \frac{30}{s+1}, \frac{100}{s+1}\right) \tag{26}$$

- Selection of weight W_2

For W_2, k_m is determined so as to restrict the control input, and l_f and h_f are determined so as not to be affected by noise.

$$W_2 = \frac{s+l_f}{s+h_f}\cdot k_m \;, l_f = 50 \;, h_f = 100 \;, k_m = 30 \tag{27}$$

- Selection of weights B_d and D_{rp}

B_{d11}, B_{d22}, B_{d33} and B_{d44} are respectively weights for a trolley velocity, torque disturbances and a load velocity and torque ones. And we chose D_{rp} so as to become zero, because D_{rp} corresponding D_{21} in the conventional H2 control is not necessary to satisfy row full rank.

$$B_d = diag(-1\,, -1\,, -30\,, -30) \;, D_{rp} = 0 \tag{28}$$

3.3 Experimental & simulation results

Figure 3 and Fig. 4 show the simulation and experimental results by the reduced order H_2 control system based on the minimal order observer that we propose. In these results, the desired position of 0.5 [m] was given first, and after about 25 seconds an impulse disturbance 4 ~ 5 [deg] was given. As a result, we can obtain similar results for simulations and experiments. And, you can see the position quickly followed the reference value, and even if a disturbance was added to the system, the error from the reference value was small, and the sway of the load was quickly reduced.

Next, fig. 5 shows experimental results by the H_2 control system with the full order observer (conventional system). From the comparison Fig.3 and Fig. 5, you can see that both positioning controls are almost same performances, but the anti-sway for disturbance is better performance in the reduced order H2 control system rather than in the conventional system. Finally, fig. 6 shows that the crane is in control after 15 [sec]. You can see that the sway of a load is not quickly reduced in the non-control case, however done in the control case.

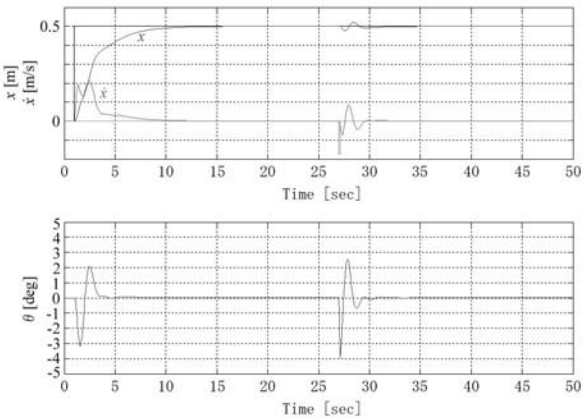


Fig. 3 Simulation results of minimal order observer

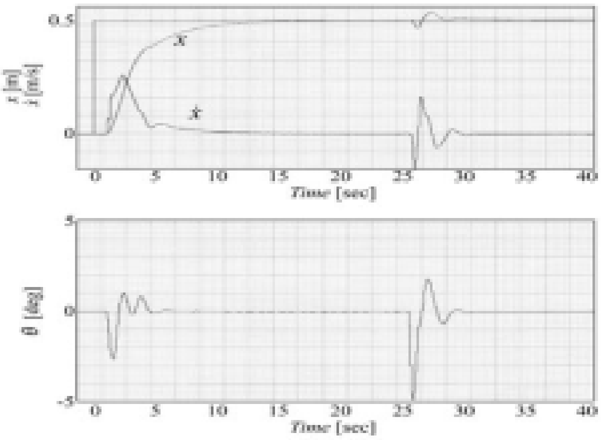


Fig. 4 Experimental results for full-order observer

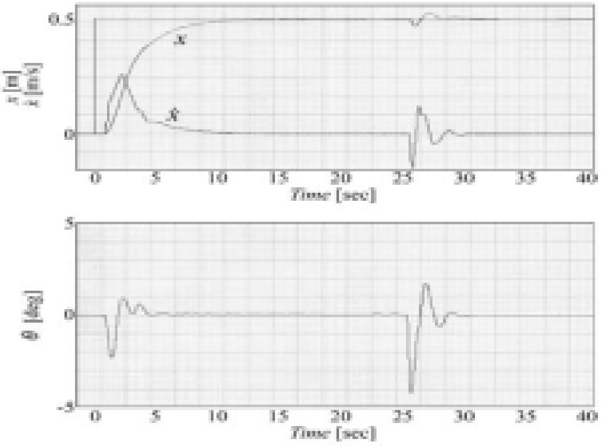


Fig. 5 Experimental results for minimal-order observer

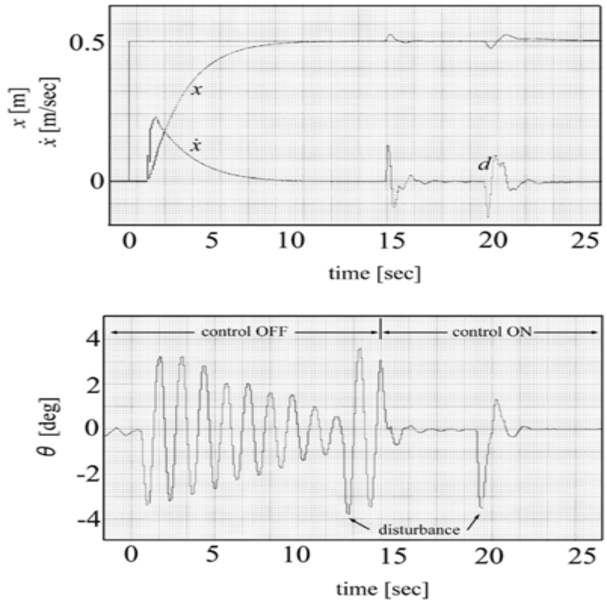


Fig. 6 The responses of non-control and control cases

4. CONCLUSIONS

We proposed a reduced order H_2 controller by using the minimal order observer for a system with disturbances. We confirmed that the responses by the H_2 controller based on the minimal order observer are the same as those by the conventional H_2 controller in the experiments. Future tasks are to apply other systems and design H infinity control system using the minimal order observer.

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